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FLEXIBILITY AND COST IN INFORMATION TECHNOLOGY OUTSOURCING: BALANCING OPPOSING GOALS

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Abstract

The growth in the importance of IT outsourcing arrangements combined with the rate of failure of such arrangements makes it imperative to address issues of contracting in the IS literature. In this work, we examine the contract directly and show how the costs stipulated in the contract affect the nature of the outsourcing relationship. The work develops a mathematical model of IT outsourcing contracts to explore the relationship between costs of the anticipated IT needs and the costs of additional resources in the face of changing IT needs. The model exposes the underlying tension between the baseline cost of an anticipated bundle of services and the flexibility to modify that bundle in response to changing environmental conditions. Additionally, we draw testable propositions about the relationship between contractually specified costs of IT services and the costs inherent in both client and vendor production of IT services. The model shows that client can never get the same cost structure as the vendor. Rather the client achieves contractually specified cost structure that will be a compromise between vendor and client costs. The model also proposes that contractual flexibility is directly related to the contractually specified cost of the anticipated bundle of IT services. Thus, contracts that stipulate a very low baseline cost for the anticipated bundle of IT services are less flexible so that changes beyond the baseline are more expensive.

Keywords: Contracts, outsourcing, economic models, flexibility, base cost

Introduction

Information technology (IT) outsourcing is one of the most significant management practices to arise in the past century. Industry observers have gone so far as to suggest that the ability to outsource is the primary driver of IT productivity (Kirkpatrick, 2002). In the United States alone, IT outsourcing accounted for more than $100 billion in revenue in 2000, and is expected to grow to $160 billion by 2005 (Vijayan, 2002). Progressively larger contracts are being inked everyday, so that “outsourcing arrangements once considered large are being dwarfed by recent deals (Gurbaxani, 1996).” Firms have now started considering IT outsourcing as a strategic activity. Firms that earlier outsourced only minor information system (IS) services, are now outsourcing entire IS departments (Mazzawi, 2002).

Though firms spend considerable time and effort in drafting IT outsourcing agreements—the average contract takes 15-18 months to prepare—contracts still fail with alarming regularity (Barthelemy, 2001). The majority of outsourcing arrangements have to be renegotiated and one in eight is prematurely terminated as expensive failure (Caldwell, 1997; Lacity and Willcocks, 2001). Why do these arrangements, which showed every sign of success initially, fail with such disturbing regularity? One of the most frequently cited reasons is a lack of flexibility (Lacity and Willcocks, 2001). Clients often believe that the benefits from the arrangements are eaten away by the additional costs they did not anticipate early in the contract (Barthelemy, 2001).

This work develops a mathematical model of IT outsourcing contracts to explore the relationship between costs of the anticipated IT needs and the costs of additional resources in the face of changing IT needs. Additionally we draw, testable propositions about the nature of IT outsourcing contracts and their relationship to vendor and client capabilities. The model shows that client’s focus on baseline cost savings necessarily results in decreased flexibility. The model also clarifies the fact that an outsourcing
relationship does not simply replace the client’s capabilities with the vendor’s superior capabilities. Rather, IT outsourcing replaces the client’s capabilities with contractually specified capabilities that will be a compromise between vendor and client capabilities. Hence, the model offers a reasonable explanation of the stylized facts of IT outsourcing contracts and the problems inherent in them.

The paper proceeds as follows: In the next section, we review the literature on contracting for IT outsourcing. Following that, we develop a model of IT outsourcing based on contracts. We then derive the propositions of this model and conclude with a discussion of the model and directions for future research.

**Literature Review**

Prior economic based research into outsourcing arrangements has focused on problems of observability and structuring arrangements to overcome such problems. Most of the research has been directed towards the vendor selection under uncertain vendor quality and inability to verify vendor performance or divergent interests of contracting parties (Chaudhary, et al., 1995; Snir and Hitt, 2001; Whang, 1992).

