Abstract

Consumer review systems have become an important marketing communication tool through which consumers share and learn product information. This paper aims to analyze the review system design as firms’ strategic decision to facilitate consumer sharing and learning about their products. We show that firms’ optimal pricing and review system design decisions critically depend on contextual characteristics including product quality, product popularity, and consumer misfit cost. Our results suggest that firms choose a low rating scale for niche products and a high rating scale for popular products. Different pricing strategies should be deployed during the initial sale period for different product types. For niche products, firms are advised to adopt lower-bound pricing for high-quality products to take advantage of the positive word-of-mouth. For popular products, firms are advised to adopt upper-bound pricing for high-quality products to enjoy the direct profit from the initial sale without damaging the product review outcomes.

Keywords: Economic modeling, e-commerce, consumer reviews, online word of mouth, product uncertainty
Introduction

With the prevalence of the internet and the success of e-commerce, consumers increasingly resort to the Web to gather information about products and services before making their purchasing decisions. There are abundant online sources for product information. In addition to manufacturer-provided product specification, which is usually available on e-tailers’ sites, numerous websites offer third-party professional reviews and consumer reviews. According to the E-tailing Group, 71% of online shoppers said their choices are most influenced by customer reviews, followed by discussion forums (Dodes 2010).

The main reason consumers use online review systems is because they are uncertain about whether the products will meet their expectations in terms of product quality, functionality, and other attributes (Chen and Xie 2008; Sun 2012). To tackle the big challenge of online shopping – consumers cannot see or touch the products, more and more companies employ IT enabled online review systems as an effective extension of the traditional word-of-mouth network. The perception is that users of online review systems can learn something about the products of interest and as they develop a better idea of whether the products might meet their expectations they can make better purchasing decisions with greater ease (Mudambi and Schuff 2010).

Many firms choose to host their own online review systems within their e-commerce sites. In this way, customers of the firms can conveniently rate products after their purchases and new customers can go through online reviews before making their purchasing decisions, all within the firms’ websites. Information technology enables firms to directly observe what customers think about their products as they design, host, and mine such review systems. This helps them improve customer service, lower return rate, increase conversion from browsers to buyers, and better respond to consumer expectation through product improvement, logistic planning, assortment or price adjustment (Dodes 2010; Fowler 2009). In addition, firms can also benefit by internalizing the review system design decision instead of letting a third party take control, and more importantly they can integrate the review system design decisions with other operational decisions such as logistics and pricing. Nevertheless, to fully benefit from hosting such review systems firms need to understand various issues associated with consumer reviews and review system design.

An online consumer review system often displays aggregated user rating information such as average rating, number of ratings, and detailed rating distribution in prominent places on the site to give viewers an overall view of how past consumers value the product or service. However, we observe many different review system design features in existing consumer review systems. For example, IMDb allows customers to rate a movie on a scale of one to ten. Many retailers such as Amazon, L.L.Bean, Macy’s, Target, and Wal-Mart let customers rate a product on a scale of one to five. These firms also ask customers whether they would like to recommend the product to other customers or not. Similarly, YouTube’s “like/dislike” button enables a binary positive or negative rating. Several interesting practical questions arise: What accounts for these design differences? How do review system design choices affect review outcomes and consumers’ perceived usefulness of the reviews? How do users of the review systems interpret review outcomes and make their purchasing decisions accordingly? What are the optimal review system design choices?

There is abundant empirical evidence that online review outcomes affect consumer purchasing decisions in various domains including movies (Eliashberg and Shugan 1997), beer (Clemons et al. 2006), Broadway shows (Reddy et al. 1998), eBay (Resnick and Zeckhauser 2002), TV shows (Godes and Mayzlin 2004), books (Chevalier and Mayzlin 2006; Sun 2012), and video games (Zhu and Zhang 2010) and so on. Most of these studies associate high mean rating with high quality or high overall satisfaction. However, the impact of review design features on current customers’ ratings and future consumers’ purchasing decisions has not been studied.

In this paper, we investigate the interaction of a firm’s pricing and review system design decisions and examine the impact of these decisions on consumer ratings and purchasing decisions. We propose a formal analytical model to address a series of research questions: How do consumers rate the product after consumption? How do prospective customers interpret the product reviews and make their purchasing decisions accordingly? What are the impacts of a firm’s review system design and pricing decisions on consumer reviews? What are the firm’s optimal design choices for their online consumer.
review systems and how do these design choices interact with its pricing decision? How do product and consumer characteristics moderate the firm’s design and pricing strategies?

The objective of this paper is to study the optimal design of online consumer review systems while accounting for product and consumer characteristics. We formally analyze two interrelated processes: product rating process and rating interpretation process. In the product rating process, product and consumer characteristics influence a firm’s product pricing and review system design decisions. The firm’s pricing and review system design decisions collectively determine review outcomes such as review volume and mean rating. In the rating interpretation process, prospective consumers update their expectations based on the review outcomes and make their purchasing decisions accordingly.

We model one particular design feature of consumer review systems – the scale of user ratings. Consumer ratings of a product based on the given scale levels reflect their overall evaluations of the product. Potential consumers rely on these ratings to update their perception of the product quality. The scale of user ratings has a direct impact on user ratings which further influence potential customers’ purchasing decisions. In addition, since firms’ pricing and review system design decisions are affected by product and consumer characteristics, such as product popularity and consumer taste preference, these characteristics moderate the impact of online review system design on consumers’ purchasing decisions.

We show that consumer review systems can serve as a marketing communication tool to reduce consumers’ uncertainty about the product quality. We find that a firm can effectively manage consumers’ perceptions of a product by strategically selecting the rating scale. The firm’s optimal pricing and review system design decisions critically depend on contextual characteristics including product quality, product popularity, and consumer misfit cost. Our results suggest that the firm should choose a low rating scale for niche products and a high rating scale for popular products. Our results also suggest different pricing strategies for niche products versus popular products. Lower-bound pricing is optimal for high-quality niche products but low-quality popular products. Upper-bound pricing is optimal for low-quality niche products but high-quality popular products.

