Analysis of Book Resale in Amazon Upgrade Framework: A Game-Theory Approach

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ABSTRACT

Amazon recently began providing their customers with an innovative and contemporary service. This service supplies customers with the option to purchase an electronic version of a book after they have purchased a physical copy. This is similar to bundling an option of electronic book purchase to a physical one. Such a business practice provides consumers with an opportunity to resell brand new books, which will influence the welfare of publishers, retailers, and consumers. Using a two-period game theory model we discuss these impacts under two scenarios: monopoly of a retailer, as well as competition between retailers. We find that (1) Under a monopoly, the publisher sells at a higher rate, and the retailer sells at a lower rate in the equilibrium; consumers also see an increase in their total surplus; (2) Under competition, retailers who provide this bundle will gain a competitive advantage over the retailers who do not. Our research indicates that bundling a physical book with its digital version yields a win-win strategy for all players.

Keywords (Required)
Resale, E-book, retailer welfare, price competition

INTRODUCTION

Digitalization has become the trend of publication for several years. Bill Gates predicted that “In the future, students' textbooks would be replaced by individual electronic tablets that would be cheap, multimedia, and connected to the Internet” [13]. According to IDPF (International Digital Publishing Forum) [14], “e-Book publisher reported a 23% increase in e-Book revenues and a 20% increase in e-Book titles over 2004.” Despite the trend of growth, electronic books continuously fail to meet the expectation of sales, while online access to physical books may be easily found. An explanation of the slow growth of e-book industry may be due to the fact that both publishers and large online retailers do not have incentives to promote e-books. They worry that they will experience catastrophic revenue losses if the demand of e-books replaces the demand of physical books. However, companies continue their efforts in incorporating digitalization into their business models. Recently, Amazon quietly began its Amazon Upgrade program, allowing consumers who purchase a physical book have online access to its digital version with only a small fee, usually between $1 and $5. Consumers can search items, insert bookmarks, highlight texts, and make tags and notes in the online electronic version.

We consider such business practice as selling a bundle of physical book with its electronic version. This business model brings several interesting insights in the current retail industry of books. From the perspective of consumers, they now have a convenient access to the book whenever and wherever internet access is available. There is no need to bring a heavy hardcopy all around. Besides, it reduces the risk of over-estimating the value of a book. Suppose a consumer pays $50 for the bundle and has access to the e-version before the physical book is delivered to her. However, she finds that her utility of the book after reading the first chapter is now lower than $50, say $25. She then can sell the physical book as brand new item for say $30. In this case, she only pays $20=$50-$30 for the book, which is lower than her utility $25. Otherwise, she will have a utility loss of $25-$50=-$25. From the perspective of publishers, digitalization of books brings two effects: the revenue effect and substitution effect. On one hand, publishers can earn more revenue by signing contract of copyright with retailers who are interested in providing such bundles, which increases publisher’s profit. On the other hand, publishers need to face the cannibalization which arises from the new books sale. Similar to what Waldman (1997) and Hendel and Lizzieri (1999) argue on secondary market where buyers can sell used books after they purchase new books, rational consumers who purchase the bundle can sell the physical book as brand new books if they find that resale can increase their utility. From the perspective of retailers, this business model brings the following benefits: (1) Attract consumers with a lower valuation. Retailers are able to attract consumers whose valuation is lower than the price of new physical book but higher than the difference between the
price of physical book and resale cost. Following the previous example, suppose the bundle is $50 and resale price is $30 for brand new physical book. A consumer whose valuation is $28 will not purchase the physical book because her valuation ($28) is less than price of a new book ($30). However, she can own the e-version by purchasing the bundle and resell the physical book because her valuation $28 is greater than the difference $50-$30=$20. (2) Set higher prices. Since the retailer needs to pay for the copyright fee, he now can charge consumers more on the bundle. (3) Increase consumer satisfaction level since consumers now have easier and quicker access to the book.

We focus on the research setting of retail industry of books in which the retailer provides a bundle of both a brand new physical book with its electronic version.1 In particular, we want to find answers for the following research questions: (1) What is the impact on publishers, retailers and customers? (2) What is the optimal pricing strategy for the retailers? (3) What will happen under the competition between a retailer who provides an e-version and a retailer who does not?