The prior literature described above is based on an incomplete contract paradigm, which essentially discusses business arrangements that allow firms to compensate for issues that can not be directly addressed in contracts. However, we take a slightly different approach, based on Tirole’s suggestions that, “…standard complete contracting tools may have been too hastily dismissed…” (Tirole, 1999 p. 744)” and Lacity and Willcocks observation that “…detailed contracts are a critical success factor to successful deals (Lacity and Willcocks, 2001 p. xiii).” We examine the contract directly and study how the costs stipulated in the contract determine the nature of the outsourcing relationship.

Prior literature identifies various drivers of IT outsourcing value. Most important economic drivers of IT outsourcing value are cost and flexibility. While cost reduction is clearly high on most clients’ priorities, practitioners and academicians have stressed on the importance of flexibility in a contract (Berg, 1999; Goolsby, 2002; Harris, et al., 1998; Hoffman, 1996; Lacity, et al., 1995; Lang, 1999; McFarlan and Nolan, 1995; McGee, 1998). Some go so far as to suggest that the most important characteristic of a successful relationship is flexibility (Goolsby, 2002), while others provide a framework to maximize flexibility (Lacity, et al., 1995). However, to the authors’ knowledge, no one has worked on the interplay of baseline cost and flexibility.

**Model**

While there are many factors salient to a good outsourcing arrangement, economic considerations are among the most important reasons for outsourcing (Loh and Venkatraman, 1992; McFarlan and Nolan, 1995; Smith, et al., 1998). There are generally two types of costs involved in an IT outsourcing contract—baseline cost and additional resource charges (ARCs). Baseline cost takes into account the IS requirements that the client expects to face. The baseline cost often includes a whole bundle of services. However, in a fluid environment, like that of IT, unforeseen eventualities arise that require additional resources beyond those originally expected, and hence the need for ARC. For example, a firm that inked a 10-year outsourcing agreement in 1993, would obviously have had very different and unexpected needs when it deployed an internet storefront in 1999. ARC determines the flexibility of the contract. Following prior literature, we assume that an increase in flexibility reduces the slope of the marginal cost curve and hence the ARC (Hiebert, 1989). In this work, we show that there is a fundamental tension between baseline cost and flexibility.

We develop a two period model to analyze the tension between baseline cost and flexibility. In order to keep the model simple, we characterize IT outsourcing services as being described only by a quantity of services, which is a random variable. We then examine baseline cost and flexibility relative to traditional average cost curves. Baseline cost is the average cost at the expected quantity of services. Flexibility is defined as the inverse of the slope of the marginal cost curve as shown in Table 1. This definition is not immediately intuitive and bears some discussion.

Conceptually, flexibility is the ability of a firm to react to a changing environment. This work limits the environment to one variable—quantity. This means that flexibility is the ability to react to changes in the desired quantity of IT outsourcing services. However, reacting does not simply mean that it is possible to change quantity; rather, it means that it is possible to change quantity cheaply. Thus, one firm is more flexible than another if changing its quantity of IT outsourcing services by a given amount results in a lesser increase in marginal cost than the other firm. These ideas are illustrated in Figure 1.
The figure illustrates three hypothetical marginal cost curves. Baseline cost is the cost given by the labeled line at the baseline quantity. Flexibility is the inverse of the slope of the marginal cost curve at any given point. The curve labeled MC(1) is the most flexible because it has the smallest slope. This means that changes in the quantity of IT services can be accommodated with only small increases in the marginal cost of producing those services. It is important to note that baseline cost and flexibility are independent concepts. For example, while MC(1) is more flexible than MC(2), MC(2) has a lower baseline cost. Conversely, MC(1) is both more flexible and has lower baseline cost than MC(3).