Our paper contributes to the literature of consumer reviews in multiple ways. First, we formally model rating scale as a firm’s review system design choice and analyze the impact of rating scale on consumer ratings and the firm’s profits. Second, while most of the review literature focuses on the rating interpretation process, our paper takes a holistic approach and analyzes both the product rating and the rating interpretation processes. Third, we explicitly formulate the information role of consumer reviews. Reviews from past consumers serve as an imperfect signal for product quality to facilitate learning for future consumers. Fourth, we analyze the interaction between the firm’s pricing and its review system design and simultaneously solve for its optimal decisions. Fifth, we identify three contextual factors – popularity, misfit cost, and quality, and examine how they moderate the firm’s pricing and review system design decisions.

This paper proceeds as follows: we review related literatures in the subsequent section. We next propose a model of consumer review systems and then analyze the interaction between the firm’s review system design choice and its pricing strategy. Finally we conclude with managerial implications and directions for future research.

**Literature Review**

There is extensive literature in the online word-of-mouth systems in the fields of information systems and marketing. Researchers have identified different roles of online word-of-mouth systems (Dellarocas 2003). One research stream views online word-of-mouth systems as a reputation mechanism and papers that are written in this realm of research typically focus on building trust and reducing seller uncertainty. Another research stream views online word-of-mouth systems as a marketing communication tool and papers that are written in this research stream focus on disclosing product information and reducing product uncertainty to consumers. This work takes the product information view of online word-of-mouth systems. Thus we focus on discussing online product review systems in this paper.

Online product review systems can be characterized along two dimensions: – who provide the reviews and who manage the reviews. There are different types of online product review systems. Based on the provider of the reviews, there are professional reviews and consumer reviews. Based on the manager of the reviews, there are third-party-managed and seller-managed product review systems. Professional
reviews and consumer reviews exhibit very different features (Chen and Xie 2008). Since this paper studies seller-managed consumer review systems, we focus on discussing the literature that studies consumer reviews.

Consumer review systems have been studied both analytically and empirically, and also from different angles in the literature. We review the literature of consumer review systems from three perspectives—contextual characteristics, review results, and design of review system.

Prior studies have identified important contextual factors in determining how online consumer reviews affect consumers’ purchasing decisions and product sales. These factors include product popularity (Sun 2012; Zhu and Zhang 2010), reviewer identity (Forman et al. 2008), consumer internet experience (Zhu and Zhang 2010), consumer expertise (Chen and Xie 2008), product age (Archak et al. 2011), firm age and growth (Clemons et al. 2006; Kuksov and Xie 2010), product type (Dellarocas et al. 2007; Dellarocas et al. 2010), marketing effort (Dellarocas et al. 2007; Duan et al. 2008), professional reviews (Dellarocas et al. 2007; Duan et al. 2008; Gao et al. 2011; Liu 2006), etc. Empirical studies have shown that many contextual factors like product and consumer characteristics moderate the impact of consumer reviews on sales. However, firms’ strategic responses to specific contextual conditions remain unanswered. In this paper, we formally model three contextual characteristics and study their impact on consumer reviews as well as a firm’s pricing and review system design decisions.

Results of online consumer reviews are captured in numerical summary of rating scores in the existing literature. Most studies utilize the mean rating and the rating volume. Only a few papers look at the distribution of available customer reviews. Specifically, the variability of consumer rating scores is captured by variance (Clemons et al. 2006; Sun 2012) and coefficient of variation (Zhu and Zhang 2010).

From the perspective of consumer review system design, the existing literature in online consumer reviews either does not consider the design of consumer review systems or treat the system design as exogenously given. In this paper we endogenize the firm’s design choice by modeling the rating scale. We systematically study the impact of rating scale on consumer rating and consumer learning and derive the firm’s integrated optimal review system design and pricing decisions while incorporating effects of different product and consumer characteristics.

The Model

Firm and Consumers

Consider a firm selling a product through the online channel. Consumers value both the quality of the product and the fit of the product (e.g., how well the product fits their tastes). The quality of the product $v$ is the firm’s private information. Before consumption, consumers are uncertain about the true value of $v$ but they share a common belief that the product quality $v$ is uniformly distributed in the range of $[\bar{v}, \bar{v}]$, where $\bar{v} > v > 0$. In other words, consumers’ before-consumption perception of product quality can be represented by $U[\bar{v}, \bar{v}]$. Therefore before consumption, consumers’ expectation of the product quality is $\hat{v} = (\bar{v} + v)/2$. The difference between the two bounds of product quality $(\bar{v} - v)$ measures the degree of quality uncertainty from a consumer’s perspective.

Consumers are heterogeneous in terms of their tastes for the product and they are located on $[0, 1]$ based on their tastes. Without loss of generality, we assume that the product is located at point zero. We denote $t$ as the misfit cost parameter which represents the unit misfit cost. Thus a consumer located at $x$ incurs a misfit cost of $tx$ and customers located further away from the product incur a higher misfit cost. Customers’ tastes follow the density function of $f(x) = \theta + 2x(1 - \theta)$, where $x \in [0, 1]$ represents a consumer’s location on the unit line and $\theta \in [0, 2]$ represents the popularity of the product. In general, this density function $f(x)$ represents a series of products with different popularity levels. Figure 1 illustrates three representative examples of user taste distributions, which correspond to three different product types. When $\theta \in [0, 1)$, there are relatively fewer consumers located close to the offered product.
and therefore these cases correspond to niche products. When $\theta \in (1, 2]$, there are relatively more consumers located close to the offered product and therefore these cases correspond to popular (or mass) products. When $\theta = 1$, the density function $f(x) = 1$ represents a uniform distribution and corresponds to neutral products. Thus, $\theta$ is the product popularity parameter, with a higher $\theta$ indicating a higher popularity. We assume the density function $f(x)$ is public information.