LITERATURE REVIEW

Our research is closely related to the stream of online book industry literature. Clay et al. (2002) discussed the issue of price differentiation of online book sales. Clevelier et al. (2003) reported significant price sensitivity for online book sales in both Amazon.com and BarnesandNoble.com. Ghose et.al (2005) found that “the ‘market expansion effect’ accruing from consumers’ cross-product affinity can mitigate the losses incurred by ‘cannibalization’ effect” and they use online book industry data to test those implications. Ghose et al. (2006) made an empirical analysis of the impact of online secondary market to new book sales and concluded that used books are not substitutes for new books in most cases.

Our research also relates to the literature of product resale. Deserpa (1994) found that uncertainty on demand and risk-aversion prevents sellers from under pricing, which creates the opportunity for resale. Rudi et al. (2001) and Lee and Whang (2002) assumed that the seller who can only sell in the early period does not need to worry about demand reduction. Karp and Perloff (2005) argued that resellers have an information advantage over early sellers in terms of screening late buyers, which allows them selling tickets at higher prices to high value buyers. A recent research by Geng et al. (2007) argued that sellers may perform better using “partial resale” strategy when buyers conduct advance and spot resale in different channels.

Our research differs from the above papers in two aspects: (1) We focus on the resale impact brought by bundling with e-books. Items of such resale are homogenous to brand new items in other sales channel, which none of above papers discusses. (2) Resale under this scenario does not lead to consumers’ giving up the item which is the case in most of above papers. Instead consumers have the item in digital format. As mentioned above, adopting this model, selling a bundle of physical book with its digital version, brings benefit to the publisher. It also helps the retailer to reach consumers with a lower evaluation on the books at the beginning. Although we focus on books, this research can be applied to other information goods like digital magazine, movie and music. The rest of the paper goes as follows. We lay out the model in the next section, both under the no e-version scenario and e-version scenario where retailers provide the electronic version of physical books. In section three, we discuss the two cases under the e-version scenario: monopoly and competition. The paper concludes with a discussion in section four.

MODEL

NO E-BOOK CHOICE

In this section, we first develop a two-period game theory model of a benchmark scenario under which there is no electronic version. Let there be one publisher $S$ and one retailer $R$ in our model. Under this scenario, publisher $S$ sells books to retailer $R$ in the first time period. In the second time period, retailer $R$ sells books to consumers while consumers can sell used physical books to other consumers (see figure 1).

---

1 Other companies like Google also invest in digitalization of publications, but they usually do that for public use, which is not discussed here.
Assume consumers have a uniform distribution of their valuation $U(0, V_b)$ on the book. We standardize the size of consumer as $U(0, b)$, so demand is proportional to consumer value range. Using similar notation in Ghose et al (2006), we let $D_N$ denote the demand of new books, $w$ denote the price publisher charges on retailer and $p_b$ denote the price of new books retailer sells to consumers. Suppose $\alpha$ percentage of consumers will sell the books purchased as used books to other consumers at price $u_p$ through which the retailer earns a commission rate $k$.

We get revenue for both publisher and retailer as follows:

Publisher’s revenue:
\[
\pi_s = D_N w = (V_b - p_b)w
\]

Retailer’s revenue:
\[
\pi_R = D_N (p_N - w) + k \cdot \alpha D_N \cdot p_u
\]
\[= (V_b - p_b) \cdot (p_b - w) + k \cdot \alpha (V_b - p_b) \cdot (1 + \alpha) p_b - \alpha V_b \]

(2)

Note that retailer’s revenue consists of two parts: (1) selling new books to customers and (2) the commission from the customers’ resale.

Set $\frac{\partial \pi_R}{\partial p_b} = 0$, we get $p_b^* = \frac{(1 + k + 2\alpha k)V_b + w}{2(1 + k + \alpha k)}$ (3)

Set $\frac{\partial \pi_s}{\partial w} = 0$, we get $w^* = \frac{(1 + \alpha k)}{2} V_b$ and $p_b^* = \frac{(3 + 3\alpha k + 4\alpha^2 k) V_b}{4(1 + \alpha k + \alpha^2 k)}$ (4)

Thus, the profit for publisher is:
\[
\pi_s = (V_b - p_b)w = \frac{(1 + \alpha k)^2 V_b^2}{8(1 + \alpha k + \alpha^2 k)}
\]

(5)

And the profit for retailer is:
\[
\pi_R = \frac{(1 + \alpha k)^2 V_b^2}{16(1 + \alpha k + \alpha^2 k)}
\]

(6)