Assumptions

Consider a two period game with two risk neutral firms—a client and a vendor. The firms wish to formalize an agreement in which the vendor supplies IT services to the client. For simplicity, we assume that there is only one type of service being supplied, which can only vary in quantity. The quantity is a random variable, \( q \), which defines how much service the vendor supplies to the client. Both parties know the distribution of quantity, \( f(q) \), in period one, but do not realize the actual level of quantity until period two. Clearly, in a real outsourcing arrangement, quantity would be a vector of various services, but the model is not significantly enhanced by considering multiple services.

In period one, the client and the vendor negotiate a contract, which specifies a function mapping all possible levels of quantity into prices. We implicitly assume that contracts are binding, negotiation is free, and that both firms have full information. Thus, in period one the parties are able to negotiate a contract free of any transaction costs that might arise in period two, when the firms are in a bilateral dependency relationship (Williamson, 1985). In other words, we recognize the transaction cost problem brought on by bilateral dependency in the second period, but by allowing free negotiation of binding contracts in the first period, firms can create the arrangement in period two that would be obtained if there were no transactions costs. Thus, we are allowing for complete, Arrow-Debreu type contracts (Arrow and Debreu, 1954; Masten, 2000).

In period two, the actual required quantity of outsourcing services is realized and the price is determined based on the contract. In the first period, the quantity is not known, but the distribution of the quantity is fully known to parties. Note also that there is no room for renegotiation of the contract. The time frame of the model is illustrated in Figure 2.

IT services can be produced at cost \( C_c(q) \) by the client and \( C_v(q) \) by the vendor. Assume that the vendor has expertise in IT services and can accomplish the same tasks at a lower cost than the client. Owing to its experience, the vendor has a better cost-control for the changes in quantity over time than the client. The vendor can provide a better level of service at a lower cost than the internal IS department because of economies of scale, tighter control over fringe benefits, better access to lower cost labor pools, and more focused expertise in managing IS (Smith, et al., 1998). Thus, we make the assumption that the vendor is both more flexible and less costly than the client. This means that the vendor’s marginal cost curve has a lower value and a lower slope at every possible level of quantity. These assumptions are detailed in Table 1.
Outsourcing IS Development, IT Operations, and IT-Enabled Services

Outsourcing will generate a surplus, $S = C_c(q) - C_v(q)$. The contract will be a mechanism by which to divide this surplus between the client and vendor. The contract will specify a price, $P(q)$, for each level of $q$. Expected values of the contract to the client and the vendor have been defined as the share of surplus received by accepting the contract. The contract will provide an expected value to the client equal to

$$E(\pi_c) = \int q (C_c(q) - P(q)) f(q) dq$$  \hspace{1cm} (1)

The expected value of the contract to the vendor is

$$E(\pi_v) = \int q (P(q) - C_v(q)) f(q) dq$$  \hspace{1cm} (2)

**Specification of Price and Quantity**

In order to solve the general form of the model and develop propositions requires additional assumptions. For tractability, we will assume that costs and contractually specified prices are linear. This allows us to have a reasonable approximation to reality and enables us to solve the model. It is a reasonable approximation to reality because outsourcing contracts frequently stipulate some baseline quantity of services for a fixed price, plus a variable cost to cover additional service requirements. Thus, a contract specifies a baseline price, $b_p$, for the target quantity of IT services, plus an ARC, $m_p$, which determines variable costs.

The other assumption that must be made is the distribution of $q$. There is no real a priori choice for the distribution of quantity that we can rely upon. Two common choices would be a normal or a uniform distribution. The normal distribution is computationally complex and relies upon two parameters. The uniform distribution is much simpler analytically. Because the focus of this work is on the negotiation of price rather than the underlying distributions we assume an uniform distribution for $q$. These assumptions are shown in Table 1.
Table 1. Assumptions