Before consumption consumers do not know exactly how well the product fits their own tastes and they share a common belief that the product misfit follows the density function of $f(x)$. Therefore before consumption consumers’ expectation of the product misfit is $\hat{x} = \int_{0}^{1} x f(x) dx = \frac{4 - \theta}{6}$. Consequently, consumers have a higher expected misfit cost for niche products and a lower expected misfit cost for popular products. The firm charges a price $p$ for the product. Therefore before consumption, consumers’ expected net utility is $\hat{u} = \hat{v} - \hat{x} - p$.

In summary, there are two types of product uncertainty – quality uncertainty and fit uncertainty. In the next subsection, we discuss how consumer review systems can help reduce these product uncertainties.

**Consumer Review Systems**

The firm hosts an online product rating system to facilitate information sharing among their customers. We study the firm’s product review system design choice and its pricing strategy in a two-period model. First-period and second-period consumers are two independent groups of consumers with identical...
characteristics. Consumers arrive independently in each period and each consumer has unit demand for the product. The total number of consumers in each period is normalized to 1. At the beginning of the first period, the firm makes its pricing and review system design decisions. In the first period, consumers are uncertain about their valuations of the product quality and how well the product will fit their tastes. First-period consumers make their purchasing decisions based on their expected valuation of the product quality and their expected misfit cost. After consuming the product, consumers learn the true product quality and the true product fit. Based on their realized net utility, first-period consumers rate the product in the review system. In the second period, consumers learn more about the product quality and the product fit from the posted reviews. Second-period consumers update their beliefs for the product and make their purchasing decisions accordingly.

We use $p_i$ to represent the product price, $\hat{v}_i$ to represent consumers’ expected valuation of the product quality, and $\hat{x}_i$ to represent their expected misfit in period $i$, where $i = 1, 2$. In the first period, consumers’ expected misfit is $\hat{x}_i = 2/3 - \theta/6$ and their expected valuation on product quality is $\hat{v}_i = (\bar{v} + v)/2$. The first-period consumers’ before-consumption expected utility is given by $\hat{u}_i = \hat{v}_i - t\hat{x}_i - p_i$. After consumption, the realized utility for a customer located at $x$ is $u(x) = v - tx - p_i$.

After consumption, first-period consumers rate the product in the review system. There are two key modeling components regarding online consumer product rating systems. The first component is the product rating process. In other words, after the first-period customers consume the product, how do they rate the product? The second component is the rating interpretation process. In other words, how do product ratings affect consumers’ expectations of the product in the second period?

We start with the product rating process. To simplify the exposition, we normalize customer ratings to a vertical unit line segment where the highest rating is 1, the lowest rating is 0, and other rating levels are evenly spaced out along the unit line segment. Thus, in a system with $s$ rating levels, the available rating levels correspond to points $0, \frac{1}{s-1}, \ldots, \frac{s-2}{s-1}$, and 1 on the vertical line. We would like to find a rating function $R(u(x), s)$, which maps a customer’s after-consumption utility to one of the rating levels. We assume that consumers rate the product truthfully, based on their after-consumption utility to one of the rating levels. We first transform consumers’ utility $u(x) \in R$ to a utility score $w(x) \in (0, 1)$ according to a logistic function $w(x) = \frac{e^{u(x)}}{e^{u(x)} + 1} = \frac{1}{1 + e^{-u(x)}}$. This type of logistic transformation is widely adopted in the marketing literature and this method captures the fact that consumers with extremely high and low net utility are more likely to rate. The logistic transformation converts consumers’ after-consumption utility to a utility score which has the same scale as the product ratings. Consumers rate the product by matching the converted consumer utility score to a product rating level according to the rating function $R(u(x), s)$:

$$ R(u(x), s) = \begin{cases} r/s-1, & \text{if } \min \left\{ \frac{i}{s-1} - w(x), \ i = 0, \ldots, s-1 \right\} \leq \varepsilon \\ \text{does not rate, otherwise} \end{cases} $$

(1)

where $r = \arg\min \left\{ \frac{i}{s-1} - w(x), \ i = 0, \ldots, s-1 \right\}$ and $\varepsilon$ is the rating participation parameter. The rating function $R(u(x), s)$ is defined such that the consumer selects the rating level closest to her utility score. Some consumers choose not to rate the product because none of the available rating levels closely reflect their evaluation of the product.

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1 We do not model how to elicit sufficient and honest feedback to make the online rating system sustainable but assume that consumers will rate the products honestly based on their after-consumption utility. See (Mayzlin 2006) for potential manipulation of consumer rating.
As a result, this rating function creates a unique mapping between the consumer misfit and consumer ratings. For example, a consumer with misfit \( x \) generates a utility score \( w(x) \) based on her net utility \( u(x) \) and she will rate \( \frac{i}{s-1} \) if \( w(x) \in \left[ \frac{i}{s-1} - \varepsilon, \frac{i}{s-1} + \varepsilon \right] \). Parameter \( \varepsilon \) measures consumers’ propensity to participate in rating. When \( \varepsilon < \frac{1}{2(s-1)} \), not all customers will rate; otherwise, all customers will rate.

The consumer located at 0 (1) on the taste line has the highest (lowest) net utility and therefore gives the highest (lowest) rating. Other consumers choose the available rating level closest to their utility scores.