**E-version scenario**

Under this scenario, we assume a free e-version for simplicity. We argue that this assumption does not deviate from reality because expense for digital version is relatively insignificant compared with the price of physical books. The difference is that now consumers have an opportunity can resell their physical book, either in the condition of new books or used books. (see figure 2). We analyze the impact of bundling a physical book with its e-version on publishers, retailers and consumers under two cases: Monopoly and Competition.
Free E-version scenario under monopoly

Following the setting of previous example and suppose there is one publisher $S$ and one monopolistic retailer $R$ who sells a new physical book bundled with a free e-version. However, only consumers who purchase the new book from the retailer $R$ have access to the e-version. We here assume that having an electronic version will not bring disutility to the consumer, thus every rational consumer will ask for a free electronic version.

Assume consumers’ valuation $\theta$ follows a uniform distribution $U(0, bV_e + V_b)$, where $V_b$ refers to the upper bound of valuation for a physical book alone and $V_e$ refers to the upper bound of valuation for an electronic version alone. We also assume that among all combinations of versions of books, the bundle of physical book with its digital version gets the highest valuation, which implies that retailer price is higher than resale price: $p_b > p_r$. In addition, used physical version possesses a higher valuation than electronic version, which means that prices of used books are higher than the upper bounds of electronic versions. In other words, we have $p_b > p_r > p_u > V_e$ (see figure 3). Finally, we assume that consumers always purchase the closest version with a lower price than her valuation.

Similarly, suppose $\alpha$ percentage of consumers who purchase new books will resell them as used books to other consumers at price $p_u$ and consumers who purchase the bundle of new physical book and its e-version resell the physical book alone as new book at price $p_r$. Both types of resale are through the retailer with a commission rate $k$.

Consumers have the following strategies according to their valuation:

1. **Buy and Keep** ($1 - \alpha$)($p_r < \theta \leq V_b + V_e$)
   For consumers whose valuation $\theta$ lies between the retailer’s price of the bundle $p_b$ and upper bound $V_b + V_e$, their strategy is to purchase the bundle of both physical version and e-version of the book. Assume after reading, $\alpha$ percent of the consumers who purchased the bundle will resell their physical books as used books, then the $(1 - \alpha)$ percent of consumers who will not sell the physical version will adopt “Buy and Keep” strategy. Note that electronic version cannot be sold.

2. **Buy and Resell as used** $\alpha (p_r < \theta \leq V_b + V_e)$

![Figure 2: Model under free E-version case](image)

![Figure 3: Consumer segmentation by valuation](image)
As mentioned above, \( \alpha \) percent of the consumers whose valuation \( \theta \) lies between the retailer’s price of the bundle \( p_b \) and upper bound \( V_e + V_e \) will sell their physical books as used books.

(3) **Buy and Resell as new** \( p_b - (1 - k)p_r \leq \theta < p_u \)

Incorporating consumers who take this strategy is one major contribution of our research. We argue that under this scenario, even consumers who have a much lower valuation than the price of the bundle of physical book and its e-version may make a purchase. For consumers whose valuation \( \theta \) lies between \( p_b - (1 - k)p_r \), the difference of the price of bundle plus commission and resale price, and consumers resale price of used books \( p_u \), their strategy is to buy the bundle of physical book and its e-version and sell the physical book as new book at a lower resale price \( p_r \).

(4) **Buy New from Resale** \( p_r \leq \theta < p_b \)

For consumers whose valuation \( \theta \) lies between the resale price of new physical book \( p_r \) and retailer’s price of the bundle \( p_b \), they will not buy a physical new version bundled with an e-version from the retailer. Instead, they will buy the new physical book sold from consumers who take strategy “Buy and Resell as new”.

(5) **Buy Used from Resale** \( p_u \leq \theta < p_r \)

For consumer whose valuation \( \theta \) lies between the consumers’ resale price of used physical book \( p_u \) and consumers’ resale price of new physical book \( p_r \), they will buy a resale of new physical book alone from consumers who take strategy “Buy and Resell as new” but an used physical book alone.

(6) **Not Buy** \( \theta < p_b - (1 - k)p_r \)

This type of consumers will not buy any version of the book because of their low valuation on the book.

