The Impact of Digitization on Information Goods Pricing Strategy

Full paper

Shivendu Shivendu  
University of South Florida  
shivendu@usf.edu

Ran Zhang  
University of California, Irvine  
ranz2@uci.edu

Abstract

The widespread adoption of the Internet and digital technologies has transformed the distribution and consumption of information goods. We develop a parsimonious model to study pricing strategies of a publisher who offers information good in dual medium (physical, digital) as well as in bundled medium. Consumers are heterogeneous in both valuation for content and preference for medium. We develop optimal pricing strategies and identify the interactive effect of different market characteristics on optimal pricing schemes. We show that offering digital medium only (single component) is optimal under some market conditions, while offering bundle of mediums and digital medium only (partial mixed bundling) is optimal under other market conditions. We find that offering information good in physical medium and in digital medium (pure component) is not optimal when the two mediums are partial substitutes. Moreover, offering only the bundle of mediums (pure bundling) is not optimal as long as physical medium has non-negligible marginal cost. Interestingly, it is always profit enhancing to offer digital medium, even if most consumers in the market prefer physical medium.

Keywords

Information goods pricing, digitization, dual medium access, bundling, heterogeneous preferences of mediums, partial substitutability of mediums

Introduction

The advent of the Internet and information technology has led to digitization of content industries and, in turn, has transformed the distribution and consumption of information goods. While traditionally consumers purchased physical copy or medium to access the information goods (print newspaper, book, CD, DVD, etc.), in the digital era, an increasing number of consumers now buy and consume the same information good in digital medium (digital newspaper, eBook, digital albums, video downloads, etc.).

Prior research suggests that “moving-online” can potentially expand the consumer base by providing additional convenience and ease of use through anytime-anywhere access (Kouikova et al 2008, Bhargava 2014). On the other hand, the digital content may serve as a substitute for access through physical medium, and thereby cannibalizing the sales for the physical version (Kannan et al 2009). In practice, an extensive debate is unfolding about the pros and cons of digitization in book industry and movie industry, and content publishers have been resistant to embrace the new digital distribution formats (Harkaway 2012, Knight 2015). In addition, a noticeable variation in pricing of content over the dual mediums exists both within and across industries. For instance, The Wall Street Journal as well as The New York Times offers home delivery + digital and digital only access, but no home delivery only option. Game Informer magazine offers print only and digital only but no bundle of the two. Warner Music sells digital album only as well as CD that comes with digital copy. Independent record labels such as Soulection and Triple Pop offer only digital album and tracks (Droppo 2014). An imperative question for content publishers is to determine the optimal content pricing strategies over dual medium access under different market conditions.

While there is significant difference in marginal cost of physical medium and digital medium, consumers have heterogeneous preferences for mediums. For example, in 2013, 38% consumers preferred digital
access to video games while majority of core gamers (62%) preferred physical CD of the games (NPD Group 2015). Moreover, the content consumption in digital medium has grown by 157% from 2010 to 2014 which indicates that the proportion of consumers who prefer digital medium is growing over the time and this trend is likely to continue in future (comScore, 2015). The heterogeneous consumer preference toward medium raises new challenges for publishers on how to price information good in dual medium. Though there is growing literature in IS on information goods pricing (Chen & Png, 2003; Choudhary et al, 2005), information goods pricing under dual medium access wherein market consists of consumers who have heterogeneous preferences for content valuation and medium has not attracted much attention. In this paper, we bridge this gap in literature by studying following research questions: (a) what are the publisher’s optimal content pricing strategies with dual medium access? (b) how the cost structure and substitutability of physical and digital medium impact the publisher’s bundling strategy over dual medium? (b) under what conditions is offering digital medium access a profit-enhancing strategy?