In this paper, we capture the rating results by the mean \( \mu(p_i,s) \) and volume \( n(p_i,s) \) of customer ratings. Given \( 0 < w(x) < 1 \), the inverse function is \( w^{-1}(w) = \frac{1}{i} \left( -\ln \left( \frac{w}{w+\varepsilon} \right) + p_i \right) \), which represents the customer’s misfit. Consider the case in which there are customer ratings for each of the rating scales \( \left\{ \frac{1}{s-1}, \ldots, \frac{s-2}{s-1}, 1 \right\} \). The mapping from customer rating score \( w(x) \) to customer misfit is as follows: consumers who give the highest rating (1) have a misfit value in \( \left[ 0, w^{-1}(1-\varepsilon) \right] \), where \( w^{-1}(1-\varepsilon) = \frac{1}{i} \left( -\ln \left( \frac{1-\varepsilon}{\varepsilon} \right) + p_i \right) \); consumers who give the lowest rating (0) have a misfit value in \( \left[ w^{-1}(\varepsilon), 1 \right] \), where \( w^{-1}(\varepsilon) = \frac{1}{i} \left( -\ln \left( \frac{\varepsilon}{1-\varepsilon} \right) + p_i \right) \); consumers who rate \( \frac{i}{s-1} \) have a misfit value in \( \left[ \frac{w^{-1}(\varepsilon)}{s-1} + \varepsilon, \frac{1}{s-1} - \varepsilon \right] \), where \( w^{-1} \left( \frac{i}{s-1} + \varepsilon \right) = \frac{1}{i} \left( -\ln \left( \frac{1-\varepsilon}{\varepsilon} \right) - i \right) + p_i \) and \( w^{-1} \left( \frac{i}{s-1} - \varepsilon \right) = \frac{1}{i} \left( -\ln \left( \frac{1+\varepsilon}{\varepsilon} \right) - i \right) + p_i \). The case when all ratings exist implies that \( w^{-1}(1-\varepsilon) \geq 0 \) and \( w^{-1}(\varepsilon) \leq 1 \), i.e., \( p_i \leq \ln \left( \frac{1-\varepsilon}{\varepsilon} \right) \) and \( p_i \geq \ln \left( \frac{1-\varepsilon}{\varepsilon} \right) - v \). In the following analysis, we assume the above parameter conditions hold so that we can focus on the all ratings existing case. Therefore the review volume and the mean rating can be characterized as:

\[
n(p_i,s) = \int_0^{w^{-1}(1-\varepsilon)} f(x)dx + \sum_{i=1}^{s-2} \int_{w^{-1}(\varepsilon/s+\varepsilon)}^{w^{-1}(i/(s-1)+\varepsilon)} f(x)dx + \int_{w^{-1}(\varepsilon)}^{1} f(x)dx \tag{2}
\]

\[
\mu(p_i,s) = \frac{1}{n(p_i,s)} \left[ \int_0^{w^{-1}(1-\varepsilon)} f(x)dx + \sum_{i=1}^{s-2} \int_{w^{-1}(\varepsilon/s+\varepsilon)}^{w^{-1}(i/(s-1)+\varepsilon)} f(x)dx + \int_{w^{-1}(\varepsilon)}^{1} f(x)dx \right] \tag{3}
\]

where \( f(x) = \theta + 2x(1-\theta) \). These two key rating results \( n(p_i,s) \) and \( \mu(p_i,s) \) can be further simplified and detailed derivations are provided in Appendix A. The range of both review volume and mean rating is between 0 and 1.

Next we model the rating interpretation process and demonstrate how second-period consumers learn from the product review results. In the second period, before consumption customers observe the first-period ratings and update their beliefs on the product quality \( v \) accordingly. However, customers cannot separate the impact of product quality and misfit on first-period customers’ ratings. Given a review scale level \( s \) and observing the mean rating \( \mu(p_i,s) \) and the volume of reviews \( n(p_i,s) \), the second-period consumers form their expected valuation on quality as:

\[
\hat{v}_z = n(p_i,s) \left[ \mu(p_i,s)(\bar{v} - v) + v \right] + \left[ 1 - n(p_i,s) \right] \left( \frac{\bar{v} + v}{2} \right) \tag{4}
\]
where \( \mu(p_i,s) \) and \( n(p_i,s) \) are given by equations (2) and (3). Second-period consumers’ updated belief of the product quality is a weighted average of the review-based belief and the no-review belief. Before consumption, consumers’ belief of the product quality is \( \mu(p_i,s)(\bar{v} - v) + v \) based on the review results and it is \( (\bar{v} + v)/2 \) without considering the review results. When updating their belief, the weight that consumers put on the review results is the review volume. In other words, consumers rely on the review results more if there are more reviews. Prior studies have identified several reasons for this type of consumer behavior such as awareness (Duan et al. 2008; Liu 2006), as well as credibility and trust (Fowler 2009).

The review system facilitates consumer learning. However, consumer learning is not perfect. Although the mean rating result indicates first-period customers’ overall evaluation of the product, second-period consumers cannot separate the effects of true quality and misfit in ratings. In addition, consumers may not fully rely on review results.

**Design of Consumer Review Systems**

We use \( s \in \{2,3,...,\bar{s}\} \) to denote the number of rating scale levels. For example, \( s = 2 \) corresponds to the like/dislike or recommend/not recommend case and \( s = 5 \) corresponds to the 5-star rating case. As the number of scale levels increases, it becomes costly for consumers to select the appropriate rating scales that correspond to their evaluations of the product and it becomes overwhelming for consumers who wish to learn from the review system to comb through the rating distribution. Thus we only observe limited options of rating scales in practice. In other words, rating levels are bounded by consumers’ capacity to evaluate the product\(^2\). Therefore we examine the firm’s optimal choice of the rating scale from a finite set \( s \in \{2,3,...,\bar{s}\} \), where \( \bar{s} \) is the maximum number of rating levels.

**Firm’s Review System Design and Pricing Strategies**

We consider a review system that solicits and displays consumers’ product ratings. The firm’s review system design decision is to choose the scale of consumer ratings.

**Formulation**

In the first period, anticipating consumers’ before-consumption expected utility function, the firm sets its price \( p_i \leq \bar{p}_i = \bar{v}_i - \bar{\xi}_i = \frac{\bar{v} + v}{2} - \left( \frac{2}{3} - \frac{\theta}{6} \right) \) to achieve a positive sale, where \( \bar{p}_i \) is the maximum price the firm can charge such that first-period consumers will purchase. The value \( \bar{p}_i \) is determined by consumers’ expected before-consumption gross utility. We assume that the expected quality is higher than the expected misfit cost, i.e., \( \bar{p}_i = \frac{\bar{v} + v}{2} - \left( \frac{2}{3} - \frac{\theta}{6} \right) \geq 0 \). This assumption ensures that it is feasible for the firm to set a positive price and make a positive sale. As a result, all consumers purchase the product in the first period and the firm’s profit is \( \pi_1 = p_i \). After consumption, customers learn about their realized utility given by \( u(x) = v - tx - p_i \) and rate the product according to the rating function \( R(u(x),s) \) defined in (1).