<table>
<thead>
<tr>
<th>#</th>
<th>Assumption</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>( C_v(q) &lt; C_c(q) )</td>
<td>Vendor has absolute cost advantage over the client at all levels of quantity.</td>
</tr>
<tr>
<td>#2</td>
<td>( \frac{dC_v(q)}{dq} &lt; \frac{dC_c(q)}{dq} ) ( \Rightarrow ) ( \frac{1}{\frac{dC_v(q)}{dq}} &gt; \frac{1}{\frac{dC_c(q)}{dq}} )</td>
<td>The slope of the vendor’s marginal cost curve is less than the slope of the client’s. This implies that the vendor has greater flexibility than the client.</td>
</tr>
<tr>
<td>#3</td>
<td>( C_v(q) = b_v + m_vq ) vendor cost&lt;br&gt;( C_c(q) = b_c + m_cq ) client cost&lt;br&gt;( P(q) = b_p + m_pq ) contract price</td>
<td>The cost to produce the services and the contractually specifies price are linear functions of quantity. The baseline quantity is normalized to zero so the baseline cost is the intercept.</td>
</tr>
<tr>
<td>#4</td>
<td>( f(q) = \begin{cases} 1 &amp; 0 \leq q \leq 1 \ 0 &amp; \text{otherwise} \end{cases} )</td>
<td>The possible range of quantity is uniformly distributed.</td>
</tr>
<tr>
<td>#5</td>
<td>( \text{flex}_i = \frac{1}{\frac{dC_i(q)}{dq}} )</td>
<td>Flexibility is defined as the inverse of the slope of the marginal cost curve. Thus, a steeper curve implies less flexibility.</td>
</tr>
</tbody>
</table>

Generating the Constraint - Base Case (50-50 Split)

As we noted above, free negotiation in the first period circumvents transaction costs in the second period. This means that one possible contract negotiated in the first period will be equivalent to what would be obtained by free negotiation in the second period. By equivalent, we mean that both parties must be at least as well off with the final contract as they would be with a contract that stipulates the second period free negotiation outcome. Thus, it is necessary to define what the second period, free negotiation outcome would be.

The Nash bargaining solution for dividing surplus between two parties with equal negotiating abilities is to split the surplus in half (Bakos and Nault, 1997; Grossman and Hart, 1986). Such practices of 50-50 surplus split are common (Bhattacharyya and Lafontaine, 1995; Chao, 1983). Thus, one viable contract is the contract which awards ½ of the surplus to each firm at each level of \( q \). To establish a base case we first define a contract that divides the surplus in half for all possible realizations of \( q \).

Given this sharing rule, the price function is

\[
\bar{P}(q) = C_v(q) + \frac{1}{2}(C_c(q) - C_v(q))
\]

The hat “~” indicates the solution to the 50-50 split base case. It can be shown that this is

\[
\bar{P}(q) = \left( \frac{b_v + b_c}{2} \right) + \left( \frac{m_c + m_v}{2} \right) q = \bar{b}_p + \bar{m}_p q
\]

Applying the assumed distribution of \( q \), this entails an expected value of the contract of

\[
E(\bar{\pi}_i) = E(\bar{\pi}_c) = \int_0^1 \left[ \left( \frac{b_v - b_c}{2} \right) + \left( \frac{m_c - m_v}{2} \right) q \right] q dq
\]

for both the client and the vendor.
General Case

The 50-50 split case represents only one of an infinite number of possible contracts. Given that the 50-50 split case is solved, it is now possible to solve for the other contracts. Because the 50-50 split case contract is possible, neither party would be willing to agree to a contract that offered less expected value than the possible contract. Thus, the other contracts must be subject to the constraints

\[ E(\pi_c) \geq E(\tilde{\pi}_c) \] (6)

for the client, and

\[ E(\pi_v) \geq E(\tilde{\pi}_v) \] (7)

for the vendor.

Applying these constraints to (2) yields the solution

\[ m_p = 3 \left[ -\frac{b_p}{2} + \frac{1}{4} (b_v + b_c) + \frac{1}{6} (m_c + m_v) \right] = -\frac{3b_p}{2} + k \] (8)

Here, \( k \) is a constant whose value depends upon \( m_c, b_c, m_v, b_v \). For a proof see appendix.