We assume that the consumers who adopt “Buy and Sell as New” and “Buy new from Resale” will take their action before the consumers who adopt “Buy used from Resale”. The rational here is that for consumers who just want an e-version, they will sell the physical book as new book as soon as possible. Only until then can they release the worry that they will have a disutility of \( p_b - (1 - k)p_r \) and can not get their money back. Since this type of resale happens early, we assume that for each consumer who takes the strategy of “Buy and Sell as New”, there will be always another consumer who takes “Buy new from Resale” to complete the resale. In equilibrium, we get the following equations:

\[
\begin{align*}
\{ & p_b - p_r = p_u - (p_b - (1 + k)p_r) = V_e \\
& p_u = p_r - \alpha(V_b + 2V_e - p_b)
\end{align*}
\]  

(7)

The first equation comes from the fact that the number of consumers who take “Buy and Sell as New” is the same as those who “Buy new from Resale”. Based on the discussion above, this equation implies that the demand and supply of new physical book alone will be the same. Besides, the portion of consumers who want to have e-version will be \( V_e \) because of the range of e-version valuation \((0, V_e)\). The second equation implies that the supply of used books is guaranteed to be met by the demand of used books.

Using the similar approach of the benchmark scenario, we compute the revenue of retailer and publisher and the surplus of customers under the free e-version scenario under monopoly. We derive our findings in the following propositions:

**Proposition 1**: At equilibrium of free e-version scenario under monopoly, the publisher sells to retailer at a lower price \( w^* < w^* \) and the publisher’s profit will increase if and only if the price for digital rights satisfies the following condition:

\[
C > \frac{(1 - \alpha)k(2 + k + \alpha k)V_b^2 + 2k(1 + \alpha k)(1 + \alpha)^2V_bV_e - (1 + \alpha)^4k^2V_e^2}{8(1 + \alpha k + \alpha^2k)},
\]

where \( C \) is the copyright expense. (Proof is provided in Appendix)

The managerial insight here is that although the publisher may experience lower revenue on new book sales, she will be compensated by the revenue of signing copyright contracts which make digitalization available. Thus, there is no incentive from the publishers’ point of view to impede such business model. In fact, from what we see from Amazon, the number of books this model covers increases steadily.

**Proposition 2**: Under free e-version scenario under monopoly, the retailer sells to consumers at a higher price \( p_b^* > p_b^* \) and has a higher profit with a larger segment of consumers covered. (Proof is provided in Appendix)
The managerial insight here is that since retailer adds value to the book by providing its digital version, the retailer can charge a higher price on consumers. Besides, such business practice also attracts extra segment of consumers because digitalization makes it possible for resale of brand new books. Thus, retailers need not worry about the substitution effect that this model may bring to the market. Although in some senses, resale of new physical book will substitute demand of the bundle, but the total revenue will increase due to the new market share.

**Proposition 3**: Under free E-version scenario, consumers have a higher total surplus compared with no E-version scenario. In particular, the surplus of consumers due to e-version out weights the loss of surplus due to the price change. (Proof is provided in Appendix)

This result shows that total surplus of consumers will benefit from this model regardless of their evaluation of the books. Thus, we argue that under the scenario of monopoly, all players will benefit from introducing this new bundling model into the market.

**Impact of free E-version under competition**

In this subsection, we focus on the scenario of competition between two retailers. Following the previous example, we still have one publisher $S$ and two online retailers: $R_1$ provides free e-version while $R_2$ does not. Therefore, consumers’ valuation distribution from $R_1$ is $U(0, V_b + V_e)$ which is wider than that from $R_2$: $U(0, V_b)$. Besides, we assume that consumers from both retailers can resell physical books. However, $R_1$’s consumers can resell physical books as new and used books and keep the e-version while $R_2$’s consumers can only resell physical books as used books. We also assume that price for new book from $R_2$ is less than the price for the bundle of new physical version and e-version $p_{b1} > p_{b2}$ and resale price of used books through both retailers are the same: $p_u^1 = p_u^2$ (see figure 4).

![Figure 4: Consumer segmentation by valuation under competition](image)

**Proposition 4**: Under the competition scenario, the upper bound of price of new book $p_2$ from retailer $R_2$ who does not provide free e-version sells is $p_1 - V_e$. Besides, consumers who take “Buy and sell as New” strategy will sell the new physical book at the same price of retailer $R_2$, $p_c = p_b^2$.