Note that the firm may charge a price lower than \( \bar{p}_i \) such that customers will have higher after-consumption utility, which in turn will positively impact first-period reviews. Lemma 1 summarizes the properties of consumers’ rating results – rating volume \( n(p_i,s) \) and mean rating \( \mu(p_i,s) \). The proofs of all lemmas and propositions are delegated to Appendix B.

\(^2\) Commonly observed scale levels include 2, 5, and 10.
Lemma 1 (Properties of rating volume and mean rating):

(a) Mean rating $\mu(p_i, s)$ decreases in the first-period price ($p_i$) and increases in the product quality ($v$) regardless of the product type.

(b) For niche products, rating volume $n(p_i, s)$ increases in the first-period price ($p_i$) and decreases in the product quality ($v$);

For popular products, rating volume $n(p_i, s)$ decreases in the first-period price ($p_i$) and increases in the product quality ($v$);

For neutral products, rating volume $n(p_i, s)$ is independent of the first-period price ($p_i$) and product quality ($v$).

When the product quality increases, consumers' after-consumption utility increases and therefore the mean rating $\mu(p_i, s)$ increases. When the first-period price increases, consumers' after-consumption utility decreases and therefore the mean rating $\mu(p_i, s)$ decreases. As a result, a higher mean rating signals a better-quality product to second-period consumers.

Decreasing the first-period price and increasing the product quality have similar impacts on the review volume and their impacts depend on the product type. Lowering the first-period price results in higher after-consumption utility for all consumers. In terms of the mapping from consumer utility score to customer taste location, the corresponding thresholds of consumer taste locations for each rating level shifts to the right. That means there are more consumers located close to the product on the taste line give the rating 1, and fewer consumers located close to 1 on the taste line give the rating 0. For popular products, because more consumers are located close to the product and fewer are located close to 1, the net result is that the total rating volume increases. In contrast, for niche products, more consumers are located close to 1, and the net result is that the total rating volume decreases with fewer consumers giving low ratings. For neutral products, because the consumer taste density is a constant, the first-period price and product quality have no impact on rating volume for neutral products.

In the rating interpretation process, second-period consumers update their beliefs on product quality based on rating results. Lemma 2 describes the properties of the updated belief on quality.

Lemma 2 (Properties of the second-period consumers' expected valuation on quality):

The second-period consumers' expected valuation on product quality ($\hat{v}_2$):

(a) increases in the true product quality ($v$) and decreases in the first-period price ($p_i$);

(b) increases in the rating scale ($s$) for popular products, decreases in the rating scale ($s$) for niche products, and is independent of the rating scale ($s$) for neutral products;

(c) increases in the product popularity ($\theta$) and decreases in the unit misfit cost ($t$).

As shown in Lemma 1, a higher product quality or a lower price leads to a higher mean rating which signals a higher product quality. Lemma 2 shows that second-period consumers observe this signal and their perception of the product quality increases. The firm can manipulate the first-period price to influence first-period consumer reviews and therefore second-period consumers' expected valuation on quality.

For a given product quality level and a given price, consumers' perception of the product quality is higher for a popular product. Interestingly consumers' perception of the product quality is negatively related to unit misfit cost. This relationship is due to the fact that consumer ratings reflect their evaluations of both the quality and the goodness-of-fit of the product. When unit misfit cost decreases, new consumers observe an increased overall rating. However they cannot attribute this increase in rating to better quality or better fit.
Since product ratings reflect an overall evaluation by first-period customers, second-period consumers cannot learn about the product fit and thus they face the same level of fit uncertainty as first-period consumers, i.e., \( \tilde{x}_2 = \tilde{x}_1 = 2/3 - \theta/6 \). Thus the second-period consumers’ before-consumption expected utility is \( \tilde{u}_2 = \tilde{v}_2 - \hat{t}_2 - p_2 \). For second-period consumers to participate, the second-period price has to be \( p_2 \leq \tilde{v}_2 - \hat{t}_2 \) such that the second-period consumers’ before-consumption expected utility is nonnegative. In response, the firm sets the second-period price to \( p_2 = \tilde{v}_2 - \hat{t}_2 \) to maximize its profit. We focus on the more interesting case in which the true quality is high enough such that \( \hat{v}_2 > \hat{t}_2 \). As a result, all consumers will purchase in the second period and the second-period profit is given by \( \pi_2 = p_2 = \tilde{v}_2 - \hat{t}_2 \).

Therefore the firm’s overall decision problem can be specified as:

\[
\max_{p_1,s} \pi(p_1,s) = \pi_1 + \pi_2 = p_1 + \hat{v}_2 - \hat{t}_2 \\
\text{s.t. } 0 \leq p_1 \leq \bar{p}_1 \\
2 \leq s \leq \bar{s}, \ s \in \mathbb{Z}
\]

(5)

The firm sets the first-period price and selects the rating scale for the consumer review system to maximize its total profit of the two periods.

**Optimal Design of the Rating Scale**

Proposition 1 delineates the firm’s optimal design choice for rating scale of the review system.

**Proposition 1 (Optimal rating scale level):**

(a) For a popular product (\( \theta > 1 \)), it is optimal for the firm to offer \( s^* = \bar{s} \), the maximum number of rating levels;

(b) For a niche product (\( \theta < 1 \)), it is optimal for the firm to offer \( s^* = 2 \), the minimum number of rating levels;

(c) For a neutral product (\( \theta = 1 \)), rating levels have no impact on the firm’s profit.

We find that the firm’s optimal design for rating scale is contingent on the popularity of the product. For a popular product, a higher rating level \( s \) has a positive effect on the second-period consumers’ perception of the quality of the product (as shown in Lemma 2) which leads to a higher overall profit for the firm for a given first-period price. Therefore it is optimal for the firm to offer the maximum number of rating levels that is conventional. In contrast, for a niche product, a higher rating level \( s \) has a negative effect on the second-period consumers’ perception of the quality of the product. Therefore it is optimal for the firm to offer the minimum number of rating levels.

