Valuation of Discount Options in Software License Agreements

Contractually agreed discounts on optional services in a software license agreement, so-called discount options, are often valued inaccurately or too highly, whereby the purchase of a discount option can be disadvantageous for businesses. A discount option is similar to the option structure capped call option on the financial market, the value of which can be determined through established financial valuation methods (analytically or numerically) from the (real) option pricing theory. However, the value of a discount option at the time of service consumption only corresponds to the present value of the maximum discount granted by the provider in certain cases. A basic understanding of the economic effect is a necessary condition for optimizing the contract in terms of utility maximization, not only in the context of software licensing.

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1 Introduction

The expenditure on software increased further in 2009 despite the economic cri-
the options are not always exercised to the extent that is possible, even though the benefits are free and available. This is partly because the advantage achieved from the training does not in all cases compensate the necessary expenditures of the company (travel costs, lost productivity, ...).

Similarly to the software license agreement, discount options can also be present in many other contractual relations. The economic advantage of such an option can only be quantified for the supplier and the beneficiary if the included uncertainties are realized and their effects are understood and evaluated with suitable methods. Previous approaches and contributions to the evaluation of options in software agreements are not established very methodically, they underestimate the uncertainties in the factors for the licensee, and they falsely determine the value on basis of the discount. A survey (Giera 2007) shows that as a result the saving potential is systematically overestimated and licensees frequently pay more for the options than they profit from them in the long run.

On the basis of the real options theory, this article presents a quantitative approach where the discount options in a software license agreement can be valued under the assumption of certain conditions. Through the understanding gained, a licensee can understand the economic impact of these contract components, interpret option values, and consult these during decisions. The developed model enables the application of classical, analytical evaluation methods and numerical solutions. The second chapter introduces the subjects of software licensing, finance and real options based on literature sources. Following this, the model and the results it has produced are presented. The fourth chapter explains a numerical solution method using a case study, before the limitations and possible extensions of the valuation model are discussed critically in the summary and the outlook.

2 Software License Agreements and Real Options

Software functionality is an IT service which has to be specified in the company according to the process requirements and then has to be developed or procured from the market. Since the software deals with protected intellectual property as opposed to personnel services or hardware, the use of license-requiring standard software is often governed by a legally binding software license agreement.

2.1 Software License Agreements

Software licensing is different from buying software in that the licensee will only be granted the right to use a copy of the software for a specific period of time, as agreed in the license contract (see Stapperfend 1991, pp. 87–94; Sedlmeier 2006, p. 10). An open-ended license (perpetual), a temporary license (subscription), and a strictly usage-based license (on-demand) are distinguished depending on the length of the usage period (Zhang and Seidmann 2009). Different usage periods and terms of use are offered by the provider in the form of different types of licenses. Approaches to the optimal choice and combination of different types of licenses for different requirements in the company are presented in the work of Järvinen et al. (2007) and Gull and Wehrmann (2009). A few articles and studies have also already dealt with the advantageousness of discounted contractual services and discount options in license agreements. Thus, Giera (2004, pp. 4–8) defines a flat-rate percentage or amount for various services independent of the company which can be saved with the option, and derives qualitative recommendations for action. Gartner (2006, p. 5) goes a step further and estimates the required length of the usage period based on the point in time the service is used, on the amount of the discount, and on the option price, without taking other uncertainties into account. Microsoft provides its customers with the “Software Assurance Renewal Planning Guide” (Microsoft 2008) which only indicates the maximum achievable savings with “…may save up to…” or “…may reduce costs…” The same applies to the “Benefits Calculator” (Microsoft 2009) offered on the website, which calculates the sum of the type and amount of services included in the contract with the full discount. This ultimately leads to the exaggerated prices for the services causing a high option value. Regardless of the type of service or service provision, a discount option in a contract is defined as follows: A discount option grants the recipient, directly or by paying a premium, the right, but not the obligation, to obtain a specific service provided by the supplier at a reduced price within the contract period. The model presented in chapter three is based on this general structure.
and thereby enables a systematic valuation of different service characteristics. Services for which a discount option in a license agreement is available or can be agreed upon are presented in excerpts in Table 1 according to their characteristics.