In order to study the information goods market under dual access, we develop an analytical model wherein a monopolist publisher (Sundararajan, 2004) has infrastructure to offer information good (like newspaper, magazine, music, movie etc.) in physical as well as in digital medium and needs to develop optimal content-medium pricing strategy. While the marginal cost of physical medium is non-negligible, marginal cost of digital medium is zero (Bhargava & Choudhary, 2001; Venkatesh & Chatterjee, 2006). Consumers are vertically differentiated in their valuation for information good or content (Choudhary et.al., 2005; Lahiri & Dey, 2013) and have heterogeneous preferences for medium (Venkatesh & Chatterjee, 2006). Specifically, if a consumer obtains the information good in the medium she prefers, then her willingness-to-pay is the same as her valuation for the information good, but if she obtains the information good in the non-preferred medium, then she incurs disutility and her willingness-to-pay is lower than her valuation for the information good. We abstract this medium-mismatch disutility through a mismatch cost parameter. In the market some consumers prefers digital medium (i.e. digital-savvy consumers) while the others prefer physical medium (i.e. traditional consumers). If the two mediums are partial substitutes, then the willingness-to-pay for the information good through bundled access (physical medium as well as digital medium) is lower than the sum of the willingness-to-pay for each of the two mediums. We abstract this partial substitutability of the two mediums through a sub-additive parameter.

Our results show that offering digital medium (single component) only is optimal under some market conditions, while offering bundle of mediums and digital medium (partial mixed bundling) is optimal under other market conditions. Offering physical medium and digital medium separately (pure component) or offering bundle as well as physical and digital medium (full mixed bundling) is not optimal when the two mediums are partial substitutes. Moreover, we find that offering only the bundle of mediums (pure bundling) is not optimal as long as physical medium has non-negligible marginal cost. In addition, we identify three optimal pricing schemes and identify the interactive role of marginal cost with mismatch parameter, sub-additive parameter, and proportion of digital-savvy consumers in publisher's optimal pricing strategy.

We contribute to literature in several streams. This is one of the few papers in the content pricing literature and to our knowledge, the first paper that develops an analytical model to study the optimal pricing strategies and profitability of offering content in physical, digital, and bundle of mediums. First, we show that offering digital medium only (single component) is optimal under some market conditions, while offering bundle as well as digital medium (partial mixed bundling) is optimal under other conditions. This finding is new in content pricing literature (Venkatesh and Chatterjee 2006, Simon and Kadiyali 2007). Our research contributes to bundling literature by identifying conditions under which single component strategy or partial mixed bundling is optimal (McAfee et al. 1989, Venkatesh and Kamakura 2003, Armstrong 2013).

Second, our analytical results suggest that offering digital medium is always optimal, even when most consumers prefer physical medium. This informs the debate about adoption of digital distribution formats in content industries and posits that the publishers’ fear of negative impact on their profits due to adoption of digital technologies is rather untenable. The key to using digital distribution to a publisher's
advantage lies in formulating appropriate pricing strategy for access of information good or content in the bundle of mediums of physical + digital as well as only in digital medium.

Literature Review

In the literature of content pricing in dual medium, there is some empirical research that informs our work. Simon and Kadiyali (2007) study how offering digital content affects its print circulation and find that digital medium cannibalizes demand in print medium. Kannan et. al (2009) examine digital content pricing for National Academies Press and empirically identify the substitutability of print and PDF forms. They validate that when the two forms are viewed as almost-perfect substitutes, the incremental preference of the bundle is not different from zero. Kouikova et al (2008) employ experimental methodology and find that physical and digital product formats have advantages over one another in specific usage situations. Along this direction, Kouikova et al (2012) find that when different product formats have distinct attribute, consumers perceive the formats as more complementary and are more likely to buy the bundle. Our conceptualization of partial substitutability between physical and digital medium is informed by these empirical evidences. Although these empirical studies form some of our theoretical basis, they only examine the optimality of full mixed bundling pricing for the publisher and pure component pricing. Their studies do not examine the optimality of partial mixed bundling, offering single product, and pure bundling.