**Pricing Strategy**

The firm’s first-period pricing has two countervailing effects on its overall profit. Increasing the first-period price directly increases the firm’s first-period profit but indirectly decreases its second-period profit through its impact on consumer reviews. The consumer review system provides a mechanism for the firm to manipulate second-period consumers’ perception of the product quality. Specifically, the second-period consumers’ updated belief on the quality of the product is at its maximum when the firm offers the product for free (\( p_1 = 0 \)), and it is at its minimum when the firm sets the price to the maximum price such that first-period consumers will purchase (\( p_1 = \bar{p}_1 \)). Second-period consumers learn about the quality of the product through the review system. Note that since the review system does not disclose product attribute information, second-period consumers’ uncertainty about the product goodness-of-fit remains the same. In other words, consumers have the same expected misfit cost in both periods. First-period pricing only reduces product quality uncertainty and has no impact on product fit uncertainty.
The firm has to balance these two effects to maximize its total profit and the firm’s optimal pricing strategy depends on the product quality and the popularity of the product.

**Proposition 2 (Optimal pricing in the first period):**

(a) For a popular product ($\theta > 1$), if the true product quality is high with $v > \bar{p}_1 + \frac{t^2 - t\theta(v - \bar{v})}{2(1 - \theta)(v - \bar{v})}$, the firm will charge $p'_1 = \bar{p}_1$ in the first period; if the true product quality is medium with $t^2 - t\theta(v - \bar{v}) \\leq v \leq \bar{p}_1 + \frac{t^2 - t\theta(v - \bar{v})}{2(1 - \theta)(v - \bar{v})}$, the firm will charge $p'_1 = v - \frac{t\theta(v - \bar{v}) - t^2}{2(\theta - 1)(v - \bar{v})}$; if the true product quality is low with $v < \frac{t^2 - t\theta(v - \bar{v})}{2(1 - \theta)(v - \bar{v})}$, the firm will offer it for free in the first period.

(b) For a niche product ($\theta < 1$), if the true product quality is high with $v > \bar{p}_1 + \frac{t^2 - t\theta(v - \bar{v})}{2(1 - \theta)(v - \bar{v})}$, the firm will offer it for free in the first period; if the true product quality is low with $v \leq \bar{p}_1 + \frac{t^2 - t\theta(v - \bar{v})}{2(1 - \theta)(v - \bar{v})}$, the firm will charge $p'_1 = \bar{p}_1$.

(c) For a neutral product ($\theta = 1$), if the misfit cost is high relative to the product uncertainty $t > \bar{v} - \bar{v}$, the firm will charge $p'_1 = \bar{p}_1$; otherwise it will offer the product for free in the first period.

The direct effect of the first-period price on the first-period profit is straightforward – the first-period profit linearly increases in the first-period price with a fixed rate for all products ($\partial \pi_1 / \partial \bar{p}_1 = 1$). The indirect effect of the first-period price on the firm’s second-period profit is more nuanced. Overall the second-period profit decreases in the first-period price for all products ($\partial \pi_2 / \partial \bar{p}_1 < 0$). However, product characteristics (product popularity $\theta$ and product true quality $v$) moderate the magnitude of this indirect effect ($\partial \pi_2 / \partial \bar{p}_1$). Specifically, increasing the true product quality amplifies the indirect effect for niche products ($\partial \partial \pi_2 / \partial \bar{p}_1 > 0$ when $\theta < 1$) while diminishes the indirect effect for popular products ($\partial \partial \pi_2 / \partial \bar{p}_1 < 0$ when $\theta > 1$). For neutral products, the true product quality has no impact on the indirect effect ($\partial \pi_2 / \partial \bar{p}_1 = 0$ when $\theta = 1$) and the magnitude of the indirect effect is determined by product misfit and quality uncertainty ($\partial \pi_2 / \partial \bar{p}_1 = (\bar{v} - v)/t$). The firm balances the direct and indirect effects of the first-period price on the firm’s profit and adopts three possible pricing strategies.

The lower-bound pricing strategy by offering the product for free is optimal for high-quality niche products, low-quality popular products, and low-misfit neutral products since the negative indirect effect of the first-period price on the second-period profit dominates the positive direct effect of the first-period price on the first-period profit. The upper-bound pricing strategy is optimal for low-quality niche products, high-quality popular products, and high-misfit neutral products since the positive direct effect of the first-period price on the first-period profit dominates the negative indirect effect of the first-period price on the second-period profit. The interior pricing strategy is optimal for medium-quality popular products and the price is set at such a level that the positive direct effect equals the negative indirect effect.

We find that the firm’s pricing strategies serve different objectives. Through upper-bound pricing, the firm pursues the maximum first-period profit. Here the firm takes advantage of the information asymmetry on quality by charging the maximum possible price in the first period. Through lower-bound pricing, the firm aims to maximize second-period profit by sacrificing its first-period profit. Here the firm
manipulates the first-period price to its lowest possible level to signal a higher quality to future consumers through the review system.

Another interesting finding is that the firm’s optimal design of rating scale and optimal pricing strategy are different for popular and niche products. The firm utilizes a high rating scale for popular products but a low rating scale for niche products. When the product quality is relatively high, the firm selects upper-bound pricing for popular but lower-bound pricing for niche products. When the product quality is relatively low, the firm selects lower-bound pricing for popular but upper-bound pricing for niche products.

**Concluding Remarks**

Consumer review systems have become an important marketing communication tool through which consumers share and learn product information. This paper aims to analyze the review system design as firms’ strategic decision to facilitate consumer sharing and learning about their products. We explore the information role of consumer review systems. Before consumption, consumers are uncertain about the product quality as well as the product fit. Consumers rely on the product rating scores to learn the product quality. Product rating scores serve as an imperfect signal for product quality since consumers cannot separate the effects of product quality and product fit from the overall rating scores. Our research contributes to consumer reviews literature by systematically modeling product and consumer characteristics, a firm’s pricing and review system design decisions, and their impact on consumer review outcomes as well as future consumer learning about a given product. Our results have important implications for the design of online consumer review systems and firms’ corresponding strategic responses.