Upgrade and downgrade rights also allow the licensee to use previous or newer versions. Furthermore, some licensing agreements offer the possibility to undertake a system or product change (cross grade), as for example being able to use a Windows application license for a Macintosh or Linux system. In the context of extending the period of use, for instance, a temporary "subscription" license can be converted into an open-ended "perpetual" license at the end of the period. License types can be partly changed for an additional charge at a later date so that a single-user license may be converted into a concurrent use license. As regards the use of virtual systems, some manufacturers also allow the use of a license on multiple systems. With home-use programs, a license purchased by a business may also be used on a private computer at home. In addition, there are often also discount options for a limited amount of discounted user trainings or support services via telephone, Internet or on site for use in the event of a problem. The offered range of services may even include support for the installation and configuration of the software in the company.

Table 1 Extract of discount options in software licensing agreements

<table>
<thead>
<tr>
<th>Software</th>
<th>Terms of use</th>
<th>User support</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upgrade and downgrade</td>
<td>Change of the original license type</td>
<td>User training</td>
</tr>
<tr>
<td>Change in system or product</td>
<td>Multiple use of a license</td>
<td>Support and assistance with problems</td>
</tr>
<tr>
<td>Extension of the usage period</td>
<td>Home-Use-Program</td>
<td>Installation and configuration</td>
</tr>
</tbody>
</table>

The continuous-time Black-Scholes model (BSM) developed by Black and Scholes (1973) is based on a stochastic process (Wiener process) with reference to the growth of the underlying asset and enables, while taking idealized assumptions into consideration, the analytical valuation of European finance options without accounting for dividend payments. By contrast, the Binomial or Cox-Ross-Rubinstein model (Cox et al. 1979) is time-discrete and defines a positive and a negative growth of the underlying asset for each point in time in the form of a binomial tree. As a result of the numerical approach, this model is more flexible than the BSM and is therefore suitable for the valuation of many more types of options. If there are several risk factors combined with complicated option characteristics, then the binomial model quickly reaches its limits with the result that the option value has to be estimated, for example with the help of a Monte-Carlo simulation (Wilkens and Wilkens 2000, pp. 109–134). All option pricing models used on financial instruments are based on the fundamental principle of arbitrage equilibrium (arbitrage-free), i.e. the possibility of achieving a risk-free profit without the use of capital on the financial market is ruled out (cf. Brunner 2004, p. 11). From this principle follows

Table 2 Overview of option type specifications (see Hull 2001, p. 4)

<table>
<thead>
<tr>
<th>Option type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>European option</td>
<td>The right to exercise the option is only possible at the end of the period of use.</td>
</tr>
<tr>
<td>Bermudian option</td>
<td>The right to exercise the option is possible at multiple time points.</td>
</tr>
<tr>
<td>American option</td>
<td>The right to exercise the option is possible anytime during the entire period of use.</td>
</tr>
<tr>
<td>Call-option</td>
<td>The right to buy an underlying asset at the agreed strike price.</td>
</tr>
<tr>
<td>Put-option</td>
<td>The right to sell an underlying asset at the agreed strike price.</td>
</tr>
<tr>
<td>Long call</td>
<td>The buying position of a call option, with the right to buy the underlying asset at the agreed strike price.</td>
</tr>
<tr>
<td>Long put</td>
<td>The buying position of a put option, with the right to sell the underlying asset at the agreed strike price.</td>
</tr>
<tr>
<td>Short call</td>
<td>The selling position of a call option, with the duty to sell the underlying asset at the agreed strike price.</td>
</tr>
<tr>
<td>Short put</td>
<td>The selling position of a put option, with the duty to buy the underlying asset at the agreed strike price.</td>
</tr>
<tr>
<td>Capped call</td>
<td>Also known as “bull spread”. This is a combination of a long call or long put with a lower strike price and a short call or short put with a higher strike price.</td>
</tr>
<tr>
<td>Capped put</td>
<td>Also known as “bear spread”. This is a combination of a short call or short put with a lower strike price and a long call or long put with a higher strike price.</td>
</tr>
</tbody>
</table>
that two financial instruments on the financial market have the same value when they have an identical payment structure, and that risk-free portfolios pay interest at the risk-free market rate. If the stochastic growth of the underlying asset is known for one period (binomial model), the fair price of a finance option can be determined through the option itself when the market is arbitrage-free, through a combination of an underlying asset and a hedge (see Hull 2003, pp. 343–349). An extension of this principle to multiple periods eventually leads to a multi-stage binomial model with a risk-neutral valuation. This requires that the expected return from one period to the next must satisfy the risk-free market interest rate. In the extreme case, the binomial model can, under certain assumptions, eventually be transformed into the time-continuous BSM through infinite reduction of the time intervals (see Wilkens and Wilkens 2000, p. 138; Hilpisch 2006, p. 129; Hommel 2003, p. 261).