Venkatesh and Chatterjee (2006) examines publisher’s profitability for multiform products that can be offered in both physical and digital mediums. They utilize numerical method and show that offering digital medium is profit enhancing under some market conditions. One key distinction in our model is that while they assume perfect substitutability between physical and digital mediums, we have more generalizable setting where the two mediums can be partial substitutes. Another difference is that in their study limited analysis is provided on the impact of market parameters on market parameters, our closed-form analytical solution enables us to analyze the dynamics of how publisher’s profit and consumer surplus change with various market parameters.

Our study is broadly related to literature on bundling of two products in general and bundling of two mediums in particular. There is a vast literature on bundling of two products. The early work (Stigler 1963, Adams and Yellen 1976) illustrates that bundling can serve as a useful price discrimination technique by using a series of numerical examples of two products and discrete-type consumers. McAfee et al (1989) show that mixed bundling almost always dominates component pricing when consumers’ valuations are independently distributed. Prior research has also examined bundling of complementary or substitutable products (Lewbel, 1985; Venkatesh and Kamakura, 2003; Armstrong (2013). Although these studies provide insightful results about optimality of mixed bundling, these results cannot be readily applied in the context of dual medium access of information goods where the same content is offered in bundled mediums. Moreover, prior bundling literature has not examined the optimality of partial mixed bundling and single component strategy when the demand of the components is positively correlated and bundle has sub-additive valuation.

Model

The market consists of a publisher of information good such as a newspaper publisher, a music label, a video game developer, a movie studio or a book publisher who has the ownership rights over the content. Given that each publisher has copyright over unique content, following prior literature (Chen & Png, 2003; Sundararajan, 2004), we model content publisher as a monopolist. Moreover, we assume that the cost of acquiring or developing content is sunk (Wu & Chen, 2008). Consumers derive value from consumption of the content but they can consume content only if it is provided in a media (Nielsen, 2014). The publisher can provide the content either in physical media such as magazine, CD, or book, or she2 can provide content in digital media such as digital access to magazine content, downloadable music, or eBook. The publisher can also offer the content as a bundle in both the media, so that consumers can

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2 Throughout this paper, our publisher is ‘she’ and consumer is ‘he’.
consume it in either media as per their choosing. For example, the publisher of the New York Times offers a bundle of digital subscription and home delivery that includes print copy delivery and all digital access.

The publisher incurs a marginal cost \( c_p \in (0,1) \) to serve a consumer to whom she provides content in physical media. On the other hand, the publisher incurs zero marginal cost to serve a consumer to whom she provides content in digital media (Bakos & Brynjolfsson, 1999; Cusumano, 2007). When the publisher offers the content as a bundle (in both physical and digital media) to a consumer, then to serve that consumer, she incurs the same marginal cost as in the case of offering the content in physical media, i.e. \( c_p \). To keep the focus of this research on optimal content-media pricing strategies, we assume that all infrastructural costs for providing content in physical or digital media are sunk, and the publisher has no supply-side constrains.

**Consumers**

The market consists of unit mass of consumers. Consumers have heterogeneous valuations \( v \) for the content, which are independent and uniformly distributed, i.e. \( v \sim U[0, 1] \) (Bakos et al., 1999; Lang & Vragov, 2005). Recent industrial report shows the percentage of consumers who prefer digital game has grown to be 38 percent while the rest portion of gamers still prefers physical media to digital downloads (NPD Group 2015). Therefore, the publisher’s optimal content-medium pricing strategy should take into account consumers' heterogeneous preference for the media.