Existing literature has shown product reviews have significant impact on consumers’ purchasing decisions. However, little is known about the impact of the design of consumer review systems. This paper models the product rating and rating interpretation processes. Based on the proposed model, we show that firms’ design choices of product rating systems influence these two processes.

We find that firms’ optimal pricing and review system design decisions are contingent on contextual characteristics. Firms should carefully evaluate market conditions such as how the true product quality matches their consumers’ perception, whether their product caters to a mass market or a niche market, and how much consumers value the fit of the product. We find that a review system with low scale levels such as like/dislike is optimal for niche products and a review system with high scale levels such as 1-10 is optimal for popular products. Our results suggest different pricing strategies during the initial sale period for different product types. When the firm offers a niche product, it should set a lower price for a better-quality product to take advantage of the impact of positive word-of-mouth. When the offered product is popular, the firm is able to charge a higher price for a better-quality product to enjoy the direct profit from the initial sale without damaging the product review outcomes.

This paper studies the optimal design of seller-managed consumer review systems. There are several directions for future research. Design features other than the rating scales such as product feature disclosure might be important for consumer review systems. It would be interesting to investigate the optimal design problem if the consumer review system is managed by a third party. The current model assumes customers rate the product truthfully. However, customers may have a positive (negative) bias when rating a product with positive (negative) network externalities. The model can be expanded to address consumers’ untruthful rating behavior. There might be other relevant contextual factors such as consumers’ online experience and consumer identity. Experienced consumers may learn more from product reviews than novice consumers. Reviews written by true customers may be perceived differently from those written by anonymous users.
Appendix A: Derivations of Important Values

**Derivation of the review volume** \( n(p_i, s) \)

When there are ratings for each of the rating levels, the total review volume is given by

\[
n(p_i, s) = \int_0^{w^{-1}((s-1) \cdot e)} f(x) dx + \sum_{i=1}^{s-1} \int_{w^{-1}((s-1) \cdot i \cdot e)}^{w^{-1}((s-1) \cdot (i+1) \cdot e)} f(x) dx + \int_{w^{-1}(1-e)}^{1} f(x) dx , \quad \text{where} \quad w^{-1}(1-e) = \frac{1}{t} \left( v - p_i - \ln \frac{1-e}{e} \right),
\]

\[
w^{-1}\left( \frac{i}{s-1} + e \right) = \frac{1}{t} \left[ \ln \frac{1-e}{e} \left( (s-1) \cdot i + v - p_i \right) \right] , \quad w^{-1}\left( \frac{i}{s-1} - e \right) = \frac{1}{t} \left[ \ln \frac{1-e}{e} \left( (s-1) \cdot i - v - p_i \right) \right],
\]

\[
w^{-1}(e) = \frac{1}{t} \left( v - p_i + \ln \frac{1-e}{e} \right), \quad \text{and} \quad f(x) = \theta + 2x(1-\theta) . \quad \text{Substituting the corresponding terms into the expression of the review volume yields} \quad n(p_i, s) = 1 - \frac{2}{t^2} \left[ \frac{2(1-\theta)(v - p_i) + t\theta}{1} \right] \ln \left( \frac{1-e}{e} D_i \right),
\]

where \( D_i = \left[ \frac{1-(s-1)e}{1+(s-1)e} \right] \left[ \frac{2-(s-1)e}{2+(s-1)e} \right] \left[ \frac{(s-2)-(s-1)e}{(s-2)+(s-1)e} \right] \) for \( s \geq 3 \). When \( s = 2 \), the review volume can be simplified as \( n(p_i, s) = 1 - \frac{2}{t^2} \left[ \frac{2(1-\theta)(v - p_i) + t\theta}{1} \right] \ln \left( \frac{1-e}{e} \right). \)

**Derivation of the average rating** \( \mu(p_i, s) \)

The mean rating is given by \( \mu(p_i, s) = \frac{1}{n(p_i, s)} \left[ \int_0^{w^{-1}(1-e)} f(x) dx + \sum_{i=1}^{s-1} \int_{w^{-1}(i \cdot e)}^{w^{-1}((i+1) \cdot e)} f(x) dx \right] \). The denominator of \( \mu(p_i, s) \) is just the review volume \( n(p_i, s) \). We only need to derive the numerator of \( \mu(p_i, s) \). Let \( m(p_i, s) \) be the numerator of \( \mu(p_i, s) \). The first term of \( m(p_i, s) \) can be simplified as

\[
i \int_{w^{-1}(i \cdot e)}^{w^{-1}((i+1) \cdot e)} f(x) dx = \frac{1}{t^2} \left( v - p_i - \ln \frac{1-e}{e} \right)^2 + \frac{\theta}{t} \left( v - p_i - \ln \frac{1-e}{e} \right).
\]

The expression \( i \int_{w^{-1}(i \cdot e)}^{w^{-1}((i+1) \cdot e)} f(x) dx \) can be represented as \( \frac{2(\theta-1)(v - p_i) - t\theta}{t^2} \ln D_i - \frac{1-\theta}{t^2} \sum_{i=1}^{s-1} \left[ \ln \left( \frac{(s-1) - i}{i + (s-1)e} \right)^2 - \ln \left( \frac{(s-1) - i}{i + (s-1)e} \right) \right] \).