2.3 Real Options

Right to exercise in the real economy and the flexibility of action regarding, for example, contracts or investment projects can, as with financial instruments, be regarded as real options which may have a value for the option holder at the exercise date. The real options theory uses the finance option models, among others, to attempt to recognize, describe, and assess these rights as accurately as possible. The aim is to incorporate the effects of this flexibility of action in the project value in greater detail (Leslie and Michaels 1997) and thus to improve the level of information for decisions as opposed to an exclusively capital-value-oriented perspective (Dixit and Pindyck 1995). This conclusion is also reached by Cobb and Charnes (2007) who give a literature overview on the topic of real option value. The real options theory is discussed controversially because of the complex valuation issues; nevertheless, the economic relevance is widely accepted and is therefore obtaining a broader range of applications (see Gamba and Fusari 2009; Copeland and Keenan 1998). An application of analytical valuation methods and procedures (e.g. BSM) from the financial option theory often leads to difficulties in the case of real options, since the requirements are usually not the same as in the capital market and since the uncertain option parameters, such as the size and growth of the underlying asset, cannot always be exactly specified under the restrictive model assumptions (Hommel et al. 2001, pp. 114–121). Borison (2005, pp. 17–31) uses five approaches to show how real options can be valued practically: In the ideal case, there is a strongly correlated financial instrument (e.g. commodity price) for the real option so that the value can be determined with the help of available market parameters. Should this fail, the possibility of subjective parameter estimation and subsequent calculation by means of BSM, for example, only remains if the conditions are the same as in the financial market. If this is also problematic, the solution can be approximated using a multi-stage binomial model, a simulation, or a combination of both, which allows any desired development scenarios. Therefore, the decision maker must be constantly aware that the calculated option value may vary substantially from reality depending on the parameter estimate and the calculation method, and an interpretation of the results is required. It is a condition for any real option valuation to find the type of option at hand, which can very rarely be represented by a simple call or put option. It is noteworthy that to date the discount option is not named explicitly in any listing of real option types, which is also true, for example, for that of Trigeorgis (1995, pp. 3–4). For that reason, the following section first explains the properties and the payment structure of a discount option and then presents a general valuation model.

3 Discount Options for Software License Agreements

Discount options for software license agreements may present some differences in contrast to other rights to services, such as traditional maintenance contracts. Thus, the possibility to upgrade to a new software version within the contract period only exists after the provider has developed and made available a new version. For other services, such as user trainings, the license only actually gets up to a certain amount of discounts, but at any time during the contract period. For the valuation model, the following assumptions are made:

3.1 Assumptions of the Model

Assumption 1 A licensee of a subscription license has \( M \in N^+ \) discount options \( O_t \) with which he can realize a reduction \( D \in \{0; 1\} \) (Discount) for each, based on the regular service price \( P \), for the utilization of a specified service out of the services offered by the software provider until the option expiration date \( T \).

Assumption 2 The exercise of the \( n \)th discount option \( O_n \) with \( n \leq M \) at the time of exercise \( t_n \) causes a certain exercise price \( K \) (strike price) for the licensee at each point in time, which is composed of the reduced service price \( (1-D) \cdot P \) and an additional necessary, safe investment pay-out \( I \) (for example, test and installation of software upgrades). The licensee realizes the expected present value \( BWCF_n \) of future earnings (underlying asset) through the performance at the time point \( t_n \), e.g., increased productivity, additional profit, or cost reduction, which the licensee would also attain as a result of the acquisition of the service from the license provider. The underlying \( BWCF_n \) is a random variable and is therefore subject to uncertainty until the exercise date \( t_n \).

Assumption 3 The beginning \( z_n \) of the possible exercise period of the discount option \( O_n \) is known and is determined either, regardless of user needs, by the next possible exercise date stated by the supplier with \( z_n \geq t_n-1 \), for example, for software upgrades (independent provision of services), or is defined exclusively by the desired demand of the users with \( z_n = t_n-1 \), for example in user training or support services (dependent provision of services).

