Discrete Event Systems: A Framework for Man-made Systems

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Discrete event systems: A framework for man-made systems

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In this short paper the followings are demonstrated briefly. 1) The discrete event systems specification, DEVS for short, provides a universal representation for DES if it is reduced and reachable. (2) A business transaction system that is a multicomponent DEVS is a framework to describe operating systems of organizations that consist of human work and computer-communication systems.

1 Introduction

In recent societies, computers are so popular that they are not only tools in workplaces but indispensable components in human organizations. Database and decision support systems are incorporated in the process of R&D, production lines, management control systems, and so on. Operating systems in business organizations are thought to be "soft," because their logically possible design can be realized in many ways. In reality, many concrete forms, for example, for the way of inventory control or report documents and slips are used by different organizations even in the same business area. We will see some important areas and mechanisms of man-made systems in presentation of the paper.

A concrete description of man-made systems will not be easy to reuse in its pure form, because the same situation will never arise in human society. So theoretical abstraction in model building for methodologies is inevitable. If we could make suitable abstractions of concrete systems, then they would serve as a basis of future application of the essence of the systems mechanism. Therefore, we believe that a meta-theory for soft science and technology is important. Discrete event systems are widely observed in the control models of production lines, computer simulation programs, Petri nets, information system methodologies, and the soft systems methodology. Some model are formal while others informal. No one seems to know how one discrete event model different from others or in what extent the model is standard. This is the reason why we need a unified realization theory for discrete event systems, which we demonstrate in this short paper.

2 DEVS state space representation

In this section the state space representation for a legitimate DEVS is defined. It is shown that the resultant state system ST of the state space representation is really a discrete event system in the sense of Section 2.1. We will follow the same set-theoretic notation as Masarovic et.al.(1975, 1989).

A discrete event system specification DEVS is defined below.

Definition 1: Discrete event system specification :DEVS (Zeigler, 1976)

A discrete event system specification (DEVS) is a sextuple

\[ M = \langle A_M, S_M, B_M, \delta_M, \lambda_M, \tau_a \rangle \]

where \( A_M \) is a set called the external event set, \( S_M \) is a set called the sequential states set, \( B_M \) is a set called the output value set, \( \delta_M \) is a function called the quasi-transition function, \( \lambda_M \) is a set called the output function, and \( \tau_a \) is a function called the time advance function with the following properties:

(a) \( \tau_a: S_M \rightarrow \mathbb{Z}^+ \)

(b) \( \delta_M: Q_M \times (A_M \cup \{ \lambda \}) \rightarrow S_M \)

(c) \( \lambda_M: Q_M \rightarrow B_M \)

Some simple examples of DEVS are shown in the presentation for this paper.

A state space representation is a wide-spread framework to recognize dynamics of a time system in a causal way. Masarovic and Takahara (1975, 1989) shows that a time system is causal if and only if it has a state space representation and that if a time system is past-determined then it is causal.

Definition 2: State space representation (Masarovic and Takahara, 1989)

Let \( S \subseteq X \times Y \) be a time system with the input alphabet \( A \) and the output alphabet \( B \). Let \( C \) an arbitrary set. \( C \) is a state space for \( S \) if and only if there exist a family of functions \( \Phi \in (\Phi_t^\tau | \Phi_t: C \times X_t^\tau \rightarrow C \) and \( t, t' \in T \) \) and a function \( \mu: C \times A \rightarrow B \) such that

(i) \( S = \{(x, y) | \) there exists some \( c \in C \) such that \( y(t) = \mu(\Phi_t(c, x)), x(0) \) for any \( t \in T \})

(ii) for any \( t, t', t'' \in T \), \( t \leq t' \leq t'' \)

(a) \( \Phi_t^{t''}(c, x_{t''}) = \Phi_t^{t''}(\Phi_{t''}(c, x_{t''}), x_{t''}) \)

(b) \( \Phi_t^\tau(c, x_t^\tau) = \mathcal{E} \)

(c) \( \Phi_t(c, x_t) = \Phi_t^\tau(c, \sigma^{-1}(x_t^\tau)) \)

where \( x_t^\tau = x_t^{t''} \times x_{t''} \) and \( t = t' - t. \)

The pair \( \Phi, \mu \) is called a (time invariant) state space representation of \( S \), and \( \Phi \) a time invariant transition family.

3 Realization of general discrete event system

Below, a way how to construct state space representations for the class of general discrete event systems is presented.

A discrete event system is a special time system defined as follows:

Definition 3: Discrete event system (Sato, Praehofer, and Pichler, 1995)
If a time system $S \subseteq X\times Y$ satisfies the following four conditions then it is called a discrete event system:

(i) $S$ is strongly stationary.

(ii) $S$ is past-determined from $k \in T$.

(iii) The input space $X$ is a discrete event input space. That is,

(iii-1) The input alphabet $A$ for $X$ is $P(A')$, where $A'$ is a finite set, and $P(A')$ the power set of $A'$.