We model this heterogeneous consumer preference for media by segmenting the market wherein \( r \) proportion of consumers prefers digital media (digital-savvy consumer segment) and \( 1-r \) proportion of consumers prefers physical media (traditional consumer segment) (Kannan et al. 2009). A consumer who has valuation \( v \) for the content has willingness to pay \( v \) for the access to the content if she is offered the content in the media she prefers. On the other hand, if a consumer is offered the content in the medium that she does not prefer, then she incurs a medium mismatch cost \( \theta \in (0,1) \). This implies that a consumer who has valuation \( v \) for the content has lower willingness to pay for the access to the content, i.e., \( v(1-\theta) \), if she is offered the content in the medium she does not prefer.

When the publisher offers the content as a bundle in both the media, consumers’ willingness to pay for the content in dual media is sub-additive. This implies that in our conceptualization consumers view content offering in physical media and digital media as substitutes (PWC, 2008). This abstraction of dual media access of content is similar to consumers’ degree of contingency modeled in Venkatesh and Kamakura (2003) where the reservation price of bundle of two products is less than the stand-alone reservation price of the two products, if the products are substitutes. We abstract this sub-additive characteristic of mediums through a parameter \( \alpha \in (0,1-\theta) \) such that a consumer’s willingness to pay for content in both mediums is \( v + \alpha v = v(1+\alpha) \). Parameter \( \alpha \) can also be interpreted as additional convenience that consumers experience, if they have choice of consuming the content in either medium. For analytical tractability, we assume that sub-additive parameter \( \alpha \) is homogeneous across all consumers. While Venkatesh and Kamakura (2003) also make this assumption to get close form solution, this abstraction is similar to constant value dependence assumption in McGuire and Staelin (1983) and Bakos and Brynjolfsson (1999. In our abstraction, willingness to pay for content in dual media bundle is sub-additive, \( v(1+\alpha) < v + v(1-\theta) \) which leads to \( \theta + \alpha < 1 \).

Now, we describe utility derived by consumers by consuming content in either of the mediums or as a bundle. A digital-savvy consumer’s utility from consuming content in digital medium is \( v - p_s \), in physical medium is \( v(1-\theta) - p_s \), and as a bundle is \( v(1+\alpha) - p_s \). Similarly, a traditional consumer’s utility from consuming content in physical medium is \( v - p_s \), in digital medium is \( v(1-\theta) - p_s \), and as a bundle is \( v(1+\alpha) - p_s \).
Content-medium pricing strategies

The publisher decides its optimal content-medium pricing strategy by taking into account consumer heterogeneities and costs of offering the content in physical medium or digital medium or as a bundle of mediums. There are seven feasible content-medium pricing strategies that the publisher can adopt: (1) offer content in physical medium only (physical only); (2) offer content in digital medium only (digital only); (3) offer content in physical as well as in digital medium (physical, and digital); (4) offer content as a bundle (bundle); (5) offer content in bundle as well as in physical and digital mediums (bundle, physical, and digital); (6) offer content in bundle as well only in physical medium (bundle, and physical) and (7) offer content in bundle as well as in only in digital medium (bundle, and digital).

To focus our analysis on only those pricing strategies that may be optimal under different values of parameters and marginal cost, we first rule out those pricing strategies that are always suboptimal.

PROPOSITION 1: (a) Any content pricing strategy that includes physical medium is weakly dominated by content-pricing strategy of offering content in bundle, and digital; (b) The content-medium pricing strategy of offering content in bundle, and digital medium dominates pricing strategy of offering content only in bundle.

The intuition for the above proposition is that by bundling digital medium with physical medium (offering bundle), consumers’ willingness to pay strictly increases compared to their willingness to pay for content in physical medium only. On the other hand, the publisher incurs no additional marginal cost by bundling digital medium with physical medium. So by offering bundle of mediums rather than physical medium, publisher can generate more profits. Note that when \( \alpha = 0 \), consumers’ willingness to pay for content does not increase if the content is offered on both mediums, that is, the bundle and the preferred medium are perfect substitutes (Calzolari and Denicolò 2009, Armstrong 2013). It is easy to see that if sub-additive parameter \( \alpha = 0 \), then it is never optimal for the publisher to offer bundle.