Assumption 4 The licensee maximizes the capital value of the total investment that results from the exercise of the option: The discount option \( O_n \) is therefore not exercised by the licensee until the time point \( t_n \in [z_n; T] \), if the expected capital value \( KWV_n \) with immediate use of the service, consisting of the expected present value of cash flows \( BWCF_n \) less the exercise price \( K \), exceeds the expected capital value \( KWV_n \) for the failure to exercise or further delay of the exercising of the option.

Assumption 5 The continuous calculation interest rate per year aggregates through the whole timeframe \( r \). The
value of the discount $OW_n$ or the total value $GOW$ of $M$ discount options is considered at the time $t_0$ of the completion of the contract.

For the calculation of the option value, two relevant exercise thresholds are of significance. The discounted exercise price $K$ forms the lower threshold that has to be exceeded by the underlying $BWCF$, in order for exercising of the service to be advantageous for the licensees. If the underlying also exceeds the regular exercise price $(K + D \cdot P)$ or $(I + P)$, then exercising of the service is beneficial even without an option (upper threshold). This means that for underlyings over the upper threshold licensees with an option have no extra benefits over the value of the discount in comparison to licensees without an option, as they both profit equally from an increase in $BWCF$. The possible advantage from the option at the exercise date is therefore limited by the maximum value (cap) or discount $D \cdot P$ as the difference (spread) between the two exercise prices. The diagram in Fig. 2 illustrates the situation of exercising a service with and without options at a possible exercise date, whereby a continuous normal distribution $P(BWCF)$ was assumed exemplary for the underlying.

The flexibility of action to obtain a service from the service provider in order to generate additional cash flows is consistent with the holding of a call option (long call). Since this flexibility of action is however not exclusively available to the option holder, which advantage is just the profit from the discount granted, the option value at the time of exercise for the underlying over the regular exercise price $(BWCF > I + P)$ is limited to this discount. Consequently, there is a capped call option with a limited maximum level. This type can be constructed by combining a long call with a lower (discounted) exercise price and a short call (sale of a call option) with a higher (regular) exercise price (see Hull 2001, p. 322). For the discount option in the license contract it must be considered that the capped option is a replicated construction, which cannot actually be traded on the money market. Figure 3 shows the schematic payment structure at the time of exercise with regard to the option price.

The possible loss of the long call is initially reduced by the position of the short call. The long call, as of the discounted exercise price, continually gains value with the increasing value of the underlying until the short call position, as of the regular exercise price, compensates for any further growth of the long call. As a result, the initially increasing value of the discount option from the regular exercise price stays constant. The value of the option $OW_n$ at the start of the contract $t_0$ of the American discount option $O_n$ with exercise period $(zc_t)$, thus, corresponds to the option value of an American capped call, which can be determined through the difference between the option price of the long call $(OW^{LC}_n)$ with discount $(K)$ and the option price of the short call $(OW^{SC}_n)$ with regular exercise price $(K + D \cdot P)$ at the underlying $BWCF_n$.

$$OW_n(t_0) = OW^{LC}_n(BWCF_n, K, zc_t, T)
- OW^{SC}_n(BWCF_n, K + D \cdot P, zc_t, T) \quad (1)$$

Since the value of an option cannot be negative, $OW^{SC}_n \geq 0$ is true for the short call and thus $OW_n(t_0) \leq OW^{LC}_n(BWCF_n, K, zc_t, T)$. From this it follows that the maximum value of the option is independent from the amount of the discount and an increase in the value of the option exclusively through raising the discount is only possible to a certain extent.