Propositions of Model

Before developing propositions, it is necessary to note our measure of flexibility. Applying the definition in assumption #5, the flexibility can be defined as

\[ \text{flex}_c = \frac{1}{dC_c(q)} = \frac{1}{m_c} \] (9)

for the client,

\[ \text{flex}_v = \frac{1}{dC_v(q)} = \frac{1}{m_v} \] (10)

for the vendor, and

\[ \text{flex}_p = \frac{1}{dP(q)} = \frac{1}{m_p} \] (11)

for the contract.

The first implication of the solution presented in (8) is the relationship between contractual flexibility and base cost. This can be seen from the derivative.
This implies that baseline cost and flexibility are positively related. Increasing flexibility requires an increase in baseline cost, while reduced baseline cost leads to decreased flexibility. This is expressed in proposition one.

**Proposition 1:** If the client demands a lower baseline price, then it will lose flexibility as ARC increase.

This is a profound implication of the model, because it means that two of the fundamental goals of outsourcing—flexibility and cost—are diametrically opposed.

The model also provides for additional implications. The traditional logic for outsourcing suggests that the client outsources to take advantage of the greater capabilities of the vendor. However, the model shows that the client cannot take full advantage of the vendor’s capabilities even in the equal division of surplus. Flexibility in the case of base case contract is given by

\[
\text{flex}_p = \frac{2}{m_v + m_c} \quad \text{and} \quad m_c > m_v,
\]

this implies \(\text{flex}_p < \frac{1}{m_v}\), so that \(\text{flex}_p < \text{flex}_v\).

**Proposition 2:** Even if the client accepts the base case (i.e. contract with intercept \(\tilde{b}_p\) and slope \(\tilde{m}_p\)), it does not get the same flexibility as that of the vendor.

A similar result holds for baseline costs.

\[
\text{flex}_p = \frac{b_c + b_v}{2} \quad \text{and} \quad b_c > b_v \quad \text{this implies} \quad \text{flex}_p > \text{flex}_v.
\]

**Proposition 3:** Even if the client accepts the baseline case (i.e. contract with intercept \(\tilde{b}_p\) and slope \(\tilde{m}_p\)), it does not get the same baseline price as that of the vendor.

These propositions highlight the important point that while outsourcing may provide the client with additional capabilities, it does not give the client the capabilities of the vendor. In other words, the client will never be able to perform IT functions at the same level as the vendor, even when IT service is outsourced to the vendor. Rather the client’s capabilities are fully determined by the contract. This raises the question of how the underlying capabilities of client and vendor determine the contract.

More flexible vendors are able to control costs more efficiently for unpredicted changes in requirements. So providing additional units of quantity does not cost the vendor too much. Therefore, the outsourcing arrangements involving more flexible vendors have low ARC, leading to high flexibility. The model shows

\[
\frac{\partial m_p}{\partial m_c} = \frac{1}{2} \Rightarrow \frac{\partial \text{flex}_p}{\partial \text{flex}_c} > 0
\]

this implies proposition four.

**Proposition 4:** Ceteris paribus the flexibility of contract will be positively related to the flexibility of the vendor.

It is also interesting to consider the impact of client flexibility on the contract. As the model shows

\[
\frac{\partial m_p}{\partial m_c} = \frac{1}{2} \Rightarrow \frac{\partial \text{flex}_p}{\partial \text{flex}_c} > 0
\]

this implies proposition five.
Proposition 5: Ceteris paribus the flexibility of contract will be positively related to the flexibility of the client.

In addition to vendor and client flexibility, the model suggests that contractual flexibility is influenced by both vendor and client baseline costs. The vendor with a lower baseline cost will be able to offer a comparatively more flexible contract for a given contractual baseline cost. The model shows

\[
\frac{\partial m_p}{\partial b_v} = \frac{3}{4} \Rightarrow \frac{\partial flex_p}{\partial b_v} < 0
\]

This implies proposition six.