Proposition 1(b) shows that offering bundle only is sub-optimal. The intuition is that by offering content in the digital medium in addition to bundle, even if the publisher keeps the bundle price the same, she can offer a lower price for content in digital medium so that some low valuation digital-savvy consumers buy digital medium but no consumer shifts from buying the bundle to buying digital. This leads to increased market coverage. In other words, under some conditions, by offering digital medium, the publisher can generate additional revenue without cannibalizing bundle revenue. Since the cost of digital medium is zero, under these conditions, the publisher will be strictly better off. Therefore, from the analysis above, we need to focus on only two content-medium pricing strategies, that is, offering content in digital and offering content in bundle, and digital.

Content-medium pricing strategy for digital medium only

When the publisher offers content in digital medium only, she incurs zero cost and her profit optimization problem reduces to revenue maximization problem.

**LEMMA 1:** When the publisher offers digital medium only, the optimal price is \( p^*_d = (1-\theta)/2(1-r\theta) \), and the optimal profit is \( \pi^*_d = (1-\theta)/4(1-r\theta) \).

It is easy to see that when content is offered in digital medium only, both optimal price \( p^*_d \) and profit \( \pi^*_d \) are increasing with the proportion of digital-savvy consumers \( r \) in the market. Note that if all consumers in the market are traditional, then the optimal price is \( (1-\theta)/2 \) and if all consumers in the market are digital-savvy \( (r = 1) \), then the optimal price is \( 1/2 \).

Content-medium pricing strategy for bundle, and digital medium

When the publisher offers content in bundle as well as in digital medium, consumers purchase decision is based on their surplus from buying either of the offerings. We derive all possible market coverage profiles from incentive compatibility (IC) and individual rationality (IR) for each of the consumer types. Traditional consumer will buy bundle if the following IR constraint is met:
\[ v(1+\alpha) - p_b \geq 0 \]  
(IR: T-b)

And will buy digital medium if the following IR constraint is met:

\[ v(1-\theta) - p_d \geq 0 \]  
(IR: T-d)

Traditional consumer will buy bundle over digital medium if the following IC constraint is satisfied:

\[ v(1+\alpha) - p_b \geq v(1-\theta) - p_d \]  
(IC: T-b)

Similarly, digital-savvy consumer will buy bundle if the following IR constraint is met:

\[ v(1+\alpha) - p_b \geq 0 \]  
(IR: DS-b)

And will buy digital medium if the following IR constraint is met:

\[ v - p_d \geq 0 \]  
(IR: DS-d)

Digital-savvy consumer will buy bundle over digital medium if the following IC constraint is satisfied:

\[ v(1+\alpha) - p_b \geq v - p_d \]  
(IC: DS-b)

From these IC and IR constraints, we get four possible market coverage profiles for each type of consumers. The market coverage profiles for traditional consumers, 1) some traditional consumers buy bundle, 2) some traditional consumers buy bundle, and some buy digital, 3) some traditional consumers buy digital, 4) no traditional consumer buys bundle or digital. Market coverage profiles for digital-savvy consumers follow the similar fashion. Combining the market coverage profiles for the two types of consumers, we get overall 16 market coverage profiles. It is easy to see that any market coverage profile that includes the scenario that any one consumer segment buys nothing is never optimal. Therefore, we now need to only focus on eight market coverage profiles in which both consumer segments participate in the market.

**PROPOSITION 2:** Any market coverage profile in which digital-savvy consumers only buy bundle is never optimal.

Sketch of proof: Suppose some market coverage profile in which digital-savvy consumers only buy bundle is optimal. Traditional consumers will also only buy bundle because they have the same willingness to pay for the bundle but lower willingness to pay for the digital than digital-savvy consumers. So consumers in both segments will only buy bundle. We have shown in Proposition 1(b) that offering bundle only is suboptimal. Therefore, any market coverage profile in which digital-savvy consumers only buy bundle is never optimal. 