The total option value $GOW$ of the contracted discount option at the point in time $t_0$ can therefore determine the sum of the $M$ option values $OW_n(t_0)$:

$$GOW = \sum_{n=1}^{M} OW_n(t_0) \quad (2)$$

If there is only one fixed exercise point, like for instance in this case $(zc_t = T)$ with a discounted license conversion option at the end of the contract period $T$, a European discount option is existent. Under arbitrage equilibrium its value $OW$ can be determined analytically through the Black-Scholes model, if there is a log-normally distributed $BWCF$ with constant volatility and this follows a geometric Brownian motion. Empirically, however, it has already been proven that this condition is not even fulfilled on the financial market (Eraker 2004). Most dis-

Fig. 2 Incoming payment and pay-out at the exercise date
Fig. 3 Payment structure of a discount option at time of exercise (see Damodaran 2002)

Fig. 4 Sequence of discounted software upgrades over time

count options in a software license agreement, independent of the problems of estimating the parameters and idealized development, can also not be valued using an analytical approach because of the complexity in the structure and the elements of uncertainty. On the one hand, the difficulty is due to the possibility of being able to exercise the option only after the service is provided (see Assumption 3) in a given period (see Assumption 4). On the other hand, the licensee normally has not only one option for a service but a series with $M > 1$, which relates to the same service (see Assumption 1). However their option values can potentially influence each other: If the exercise dates of the options are interpreted as a sequence, then the duration of the option $n+1$ begins with the exercise of the option $n$ at the earliest (see Assumption 2). Compared to options on the financial market, whose exercise has basically no effect on other options, the exercise of a real option can definitely change the underlying of subsequent real options to a certain extent. This can be illustrated with a simple example, such as user trainings. The first user training is carried out if the achievable productivity gain exceeds the exercise price and a further delay of the training course would be detrimental. Due to this training, the achievable productivity gain would probably turn out much lower for subsequent trainings partially dealing with the same issues for previously trained staff, so that mainly untrained employees are eligible.

In the case of a constant personnel base, it follows that the total value $GOW$ of the discount option with increasing $M$ can not be arbitrarily large and dependencies between the options must be considered in the valuation. Figure 4 illustrates the situation using the example of software upgrades. The time points $z_n$ of providing the service are independent of the user needs and therefore do not necessarily occur together at the exercise point $t_n$.

The first upgrade at the time point $z_1$ will be carried out by the licensee not immediately at the beginning of the exercise period, but only delayed at the time point $t_1$, since in this way for example, a higher capital value can be achieved. The second upgrade provided is bypassed in order to subsequently make use of the
third upgrade with the second discount option $O_2$. The exercise period of the third option $O_3$ begins only at the time point $z_3$, at which the exercise $t_3$ follows immediately, shortly before the contract ends. The supply time points $z_n$ are safe, if the provider, for example, creates exact information about the upgrade cycle and also complies with it, or unsafe and randomly distributed, if there is no exact information and for instance it is only possible to get guidance from historical upgrade cycles. In this case, a numerical approximation, which will be presented in the next section with the help of an example, can be consulted in order to value the option.

4 Option Price Determination of Discount Options in Software License Agreements

In a licensing agreement, it is indeed theoretically possible to value a simple European discount option using the BSM. However, in practice this would rarely lead to a useful result because of the restrictions mentioned. A more appropriate method is the numerical backward induction with risk-neutral valuation for discrete exercise times (Wilkens and Wilkens 2000, p. 70; Irl and Prelle 2007, p. 44). The basic idea is to reproduce an American option $O_n$ through the use of a Bermudian option, which can only be exercised at certain times points $u_m \in [z_n; T]$. The determination of the considered expected values takes place under the risk-neutral probability measure $Q$, which is equivalent to the actual probability distribution. This takes an existing risk premium directly into account in the synthetic distribution, so that a discounting of the expected values can then take place using the risk-neutral rate (see Brunner 2004, pp. 11–13; Meyer 2006, p. 77). At each discrete time point $u_m$, the licensee decides whether an immediate exercise is beneficial in comparison to delaying until the next possible decision point $u_{m+1}$. The decision over the exercise of options in a license agreement at the time point $u_m$ is, however, not based on the directly realizable value of the option $OW_n(u_m)$ but on the immediately achievable capital value $KW_n^S(u_m)$ and the capital value in the case of a delayed exercise $KW_n^Y(u_m)$, which is not limited and can thereby turn out much higher than the maximum achievable discount (see Assumption 4). Consequently, unlike a discount option on the financial market, it can be advantageous to further delay the exercise of an option, although the option value itself has already reached its upper limit (cap) or the discount $D \cdot P$. The capital value immediately achievable at the time point $t_n = u_m$ can be calculated from the expected present value $BVCF_n(u_m)$ of the cash flows (for example, from a numerical simulation) and the certain exercise price $K$. The expected capital value $KW_n^Y(u_m)$ of the delay at the time point $u_m$, for which $KW_n^Y > 0$ always applies, because of the non-exercise, can be determined using a risk neutral valuation approach out of the discounted expectation value with the probability measure $Q$ of the maximum achievable capital value at the time point $u_{m+1}$, which is again based upon the current decision of the immediate exercising or further delays. The calculation of the expected values $KW_n$ and hence the determination of the optimal exercise time point $t_n$ take place inductively in reverse $(u_{m+1} \rightarrow u_m)$ starting from the end of the period of validity $T$ up to the earliest possible exercise date $z_n$ of the option $O_n$.