Proposition 6: Ceteris paribus the flexibility of contract will be negatively related to the baseline cost of the vendor.

A similar result holds for the client. The model shows

\[
\frac{\partial m_p}{\partial b_c} = \frac{3}{4} \Rightarrow \frac{\partial flex_p}{\partial b_c} < 0
\]

This implies proposition seven.

Proposition 7: Ceteris paribus the flexibility of contract will be negatively related to the baseline cost of the client.

Discussion

This work analyzed the relationships among flexibility and baseline cost of production of IT services by vendors, clients, and the contractually specified joining of the two. Our main results are:

- Lower baseline cost leads to decreased flexibility in a contract.
- The contractually governed outsourcing arrangement does not allow the client the same level of capability as that of the vendor.
- The flexibilities of the client and the vendor are positively related to the flexibility of the contract.
- The baseline costs of the client and the vendor are negatively related to the flexibility of the contract.

The first point is especially relevant because of the client’s reasons for outsourcing. Concerned about the current financial needs of the firm, the client seeks a contract involving lowest baseline cost not considering the effect of high ARC or low flexibility. However, as IT invariably changes quickly, flexibility is required. So initially, what looked like a good deal ends up being a very costly problem. The parties then either have to live with the problem or renegotiate the contract or terminate the arrangement; none of which is a particularly appealing prospect.

With the increase in money involved in IT outsourcing arrangements and in the rate of failure of such arrangements, it becomes very important that more attention is paid to issues that can be directly addressed in the contract. In this work, we examine the contract directly and show how the costs stipulated in the contract affect the nature of the outsourcing relationship. We develop an economic model that exposes the tension between baseline cost and flexibility of the contract. Due to financial constraints involved, it is very important for the client to strike a balance between baseline cost and flexibility. We show that outsourcing arrangement involving cheapest baseline cost is not always the most flexible and, hence, not always the best option.

The second point is also quite interesting because it illuminates a subtle point that is often ignored. Clients frequently seek out vendors to gain access to the vendor’s world class IT capabilities (Lacity, 1995). The perception is that having the vendor manages the client’s IT allows the client to enjoy the same level of capabilities the vendor possesses. However, this is not the case, as the client’s IT capabilities are precisely those determined by the contract and not by the vendor. That is to say, the contract fully mediates the vendor’s capabilities. A poorly conceived deal with an excellent vendor does not give the client world class IT capabilities.
These results imply a very significant suggestion for practice. Rather than focus on current IT needs or some target bundle of services, clients and vendors should consider a range of possible outcomes. Specifically, we recommend that IT outsourcing negotiations should develop several scenarios and compare the internal costs to the contractually specified costs for a variety of situations. Considering several such scenarios will give clients a better feel for the contract’s flexibility and give the vendor an additional selling point.

This work investigates the interplay between baseline cost and flexibility. We realize that including factors other than quantity such as quality, trust and nature of relationship will make the model more applicable to the real world. Future research can look at including these factors in the model. Future research can also look at the issue as to why the clients go for less flexible contracts as well as the effects of lack of flexibility of the contract.

References

Appendix 1

From eqn (5) we have

\[
E(\bar{\pi}_c) = E(\bar{\pi}_v) = \frac{1}{2} \left[ \left( \frac{b_c - b_v}{2} \right) + \left( \frac{m_c - m_v}{2} \right) \right] dq
\]  

(5)

Now for a general case where the contractually specified baseline cost is \( b_p \) and slope of the cost curve is \( m_p \).

\[
E(\pi_c) = \int_0^1 \left[ (b_c - b_p) + (m_c - m_p) q \right] dq
\]  

(a1)

Similarly

\[
E(\pi_v) = \int_0^1 \left[ (b_p - b_v) + (m_p - m_v) q \right] dq
\]  

(a2)

From eqn (6) and (7) we get the client and vendor constraints

\[
E(\pi_c) \geq E(\bar{\pi}_c)
\]  