(iii-2) The constant valued function $\Lambda$ is in $X$, where

$$\Lambda(t) = \Lambda$$

for any $t \in T$.

(iv) $S$ has the discrete event-determinacy: that is, for any $(x^k, y^k) \in S^k$, $(x^k, y^k) \in S^k$, $c \in S(\Lambda)$, $x' \in X$ and $y' \in Y$, if $(x^k, y^k, 0^k, c(c)) \in S$, $(x^k, y^k, 0^k, c(c)) \in S$ and $(x^k, y^k, 0^k, c(c)) \in S$ hold then $(x^k, y^k, 0^k, c(c)) \in S$.

Taking the set $A'$ as the tasks to be executed or processed by the system, the condition (iii) shows parallel execution of tasks. $\Lambda$ is simply denoted by $\Lambda$. The discrete event-determinacy in (iv) shows that $S(\Lambda)$ virtually represents the "internal state" of $S$ which is determined by the past-determinacy.

In the following of this short paper, $S \subseteq X\times Y$ is an arbitrarily fixed discrete event system that is past-determined from $k$.

Let $S(\bar{X}) = \{ y \mid \exists x^k, \exists y^k, (x^k, y^k) = S(\bar{X}) \}$. We can prove the fact that $S(\bar{X}) = S(\Lambda)$ holds. Based on this fact and the discrete event-determinacy, the following function can be defined for a discrete event system. Define the function $p_0 : S(\Lambda) \times X \to Y$ by the correspondence: $p_0(c, x) = y$ if and only if there exists $(x^k, y^k) \in S^k$ such that $(x^k, y^k) = (c, y)$ in $S$ and $(x^k, y^k) \in S$. Let $p_0$ be the initial response function of $S$. That is, $(x, y) \in S$ if and only if there exists some $c \in S(\Lambda)$ such that $p_0(c, x) = y$.

Proposition 1

The function $p_0$ defined above is an initial response function of $S$. That is, $(x, y) \in S$ if and only if there exists some $c \in S(\Lambda)$ such that $p_0(c, x) = y$.

Proposition 2

The function $p_0$ is causal and reduced.

Now we can define the state space representation for a discrete event system.

Let $t \in T$ be arbitrary. Define $\phi(t)^s : S(\Lambda) \times X_{0t} \to S(\Lambda)$ by $\phi(t)^s(c, x_{0t}) = \lambda_t(p_0(c, x_{0t}))$. Let $\phi(t)' : S(\Lambda) \times X_{t'} \to S(\Lambda)$ and $t', t \in T$, $t < t'$, such that $\phi(t)'(c, x_{t'})$ is defined as $\phi(t)'(c, x_{t'}) = \phi(t)^s(c, x_{0t})$ and $t = t'$. Define a function $\mu : S(\Lambda) \times A \to B$ as $\mu(c, a) = p_0(c, x(0))$, where $x$ is an arbitrary but $x(0) = a$. Since $p_0$ is causal $\mu$ is well-defined.

Proposition 3

The pair $(\phi, \mu)$, defined above, is a (time invariant) state space representation of $S$.

4 The uniqueness problem of representation for discrete event systems

The definition of the uniqueness problem of representation for a system $S$ and its importance are stated in Mesarovic and Takahara (1989) as follows.

The uniqueness problem of representation:

Given a time system $S$, find conditions on $S$ under which $S$ has a unique state space representation up to isomorphism.

They point out that "the state space approach is meaningless for the analysis of a system unless the uniqueness problem is solved in a positive way; otherwise the whole family of representations of $S$ which may be differ in various degrees must be used simultaneously whenever the system is being investigated." In order to provide the answer to the problem of representation for discrete event systems, the uniqueness of $S(\Lambda)$-realization up to isomorphism is shown in this section.

We need a way to compare a dynamical system representation with others.

Definition 4: Morphism (Mesarovic and Takahara, 1989)

Let $\phi_0, \phi_0'$ and $\phi_1, \phi_1'$ be time invariant dynamical system representations of $S$. Then a mapping $h : C \to C$ is called a morphism from $\phi_0$ to $\phi_0'$ if the diagrams in Fig. 1 are commutative. That is, for any $t \in T$, $c \in C$ and $x \in X_0$, it holds that $p_0(c, x_0) = \phi_0(h(c), x_0)$ and $p_0'(h(c), x_0) = \phi_0'(h(c), x_0)$. If $h$ is bijective then $\phi_0, \phi_0'$ is called isomorphic to $\phi_1, \phi_1'$.

A morphism $h$ from $\phi_0, \phi_0'$ to $\phi_1, \phi_1'$ is denoted by $h : \phi_0, \phi_0' \to \phi_1, \phi_1'$. If there is a morphism from $\phi_0$ to $\phi_1, \phi_1'$ that is not surjective, then $\phi_0, \phi_0'$ can be "embedded" into $\phi_1, \phi_1'$. Thus we can intrinsically think of that $\phi_1, \phi_1'$ is possibly bigger and then redundant than $\phi_0, \phi_0'$. If a morphism is surjective, then $\phi_1, \phi_1'$ is smaller than $\phi_0, \phi_0'$.