Proof: From previous discussion, we know that the range of consumers’ valuation on E-version follows $U(0, V_e)$. Therefore, consumers’ range of selling new physical books also follows $U(0, V_e)$. We assume that consumers always choose the version of the book whose price is lower than but the closest to their valuation. When there are multiple prices for the same version, they will always choose the lowest price. Suppose $R_1$ decides retail price $p_{b1}$ first and $R_2$ decides $p_{b2}$ later. We first discuss the upper bound of new book price of $R_2$ with the price order: $p_{b1} - V_e < p_{b2} < p_{b}$

Since the upper bound of resale price of physical books as new books $p_r$ from $R_1$’s consumers is $p_{b1} - V_e$, which is lower than the retail price of new physical book $p_{b2}$ from $R_2$, then consumers in valuation region $(p_{b1} - V_e, p_{b1})$ will always buy from $R_1$’s consumers’ resale at the price $p_r$ and $R_2$ can not make any sales unless he reduce the price to no higher than $p_{b1} - V_e$ and consumers who resell physical books as new books observe the sale price of retailer $R_2$. Thus, the new book price of $R_2$ is $p_{b1} - V_e$.

We discuss the following two cases to show that consumers’ best strategy is to sell at the same price as that of retailer $R_2$. 

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*Chen Analysis of Book Resale: A Game-theory Approach*

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In this case, resale price of physical books as new books \( p_r \) from \( R_1 \)'s consumers is higher than the retail price \( p^2_b \) from \( R_2 \). Then consumers in valuation region \((p^2_b, p^1_b)\) will always buy from \( R_2 \) and \( R_1 \)'s consumers can not make resale at the price \( p_r \). Thus, \( R_1 \)'s consumers have to reduce their price to no higher than the price of retailer \( R_2 \), which implies \( p^2_b \leq p_r \).

Case 2: price order: \( p_r < p^2_b < p^1_b \)

In this case, resale price of physical books as new books \( p_r \) from \( R_1 \)'s consumers is lower than the retail price \( p^2_b \) from \( R_2 \). Then consumers in valuation region \((p_r, p^1_b)\) will always buy from \( R_1 \)'s consumers and retailer \( R_2 \) can not make any sale, which will force retailer \( R_2 \) to lower its price. However, consumers who resell their physical book as new books do not want to experience a decrease of retailer \( R_2 \)'s price, because that will also decrease their utility because they will collect less money back. This implies \( p_r \to p^2_b \). Combined with case one, we conclude that \( p_r = p^2_b \). With deterministic values for both \( p_r \) and \( p^2_b \), it is trivial to show that the retailer \( R_2 \) will have a lower market share compared with retailer \( R_1 \) following the similar approach in previous section.

**CONCLUSION**

Conventional researches view resale market as another competitive sales channel for both book publisher and retailer in terms of profitability. However, such competition is only between new books and used books. Along with the emergence and development of electronic book in retail industry of books, such competition extends to digital channel, in which marginal cost is extremely low and the concept of used book disappears.

The conventional wisdom suggests publisher and retailer consider completely banning resale of electronic book by methods that holding copyrights. In this paper, based on Amazon's practice of introducing a new model of bundling physical book and its e-version, we consider an alternative where publisher and retailer provide a bundle of new book with its e-version to consumers, which can increase publisher and retailer's profit and consumers' surplus. In other words, bundling new book with its digital version can be a win-win solution for publisher, retailer and consumers. As for the pricing strategy, the retailer can choose the optimal price based on estimate of parameters like values of physical and electronic books. Another contribution of our paper is that we show that bundling an e-version with physical book can provide a competitive advantage for retailers in the market over those who do not have that service. As one of the first papers exploring resale of homogenous items because of digitalization, this study has several limitations that future research may address. First, this paper does not consider consumers who purchase used books resell used books. We neither incorporate book-renting market to avoid over-complexity in the model. Second, we assume a free e-version case, but a retailer could potentially adopt her own pricing strategy for on e-version based on dynamic sales. Third, future work in this research area might study scenario of more complex competition, such as two retailers provide different digitalization versions with different functions. It will be interesting to study whether our results can be applied to those electronic goods.