From the above analysis, we know when the publisher adopts bundle, and digital content-medium strategy, only five possible market coverage profiles can be potentially optimal.

**Figure 1a: Market coverage profile 1: traditional consumers buy bundle (T-b), and some digital-savvy consumers buy bundle, some buy digital (DS-b & d)**

(a) Market coverage profile 1(T-b & DS-b & d): Under this market coverage profile while traditional consumers buy content only in bundle, some digital-savvy consumers buy content in bundle and some buy in digital medium. Since traditional consumers only buy bundle, the marginal traditional consumer’s IR: T-b must be binding. Hence, the valuation of the marginal traditional consumer who is indifferent
between buying bundle and not buying is \( v = p_b / (1 + \alpha) \). Since some digital-savvy consumers buy bundle and some buy digital, the IC: DS-b of the marginal digital-savvy consumer who is indifferent between buying bundle and digital must be binding. Hence, the valuation of the marginal digital-savvy consumer who buys bundle is \( v = (p_b - p_d) / \alpha \) and his IR: DS-b is always satisfied. Similarly, the valuation of the marginal digital-savvy consumer who is indifferent between buying digital and not buying is given by \( v = p_d \). In order for market coverage profile 1 to be feasible, the following two conditions must be met. (i) \( p_b / (1 + \alpha) \leq (1 - \theta)(1 - p_b / (1 + \alpha))(p_b - c_p) + r((p_b - p_d) / \alpha - p_d)p_d \) (see Figure 1a). Now, we can write the profit function under market coverage profile 1 as:

\[
\pi_{\text{ PROF }} = \left( r(1 - (p_b - p_d) / \alpha) + (1 - r)(1 - p_b / (1 + \alpha))(p_b - c_p) + r((p_b - p_d) / \alpha - p_d)p_d \right)
\]

Similarly, we report the conditions and profit functions for the other four cases (see Figure 1b) in the Appendix A for the interest of space.

**Figure 1b**: (i) Market coverage profile 2: \( T - b \) and DS - d; (ii) market coverage profile 3: \( T - b & d \) and DS - d; (iii) 4: \( T - b & d \) and DS - b & d; and (iv) 5: \( T - d \) and DS - d.

We find that market coverage profile 4 is dominated by either market converge profile, 1 or 2, or 3 under different parameter space. In the next section, we derive optimal pricing schemes.

**Optimal content-medium pricing strategy**

We now have four market coverage profiles that may be optimal in different parameter spaces. We derive feasible optimal pricing schemes as follows.

**PROPOSITION 3**: When the publisher offers content in bundle, and digital medium (partial mixed bundling) or in digital medium only (single component), for any feasible parameter space, the optimal pricing scheme is from set \( \{ P_{i}^*, P_{II}^*, P_{III}^* \} \), where

\[
P_{i}^* = \left\{ \begin{array}{ll}
p_{i}^* &= \frac{1 - \theta}{2 - 2r\theta}, \\
p_{II}^* &= \theta + \frac{\alpha + c_p}{2} + \frac{1 - \theta}{2 - 2r\theta}
\end{array} \right. \] \quad \text{and} \quad P_{III}^* = \left\{ \begin{array}{ll}
p_{III}^* &= \frac{1 - \theta}{2 - 2r\theta}
\end{array} \right.
\]

Under optimal pricing scheme \( P_{i}^* \), under different parameter spaces the market coverage profile is either 1 or 2. On the other hand, under optimal pricing scheme \( P_{II}^* \), the market coverage profile is 3 and under optimal pricing scheme \( P_{III}^* \), the market coverage profile is 5.
We find that while price of bundle increases with marginal cost, the price of digital is independent of marginal cost. Moreover, while the prices of digital and bundle in pricing scheme $P^I$ are independent of the proportion of digital-savvy consumers, the price of digital in pricing scheme $P_u^I$ and $P_u^I$, and the price of bundle in pricing scheme $P_u^I$ increase with the proportion of digital-savvy consumers.