$$KW^V_n(u_m) = E_Q(\max(KW^S_n(u_{m+1}); KW^V_n(u_{m+1}))) \cdot e^{-r(u_{m+1}-u_m)} \quad (3)$$

Beginning at $z_n$, the option is now exercised at the time point $u_m$ if the capital value for immediate exercising is greater than for further delaying. According to this, for the optimal time of exercise $t_n^*$ of the option $O_n$, this applies:

$$KW^S_n(u_m) > KW^V_n(u_m) \quad \Rightarrow \quad t_n^* = u_m \quad (4)$$

The option value $OW_n$ corresponds to the discounted expectation value of the maximum at the time point $t_0$ from the realized capital value at the exercise time $t_n^*$, and the discount $D \cdot P$ as the upper limit:

$$OW_n(t_0) = E_Q(\min(KW^S_n(t_n^*); D \cdot P)) \cdot e^{-r(t_n^*-t_0)} \quad (5)$$

Figure 5 illustrates the procedure for the numerical backward induction and the use of formulas (3) to (5).

The optimal time of exercise $t_n^*$ for the single option $O_n$ is also optimal in the integrated treatment of the whole option.
series. This is because all discount options of the series apply for the same service. As a consequence, the optimal time of exercise of the option $O_{n+1}$ cannot be earlier than $t_n^*$, as otherwise the option $O_n$ would already have been exercised before. An exercise after the optimum time $t_n^*$ is actually a disadvantage for the option $O_{n+1}$ since the possible exercise period is shortened as a result. The following example illustrates a typical situation of a discount option for upgrades in a software license agreement:

**Situation** In a license agreement with 500 licenses, there is a discount option available for free upgrades ($D = 100\%$). The regular price for the licenses and an upgrade respectively is $P = 50$ K-EUR (100 EUR per license).

Additionally, the following assumptions apply:

- The contract period is 3 years and it will be decided each quarter whether an upgrade will be implemented.
- The manufacturer announces an upgrade ($z_1$) in the 2nd quarter of the first year and another ($z_2$) in the 4th quarter of the second year of the contract.
- The present value $BWCF$ in $t$ corresponds to the present value of the expected additional cash flows related to $t$ that can be achieved through an upgrade, where the maximum will only be reached in the 2nd quarter after the provision of the service, for example due to a reduced error rate, and then falls again because of lost deposits. The development of the $BWCF$ is indicated exemplarily here for exactly 1 upgrade discretely for each period. In the actual case of application this could be simulated, e.g., based on available data from past upgrades.
- The investment payment for the migration to a new version of the software through an upgrade amounts to $I = 75$ K-EUR (150 EUR per license).
- The calculation interest rate $r$ amounts to 4% p.a.
- A risk premium ($Q$) is not taken into account.

The valuation problem of the presented discount options can be solved using a (two-step) numerical backward induction as an example of a path of development:

**Solution method** The parameters described in Table 3 are compared and calculated in Table 4 based on an exemplary path for the twelve quarters of the contract term, showing the expected value $BWCF$, the investment payment made $I$, and the resulting capital value $KW$. Initially the optimal exercise point $t^*$ of the first option $O_1$ and thus the value of the option $OW_1$ in $t_0$ are determined. The expected present values (in K-EUR) of the attainable cash flows from the exercise in the corresponding period are chosen here as an example to illustrate the basic procedure.