**REFERENCES**


Online resources:

**Appendix**

*Proposition 1:* At equilibrium of free E-version condition, the publisher sells to retailer at a lower price \( w^e^p < w^e \) and the publisher’s profit will increase if and only if the price for digital rights satisfies the following condition:

\[
C > \frac{(1-\alpha)k(2+k+\alpha k)V_b^2 + 2k(1+\alpha k)(1+\alpha)^2 V_e V_e - (1+\alpha)^4 k^2 V_e^2)}{8(1+\alpha k + \alpha^2 k)}
\]

Proof: Under Free E-version scenario:

\[
\begin{align*}
  p^e_b - p^e_r &= p^e_v - (p^e_b - (1+k)p^e) = V_e \\
  p^e_v &= p^e_v - \alpha(V_b + 2V_e - p^e) \\
  \Rightarrow \quad p^e_r &= (1+\alpha) p^e_b - \alpha V_b - (2+\alpha) V_e
\end{align*}
\]

Publisher’s revenue: \( \pi^e_r = D^e_b w^e + C = (V_b + V_e - p^e_r)w^e + C \), where \( C \) is the copyright expense.

Retailer’s revenue includes two parts: (1) revenue from new physical books sale, (2) commission from the customers’ resale of both new and used physical books.

\[
\pi^e_r = (V_b + 2V_e - p^e_b)(p^e_b - w^e) + k \cdot V_e (p^e_b - V_e) + k \cdot \alpha (V_b + 2V_e - p^e_b) \cdot (p^e_r - \alpha(V_b + 2V_e - p^e)) - C
\]

\[
= (V_b + 2V_e - p^e_b)(p^e_b - w^e) + k \cdot V_e (p^e_b - V_e) + k \cdot \alpha (V_b + 2V_e - p^e_b) \cdot ((1+\alpha)p^e_b - \alpha V_b - (1+2\alpha) V_e) - C
\]
Set \( \frac{\partial \pi_b}{\partial p_b} = 0 \), we get
\[
p_b^* = \frac{(1 + \alpha k + 2\alpha^2 k)V_b + (2 + k + 4\alpha k + 3\alpha^2 k)V_e + w}{2(1 + \alpha k + \alpha^2 k)}
\]

Then replace \( P_b \) into publisher’s profit function.

Set \( \frac{\partial \pi_b}{\partial w} = 0 \), we get
\[
w^* = \frac{1}{2}((1 + \alpha k)V_b - (1 + \alpha)^2 kV_e) < w^* = \frac{1}{2} V_b
\]

\[
p_e^* = \frac{(3 + 3\alpha k + 4\alpha^2 k)V_b + (4 + k + 6\alpha k + 5\alpha^2 k)V_e}{4(1 + \alpha k + \alpha^2 k)} > p_b^* = \frac{(3 + 3\alpha k + 4\alpha^2 k)V_b}{4(1 + \alpha k + \alpha^2 k)}
\]

Thus, the publishers’ profit is:
\[
\pi_b = (V_b + V_e - p_e^*)w + C = \frac{((1 + \alpha k)V_b - (1 + \alpha)^2 kV_e)^2}{8(1 + \alpha k + \alpha^2 k)} + C
\]

The retailers’ profit is
\[
\pi_e = (V_b + 2V_e - p_b^*)(p_e^* - w) + k \cdot V_e (p_b - V_e)
\]

\[+ k \cdot \alpha (V_b + 2V_e - p_b) \cdot ((1 + \alpha) p_b - \alpha V_b - (1 + 2\alpha) V_e) - C
\]

\[= \frac{1}{16(1 + \alpha k + \alpha^2 k)}.\]

\[
\{V_b^2 + 2(4 + 7k)V_bV_e + (4 + k)^2 V_e + 3\alpha^4 k^2 V_e (4V_b + 3V_e) + 2\alpha^2 k^2 (2V_b^2 + 7V_bV_e + 6V_e (-\alpha V_b + V_e) + \alpha k ((4 + k)V_b^2 + 2(11 + 8k)V_bV_e + 2(12 + 7k)V_e^2 - 4\alpha k V_b (V_b + 2V_e)) + 2\alpha k (V_b^2 + (6 + 7k)V_b V_e + 2(4 + 3k)V_e^2 - 2\alpha V_b (V_b + (4 - k) V_e)))\}
\]

We compare the profit of both publisher and retailer under the No E-version case and E-version case. The profit difference for publisher:
\[
\pi_b^e - \pi_b = \frac{(1 + k)^2 V_b^2}{8(1 + \alpha k + \alpha^2 k)} - \frac{((1 + \alpha k)V_b - (1 + \alpha)^2 kV_e)^2}{8(1 + \alpha k + \alpha^2 k)} - C
\]

\[
\begin{cases}
> 0 & \text{if } C < \frac{((1 - \alpha) k (2 + k + \alpha k)V_b^2 + 2k(1 + \alpha k)(1 + \alpha)^2 V_b V_e - (1 + \alpha)^4 k^2 V_e^2)}{8(1 + \alpha k + \alpha^2 k)}
\\
\leq 0 & \text{if } C \geq \frac{((1 - \alpha) k (2 + k + \alpha k)V_b^2 + 2k(1 + \alpha k)(1 + \alpha)^2 V_b V_e - (1 + \alpha)^4 k^2 V_e^2)}{8(1 + \alpha k + \alpha^2 k)}
\end{cases}
\]