Discussion and conclusion

Pricing information goods over dual mediums is an important question for multiple information goods or content industries in the digital era. The increasingly prevalent digital medium has brought the convenience of anytime-anywhere access of content to consumers, and thus is likely to lead to larger user base. On the other hand, digital medium may also cannibalize the sales from traditional medium. Therefore, what optimal pricing and bundling strategy should content providers adopt and how various market characteristics affect the optimal pricing and profit are challenging questions for both practitioners and researchers. Prior research on content pricing over dual mediums provided limited insight on this issue and did not offer analytically tractable solution that inform managers under more general and dynamic market conditions. Insights from literature on bundling of two products are not readily applicable in the context of information goods because of the unique market characteristics in the content pricing context, namely partial substitutability of the two mediums, correlated demand for the two mediums, consumers’ heterogeneous preference toward mediums, and asymmetric cost structure of the two mediums. In order to bridge this gap, we build an analytical model that captures various firm and market characteristics of information goods or content industry, and provide optimal bundling strategy and pricing schemes under different market conditions.

Our analytical framework can be extended to other related research questions in the domain of information goods pricing. First, it is interesting to identify the optimal parameter region for each pricing scheme, and analyze the impact of different market conditions on the publisher’s market coverage, profit, consumer surplus, and social welfare. Second, when the valuation of content is unknown to consumers, publishers may want to offer free introductory period for subscription or free samples for partial or full digitized content. It would be interesting to examine how the introductory period or free samples signals the quality of contents and affects publisher’s optimal pricing strategy over the two mediums. Third, in current study we focus on content pricing over two mediums, physical and digital medium. It would be interesting to see how publisher’s pricing strategy will change when content is accessed via physical medium as well as multi-digital mediums such as computer and mobile phone.

REFERENCES


**Appendix A: Technical Results**

(b) Similarly, under market coverage profile 2 (T - b & DS - d), the conditions for this market coverage profile to be feasible are: (i) \( p_b / (1 + \alpha) \leq p_d / (1 - \theta) \) and (ii) \( (p_b - p_d) / \alpha > 1 \) (Figure 1b (i)). In this case, the publisher’s profit function is:
\[ \pi_{d}^* = (1 - r)(1 - p_{d}/(1 + \alpha))(p_{d} - c_{p}) + r(1 - p_{d})p_{d} \]  
(2)

(c) Under market coverage profile 3 (T - b & d & DS - d), the following two conditions must be met:
\[ \frac{p_{d} - p_{d}}{1 - \theta} < \frac{p_{d} - p_{d}}{\alpha} \quad \text{and} \quad \frac{p_{d} - p_{d}}{1 - \theta} \geq 1 \]  
(Figure 1b (ii)). In this case, the publisher’s profit function is:
\[ \pi_{d}^{3} = (1 - r)(1 - \frac{p_{d} - p_{d}}{\alpha + \theta})(p_{b} - c_{p}) + ((1 - r)(\frac{p_{d} - p_{d}}{\alpha + \theta} - \frac{p_{d}}{1 - \theta}) + r(1 - p_{d}))p_{d} \]  
(3)

(d) Under market coverage profile 4 (T - b & d and DS - b & d), the conditions for market coverage profile 4 to be feasible are:
\[ \frac{p_{d} - p_{d}}{1 - \theta} < 1 \quad \text{and} \quad \frac{p_{d} - p_{d}}{\alpha} < 1 \]  
(Figures 1b (iii)); Under market coverage profile 5 (T - d and DS - d), the two conditions must be met:
\[ \frac{p_{d} - p_{d}}{1 - \theta} > 1 \quad \text{and} \quad \frac{p_{d} - p_{d}}{\alpha} > 1 \]  
(Figures 1b (iv)).