The optimal exercise time $t^*$ of the upgrade, in which a further delay would be detrimental in comparison to the immediate exercise, lies in the third year at $t^* = 10$ and is therefore not identical with the period in which the discounted option value is at the maximum ($t = 9$). In

### Table 3 Description of the parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period $t$</td>
<td>Number of the time period within the contract term.</td>
</tr>
<tr>
<td>Upgrade $z$</td>
<td>Upgrade which is available in the period $t$.</td>
</tr>
<tr>
<td>Expected value $BWCF$ in $t$</td>
<td>Expected present value of the additional (uncertain) cash flows, if the upgrade is carried out in this period.</td>
</tr>
<tr>
<td>Investment pay-out $I$ in $t$</td>
<td>Certain investment pay-outs, if the upgrade is carried out in this period.</td>
</tr>
<tr>
<td>Capital value $KW^S$ in $t$</td>
<td>Sum of the $BWCF$ and $I$ in $t$, if the upgrade is carried out in this period.</td>
</tr>
<tr>
<td>Capital value $KW^V$ in $t$</td>
<td>Capital value in $t$, if the upgrade is not carried out until later. This corresponds to the discounted maximum of the capital value in $t + 1$.</td>
</tr>
<tr>
<td>Option value $OW$ in $t$</td>
<td>Benefit or value of the option at time of exercise. This corresponds to the maximum out of the capital value at exercise and the discount.</td>
</tr>
<tr>
<td>Option value $OW$ in $t_0$</td>
<td>Discounted option value $OW$ in $t$ at the time $t_0$ (beginning of the contract).</td>
</tr>
</tbody>
</table>

### Table 4 Evaluation of the exemplary path

<table>
<thead>
<tr>
<th>Contract year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<tbody>
<tr>
<td>Quarter</td>
<td>1</td>
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<td>4</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Period $t$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>Upgrade $z$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
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<tr>
<td>Expected value $BWCF$ in $t$</td>
<td>0</td>
<td>30</td>
<td>50</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
<td>100</td>
<td>125</td>
<td>150</td>
<td>130</td>
<td>110</td>
</tr>
<tr>
<td>Investment pay-out $I$</td>
<td>0</td>
<td>-75</td>
<td>-75</td>
<td>-75</td>
<td>-75</td>
<td>-75</td>
<td>-75</td>
<td>-75</td>
<td>-75</td>
<td>-75</td>
<td>-75</td>
<td>-75</td>
</tr>
<tr>
<td>Capital value $KW^S$ by exercise</td>
<td>0</td>
<td>-45</td>
<td>-25</td>
<td>-5</td>
<td>-15</td>
<td>-25</td>
<td>-35</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>55</td>
<td>35</td>
</tr>
<tr>
<td>Capital value $KW^V$ by delay</td>
<td>69,3</td>
<td>70,0</td>
<td>70,7</td>
<td>71,4</td>
<td>72,1</td>
<td>72,8</td>
<td>73,5</td>
<td>74,3</td>
<td>54,5</td>
<td>34,7</td>
<td>0,0</td>
<td></td>
</tr>
<tr>
<td>Option value $OW$ in $t$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>25</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>35</td>
</tr>
<tr>
<td>Option value $OW$ in $t_0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>18,3</td>
<td>35,1</td>
<td>33,8</td>
<td>32,5</td>
<td>21,9</td>
</tr>
</tbody>
</table>
the path shown in the example, an upgrade takes place in the second quarter of the third year only after a delay, although the first upgrade would only have been possible after one quarter and a further upgrade only in the fourth quarter of the second year. This is because the decision maker only considers the capital value (see Assumption 4) and is prepared to wait for a better time if this is advantageous.

In the event that an exercise of the first software upgrade is advantageous \((t = 2\) to \(t = 7)\), similarly the value \(O_2\) of the option \(O_2\) would still have to be considered by a second backward induction, whereby the altered \(BWCF\) for the periods \(t = 8\) to \(t = 12\) would then have to be approximated again.

The total option value \(GOW\) of the discount option is 33.8 K-EUR for the path shown in the example, and is thus equal to a third of the value for two regular software upgrades which amounts to 100 K-EUR. In a real valuation scenario, a simulation would generate and assess many paths of the uncertain BWCF in order to then eventually determine the expected option value out of the stochastic distribution.

In contrast to the valuation approaches referred to in chapter two that are based on the size of the discount, a random rise in the regular license price would only increase the option value to a maximum of 50% at 50.7 K-EUR in this method.