**Proposition 2:** Under free e-version scenario under monopoly, the retailer sells to consumers at a higher price \( p_e^* > p_b^* \) and has a higher profit with a larger segment of consumers covered.

Proof: As shown in proof of Proposition 1, the coverage of consumers in E-version case is
\[
(1 + \alpha) \frac{(1 + \alpha k)V_b - k(1 + 2\alpha + \alpha^2)V_e}{4(1 + \alpha k + \alpha^2 k)} + 2V_e.
\]
which is greater than the coverage of consumers under no E-version scenario 
\[
\frac{(1 + \alpha)(1 + \alpha k)}{4(1 + \alpha k + \alpha^2 k)} V_b
\]

The profit difference for retailer is as follows:
\[
\pi_R - \pi_R^e = \frac{(1 + \alpha k)^2 V_b^2}{16(1 + \alpha k + \alpha^2 k)} - \frac{1}{16(1 + \alpha k + \alpha^2 k)}.
\]
\[
\{V_b^2 + 2(4 + 7k)V_b V_c + (4 + k)^2 V_c^2 + 3\alpha^4 k^2 V_c (4V_b + 3V_c) + 2\alpha^3 k^2 (2V_b^2 + 7V_b V_c + 6V_c (\alpha V_b + V_c)) + \alpha^2 k ((4 + k) V_b^2 + 2(11 + 8k)V_b V_c + 2(12 + 7k)V_c^2 - 4\alpha k V_c (V_b + 2V_c))
\]
\[
+ 2\alpha k (V_b^2 + (6 + 7k)V_b V_c + 2(4 + 3k)V_c^2 - 2\alpha V_c (V_b + (4 - k)V_c)) \%
\]

After simplification of the equation above, we get \(\pi_R - \pi_R^e < 0\). Thus, retailer under free E-version scenario has a higher profit compared with the no E-version scenario.

**Proposition 3:** Under free E-version scenario, consumers have a higher total surplus compared with no E-version scenario. In particular, the surplus of consumers due to e-version out weights the loss of surplus due to the price change.

Proof: The consumers’ total surplus before E-version service is:

**Total surplus = surplus from purchasing new book + surplus from purchasing used books**

Let \(W\) denote the surplus.

\[
W = W_u + W_b = (V_b - p_b)(\frac{V_b - p_b}{2}) + \alpha (V_b - p_b)(1 + \frac{\alpha}{2}) p_b - \frac{\alpha}{2} V_b
\]

Since in equilibrium, we have \(p_b^* = \frac{(3 + 3\alpha k + 4\alpha^2 k) V_b}{4(1 + \alpha k + \alpha^2 k)}\)

Thus, \(W = \frac{(1 + 7\alpha^4 k^2 + 6\alpha + 2\alpha k + 6\alpha^3 k (1 + k) + \alpha^2 (-1 + 12 k + k^2)) V_b^2}{32(1 + \alpha k + \alpha^2 k)}\)

The total surplus after E-version is as follows:

**Total surplus = surplus from new book purchase + surplus from used books purchase + surplus from E-version**

\[
W^E = W_u^e + W_b^e + W_c = (V_b + V_c - p_b^e)(\frac{V_b + V_c - p_b^e}{2}) + \alpha (V_b + V_c - p_b^e)(\frac{\alpha}{2} - V_c^e)
\]

\[
+ \alpha (V_b + V_c - p_b^e)(\frac{\alpha}{2} (V_b + V_c - p_b^e)) + V_c^e (kp_b^e + (\frac{3}{2} - k)V_c^e)
\]

In equilibrium, we have \(p_b^e = \frac{(3 + 3\alpha k + 4\alpha^2 k) V_b + (4 + k + 6\alpha k + 5\alpha^2 k) V_c}{4(1 + \alpha k + \alpha^2 k)}\)

After simplification of the previous equation, we have \(W^E > W\). Thus, consumers have a higher surplus.