We can have the dynamical system representation defined by $S(\Lambda)$-realization. In the following the dynamical system representation is also called $S(\Lambda)$-realization and is denoted by $\phi_0, \phi_0'$. For discrete event systems the following property of dynamical system representations is imposed.

![Fig. 1. Commutative diagram](image)
Definition 5: Discrete event-response function

Let a function $\rho_0: C \times X \rightarrow Y$ be an initial response function of $S$. It is called a discrete event-response function of $S$ if it satisfies the following condition: For any $c \in C$ and $x \in X$ there exist $c^* \in C$ and $k^* \in X^k$ such that $\rho_0(c, A) = \lambda^* \rho_0(c^*, \lambda^* A_k)$, $\rho_0(c, x) = \lambda^* \rho_0(c^*, \lambda^* A_k(x))$ and $\rho_0(c^*, \lambda^* A_k(0), k_0) = \rho_0(c^*, \lambda^* A_k(0), k)$. 

If $\rho_0$ of a dynamical system representation $\langle p, \Phi \rangle$ of $S$ is a discrete event-response function then $\langle p, \Phi \rangle$ is called a discrete event dynamical system representation of $S$.

The following is the main theorem on the uniqueness problem of representation for discrete event systems.

**Theorem 1**

Let $\langle p, \Phi \rangle$ be a discrete event dynamical system representation of $S$ and $C$ its state space. Then there always exists a surjective morphism $h: \langle p, \Phi \rangle \rightarrow \langle p^*, \Phi' \rangle$, which is defined by $h(c) = \rho_0(c, A)$.

5 The uniqueness of DEVS

As an application of the theory it can be shown that a reduced and reachable DEVS is unique up to isomorphism in the class of discrete event dynamical system representations.

We can naturally define a time-invariant state space representation $\langle p, \Phi \rangle$ from a legitimate DEVS. We define the system $S_D$ for a legitimate DEVS $M$, which shows the state transition of $M$ (Sato, Praehofer, and Pichler, 1995).

$$(x, y) \in S_D \subseteq X \times (Q_M)^T \text{ iff there exists } (s, e) \in Q_M \text{ such that } y(t) = \mu_{\Phi_0}(s, e, x^t), x(t) \text{ for any } t \in T.$$

Since $S_D$ represents the state transition mechanism of a legitimate DEVS $M$, it is called the state system of $M$. The pair $\langle p, \mu \rangle$ for $S_D$ defined from a legitimate DEVS is called the DEVS state space representation (of $M$).

The importance of the legitimacy is that the DEVS state space representation is always possible. This fact can be stated as follows.

**Theorem 2 [Zeigler, 1976]**

Let $M$ be a legitimate DEVS. Then $M$ can be extended to a time invariant state space representation.

Since the DEVS formalism has been used so widely as a theoretical framework for modeling and discrete event simulation, it is believed that the definition of DEVS specification is natural and universal in some sense. This is true as we see below.

**Theorem 3**

If the dynamical system representation that is defined by the DEVS state space representation for a legitimate DEVS is reduced and reachable then it is isomorphic to $Sp\lambda()$-realization, where $S_D$ is the state system of the legitimate DEVS.

6 DEVS in man-machine systems

Sato and Praehofer (1995) formulated a dynamic mechanism of transaction processing as a special multicomponent DEVS, original form of which is defined by Zeigler (1984). The model proposed is called a business transaction system. It consists of both static and dynamic structures. The former depicts the interconnection of transactions and intermediate inventories in business tasks. The inventories are not only real materials but the records in a file system. The dynamic structure is constructed as a multicomponent DEVS. The state space consists of a file system and a schedule of internal transactions.

We can see the skeletal structure of a business transaction systems by drawing its static structure. Examples of business transaction systems are:

(a) Materials Requirements Planning (MRP)

The aim of the control by MR is to attain zero-inventory operation of a production line. The MRP controller is a mechanism to synchronize parallel processes such as subcontractors, vendors and assemblies.

(b) Kanban system

In the control by the kanban system, orders for production are issued only to the final process in the production systems.

Kanbans are used between two sequential processes and one kanban corresponds to the name and small number of parts or products. When prescribed number of kanbans are accumulated in its holding place for a kanban then an order for production is issued to the preceding process. The total number of kanbans are constants. The kanban system is also called Toyota system.

(c) Sales and invoicing subsystem

Business documents and ledgers are accumulated and then consists of part of the state of the system.

7 Conclusion

The realization theory provides an answer to the uniqueness problem of representations for discrete event systems. Business transaction system is a DEVS framework for man-made systems in organizations.

**Reference**


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