The presented example shows how complex structured discount options and option series can be valued with the help of the numerical backward induction. A realistic approximation of the expected present value \(BWCF\) of the cash flows for the different periods within the exercise timeframe in this case represents the biggest challenge, especially if the exercise of an earlier option influences it. Other uncertainties, such as the provisioning time of the upgrades, may also be considered in the context of a modified simulation. In an option series with many uncertain parameters (dimensions) and fine granular time intervals, the simulation costs can increase rapidly, so that efficient methods and algorithms must be used for the calculation and simulation itself in order to reduce the calculation costs. Applicable methods for this purpose are, for example, "sparse grids", which clearly reduce the numerical complexity of multidimensional problems (Mertens 2005), or the method of least squares by Longstaff and Schwartz (2001).

5 Summary and Outlook

The use of standard software requiring a license plays an increasingly important role in businesses, meaning that more attention has to be given to the associated costs and risks. Discount options on services included in the license agreement have significant financial impact throughout the life time of the software and should therefore be understood by the licensee and taken into account during the decision making. On the basis of valuation models for finance options, the real options method was introduced as a possible approach to describe and value these discount options. The transfer and application of assumptions specific to financial markets that concern real economic investment projects, as is necessary in the case of the Black-Scholes model, can lead to the limits of real options theory through the estimation of input parameters and thus actually exclude an analytical valuation approach. The paper has shown that a discounted service is a capped call option and that it can have extra features in the case of a software license agreement, such as the possibility of delaying exercise. If the growth of attainable cash flows can be estimated, the numerical method of backward induction is, in principle, suitable to value these contractual components in a realistic scenario, and therefore provide the licensee with useful information for making decisions on an economical basis in the management of software contracts. The model presented here has thus resolved a major weakness of existing valuation methods in this field which primarily concentrate on the rate of discount. As a result, it will become harder for manufacturers to boost the value of additional services included in their contracts through excessive service prices in order to compensate for expensive licensing fees. Furthermore, companies can develop an option strategy in order to demand, for example, more discounts on training or free support enquiries if the personnel structure requires this. To provide this flexibility, the software providers should abstain from rigid option packages that contain a fixed quota of different services for each license or conversion keys, and rely on changeable or purchasable models with more freedom of choice. An interesting extension to this would be to investigate other types of options, such as exit options, which allow the licensee to later renounce the service.

Abstract

Daniel Gull

Valuation of Discount Options in Software License Agreements

Many companies increasingly rely on licensed standard software for system software and applications. In addition to the regulation of usage conditions, software licensing agreements increasingly include services, such as software upgrades and user training, as a part of the contract or these are optional for a fee, which can be made use of by the licensee during the term of the contract at a reduced price or as a free service. This benefit entitlement is called a discount option and must be valued during the selection and designing of a contract. This paper describes the basic valuation issues as well as some weaknesses of previous approaches, and subsequently presents a model which, on the basis of the real option theory, enables an assessment of the discount options using mathematical methods. As the value of discount options can in many cases only be estimated by using analytical methods under certain conditions, a practical solution method is explained on the basis of numeric backwards induction. The procedure for applying the model and the achieved advances in knowledge are illustrated with an example.

Keywords: Real options, Discount options, Software asset management, License management
of the provider or to reduce the number of purchased licenses in the short term. This could also generate benefits for the software provider if it increases the attractiveness of the products and if the sales revenue from many such options more than covers the occurring revenue losses. In this respect, a useful next step might be an empirical evaluation of the presented model and all extensions thereof with the help of a usable data base, e.g. from a large software provider or licensee. Moreover, the further adjustment of analytical model approaches from the financial option theory to the specific requirements of real options would not only simplify the valuation of discount options for software license agreements, but also increase the acceptance of the practical use of this method for similar problems in other areas of business and information systems engineering.

Appendix

Algorithm of the numerical backward induction of the case study used in Sect. 4

for $t := T$ to 1
//capital value of delay
if $t = T$ then $KW_t^V := 0$
(because option is voided afterwards)
else
$KW_t^V := \max(\{KW_{t+1}^V, KW_{t+1}^S\} \cdot (1 + r)^{-1})$
//optimal exercise time when delaying
//is disadvantageous
if $KW_t^V > KW_t^S$ then $t^* := t$
//output option value in $t_0$ given optimal exercise
$OW := OW_0$

References

Gartner (2005) Determining the value of Microsoft software assurance. Gartner Research