(6)

\[
E(\pi_v) \geq E(\bar{\pi}_v)
\]  

(7)

Putting (a1) and (5) in eqn (6)

\[
\frac{1}{2} \left[ \left( \frac{b_c - b_p}{2} \right) + \left( \frac{m_c - m_p}{2} \right) \right] dq \geq \int_0^1 \left[ \left( \frac{b_c - b_v}{2} \right) + \left( \frac{m_c - m_v}{2} \right) \right] dq
\]  

(a3)

\[
\left[ \left( \frac{b_c - b_p}{2} \right)^2 + \left( \frac{m_c - m_p}{3} \right)^3 \right]_0^1 \geq \left[ \left( \frac{b_c - b_v}{2} \right)^2 + \left( \frac{m_c - m_v}{3} \right)^3 \right]_0^1
\]  

(a4)

\[
(b_c - b_p) \left( \frac{1}{2} - 0 \right) + (m_c - m_p) \left( \frac{1}{3} - 0 \right) \geq \left( \frac{b_c - b_v}{2} \right) \left( \frac{1}{2} - 0 \right) + \left( \frac{m_c - m_v}{3} \right) \left( \frac{1}{3} - 0 \right)
\]  

(a5)

\[
\frac{b_c - b_p}{2} + \frac{m_c - m_p}{3} \geq \frac{b_c - b_v}{4} + \frac{m_c - m_v}{6}
\]  

(a6)

\[
m_p \geq 3 \left[ \frac{b_c - b_p}{2} + \frac{m_c}{3} - \frac{b_c}{4} + \frac{m_c}{6} \right]
\]  

(a7)

\[
m_p \geq 3 \left[ -\frac{b_p}{2} + \frac{1}{4} (b_c + b_v) + \frac{1}{6} (m_c + m_v) \right]
\]  

(a8)
Putting eqn (a2) and (7) in eqn (5)

\[
\int_0^1 \left[ (b_p - b_c) + (m_p - m_c) q \right] dq 
\geq \int_0^1 \left[ \left( \frac{b_c - b_v}{2} \right) + \left( \frac{m_c - m_v}{2} \right) q \right] dq 
\]

(a9)

\[
\int_0^1 \left[ \frac{(b_p - b_c) q^2}{2} + \frac{(m_p - m_c) q^3}{3} \right] dq 
\geq \int_0^1 \left[ \left( \frac{b_c - b_v}{2} \right) \frac{q^2}{2} + \left( \frac{m_c - m_v}{2} \right) \frac{q^3}{3} \right] dq 
\]

(a10)

\[
(b_p - b_v) \left( \frac{1}{2} - 0 \right) + (m_p - m_v) \left( \frac{1}{3} - 0 \right) 
\geq \left( \frac{b_c - b_v}{2} \right) \left( \frac{1}{2} - 0 \right) + \left( \frac{m_c - m_v}{2} \right) \left( \frac{1}{3} - 0 \right) 
\]

(a11)

\[
\frac{b_p}{2} - \frac{b_c}{2} + \frac{m_p}{3} - \frac{m_c}{3} 
\geq \frac{b_c}{4} - \frac{b_v}{4} + \frac{m_c}{6} - \frac{m_v}{6} 
\]

(a12)

\[
m_p \geq 3 \left[ \frac{b_c}{2} - \frac{b_p}{2} + \frac{m_p}{3} + \frac{b_c}{4} - \frac{b_v}{4} + \frac{m_c}{6} - \frac{m_v}{6} \right] 
\]

(a13)

\[
m_p \geq 3 \left[ -\frac{b_p}{2} + \frac{1}{4} (b_v + b_c) + \frac{1}{6} (m_c + m_v) \right] 
\]

(a14)

Solving (a8) and (a14)

\[
m_p = 3 \left[ -\frac{b_p}{2} + \frac{1}{4} (b_v + b_c) + \frac{1}{6} (m_c + m_v) \right] 
\]

(8)