\[ \pi_{d}^{4} = (1 - r)(1 - \frac{p_{d} - p_{d}}{\alpha + \theta})(p_{b} - c_{p}) + ((1 - r)(\frac{p_{d} - p_{d}}{\alpha + \theta} - \frac{p_{d}}{1 - \theta}) + r(1 - p_{d}))p_{d} \]  
(4)

\[ \pi_{d}^{5} = (1 - r)(1 - \frac{p_{d}}{1 - \theta})(p_{b} - c_{p}) + r(1 - p_{d})p_{d} \]  
(5)

**Proof for Proposition 1:** (a). Both types of consumers’ willingness to pay for the bundle is \( v(1 + \alpha) \), traditional consumer’s willingness to pay for the physical is \( v \), while digital-savvy consumer’s willingness to pay is \( v(1 - \alpha) \). So \( v(1 + \alpha) > v \) and \( v(1 + \alpha) > v(1 - \theta) \). And bundle incurs the same cost as physical medium, so the publisher is strictly better off by offering bundle than physical medium. Besides, offering full mixed bundling (bundle, physical, and digital) is equivalent to offering bundle, and digital in the optimal solution, therefore, any pricing strategy that includes physical is weakly dominated by offering bundle, and digital.

(b). Suppose offering bundle only at \( p_{b} \), then the publisher’s profit is \( \pi_{1} = (p_{b} - c_{p})(1 - \frac{p_{b}}{1 + \alpha}) \), we get
\[ p_{b}^{*} = (1 + \alpha + c_{p})/2, \quad \pi_{1}^{*} = \frac{(1 + \alpha + c_{p})^{2}}{4(1 + \alpha)} \]. Now, the publisher also offers digital, at a lower price 1/2 , then the publisher’s profit is \( \pi_{2} = (1 - r)(-c_{p} + p_{b})\left(1 - \frac{p_{b}}{1 + \alpha}\right) + r\left(-c_{p} + p_{b}\right)\left(1 - \frac{p_{b} - p_{d}}{\alpha}\right) + \left(p_{b} - p_{d} - p_{d}\right)p_{d} \), we get
\[ \pi_{2}^{*} = \frac{1}{4}\left(1 + \alpha + c_{p}\left(-2 + \frac{r + \alpha}{\alpha(1 + \alpha})\right)\right), \quad \text{it is easy to see} \quad \pi_{2}^{*} > \pi_{1}^{*}. \]

**Proof for Lemma 1:** When the publisher offers digital medium only, the publisher’s profit is \( \pi_{d} = p_{d}(1 - p_{d}/(1 - \theta)) + p_{d}(1 - p_{d}) \), take F.O.C with \( p_{d} \), we get \( p_{d}^{*} = \frac{1 - \theta}{2(1 - r \theta)}, \quad \pi_{d}^{*} = \frac{1 - \theta}{4(1 - r \theta)}. \)

**Proof for Proposition 3:** Take F.O.C for profit function \( \pi_{d}^{1} \) (see equation 1) with \( p_{d} \) and \( p_{b} \), we get
\[ p_{1}^{*} = \frac{1}{2}, \quad p_{1}^{*} = \frac{1 + c_{p} + \alpha}{2}. \]  
Use same approach for profit function \( \pi_{d}^{3} \) (see equation 2), we get same prices \( p_{2}^{*} \) and \( p_{2}^{*} \); For profit function \( \pi_{d}^{1} \) (see equation 3), we get
\[ p_{3}^{*} = \frac{1 - \theta}{2(1 - r \theta)}, \quad p_{3}^{*} = \frac{1}{2}(\theta + c_{p} + 1 - \theta) \]; For profit function \( \pi_{d}^{4} \) (see equation 4), we get ; For profit function \( \pi_{d}^{5} \) (see equation 5), we get
\[ p_{5}^{*} = \frac{1 - \theta}{2(1 - r \theta)}. \]