After further simplification, we can rewrite the numerator of the mean rating \( m(p_i, s) \) as

\[
\frac{1-\theta}{t^2} \left( v - p_i - \ln \frac{1-e}{e} \right)^2 + \frac{\theta}{t} \left( v - p_i - \ln \frac{1-e}{e} \right) - \frac{2(\theta-1)(v - p_i) + t\theta}{t^2} \ln D_i - \frac{1-\theta}{t^2} \sum_{i=1}^{(s-2)/2} \frac{s - 1 - 2i}{s - 1} \ln D_i \ln D_i,
\]

where \( \text{int}(s-2)/2 \) takes the integer part of the value of \( (s-2)/2 \),

\[
D_2 = \left[ \frac{(s-1) - i}{i + (s-1)e} \right] \left[ \frac{(s-1) + i}{i - (s-1)e} \right], \quad \text{and} \quad D_i = \left[ \frac{i + (s-1)e}{(s-1) - i} \right] \left[ \frac{(s-1) + i}{i - (s-1)e} \right] \quad \text{for} \quad s \geq 3.
\]

When \( s = 2 \), we can further simplify \( m(p_i, s) \) as \( m(p_i, s) = \frac{1-\theta}{t^2} \left( v - p_i - \ln \frac{1-e}{e} \right)^2 + \frac{\theta}{t} \left( v - p_i - \ln \frac{1-e}{e} \right). \)
Appendix B: Proofs of Propositions and Lemmas

Proof of Lemma 1

We first analyze the properties of rating volume \( n(p_i,s) \). Since \( \frac{\partial}{\partial \varepsilon} \left[ \frac{i-(s-1)\varepsilon}{i+(s-1)\varepsilon} \right] \) \( \frac{-2i(s-1)}{[i+(s-1)\varepsilon]^3} \) < 0 and \( \frac{\partial}{\partial \varepsilon} \left( \frac{1-\varepsilon}{\varepsilon} \right) \) \( \frac{-1}{\varepsilon^2} \) < 0, the natural logarithm term in \( n(p_i,s) \) decreases in \( \varepsilon \) and thus rating volume increases in \( \varepsilon \). At \( \varepsilon = \frac{1}{2(s-1)} \), the natural logarithm term is zero. Thus all customers rate when \( \varepsilon \geq \frac{1}{2(s-1)} \). When \( \varepsilon < \frac{1}{2(s-1)} \), the natural logarithm term is positive and not every customer will rate after consumption. Since the total market size is 1, this implies that \( 2(1-\theta)(v-p_i)+t\theta > 0 \). Since \( \frac{\partial n(p_i,s)}{\partial p_i} = \frac{4(1-\theta)}{t^2} \ln \left( \frac{1-\varepsilon}{\varepsilon} D_i \right) \) and \( \frac{\partial n(p_i,s)}{\partial v} = \frac{4(\theta-1)}{t^2} \ln \left( \frac{1-\varepsilon}{\varepsilon} D_i \right) \), for popular products the rating volume decreases in the first-period price but increases in the product quality whereas for niche products the rating volume increases in the first-period price but decreases in the product quality. For neutral products the rating volume is independent of the first-period price and product quality.

Next we analyze the properties of mean rating \( \mu(p_i,s) \). Mean rating \( \mu(p_i,s) \) is the weighted average of the reviews where each rating level is weighted by the corresponding number of ratings and the rating volume is the total weight. Since \( \mu(p_i,s) = m(p_i,s)/n(p_i,s) \), the relationship between mean rating and the first-period price \( \frac{\partial \mu(p_i,s)}{\partial p_i} \) can be represented as \( n(p_i,s) \frac{\partial m(p_i,s)}{\partial p_i} - m(p_i,s) \frac{\partial n(p_i,s)}{\partial p_i} \), where \( \frac{\partial m(p_i,s)}{\partial p_i} = \frac{1}{t^2} \left[ -2(1-\theta)(v-p_i-\ln \frac{1-\varepsilon}{\varepsilon} D_i) - \theta t \right] < 0 \) and \( \frac{\partial n(p_i,s)}{\partial p_i} \) is given earlier. For niche and neutral products \( (0 \leq \theta \leq 1) \), \( \frac{\partial n(p_i,s)}{\partial p_i} \geq 0 \) and thus the mean rating decreases in price. For popular products \( (1 < \theta \leq 2) \), the numerator of \( \frac{\partial \mu(p_i,s)}{\partial p_i} \) is less than \( n(p_i,s) \frac{\partial m(p_i,s)}{\partial p_i} - \frac{\partial n(p_i,s)}{\partial p_i} < 0 \) since we know that \( m(p_i,s) < n(p_i,s) \), \( \frac{\partial n(p_i,s)}{\partial p_i} < 0 \), and \( t \geq v-p_i+\ln \frac{1-\varepsilon}{\varepsilon} \). Thus the mean rating decreases in the first-period price for all product types. Since \( \frac{\partial \mu(p_i,s)}{\partial v} = -\frac{\partial \mu(p_i,s)}{\partial p_i} \), the mean rating increases in product quality.

Proof of Lemma 2

Substituting the \( n(p_i,s) \) and \( \mu(p_i,s) \) terms derived in Appendix A back into formula (4) yields that second-period consumers’ perceived quality is \( \hat{v}_2 = (\bar{v}-\bar{y}) \left( \frac{(1-\theta)}{t^2} \left[ (v-p_i)^2 + C_1 - C_2 \right] + \theta t (v-p_i) \right) \), where \( C_i = \left[ \ln \left( \frac{1-\varepsilon}{\varepsilon} \right) \right]^2 \) and \( C_2 = \sum_{s=2}^{s(u-1)} \left( \frac{s-1-2i}{s-1} \right) \ln D_2 \ln D_3 > 0 \). The second-period expected quality decreases in the first-period price since \( \frac{\partial \hat{v}_2}{\partial p_i} = -\frac{(\bar{v}-\bar{y})}{t^2} \left[ (2(1-\theta)(v-p_i)+\theta t) \right] < 0 \). The second-period expected quality...
increases in the true quality $v$ since $\frac{\partial \hat{v}_2}{\partial v} = \frac{\bar{v} - v}{t^2} \left[ 2(1-\theta)(v-p_i) + t\theta \right] > 0$, inferred from the condition of $n(p_i, s) \leq 1$. To check the impact of the rating scale on the second-period expected quality, we need to compare $\hat{v}_1(p_i, s+1) - \hat{v}_2(p_i, s) = \frac{(\theta - 1)(\bar{v} - v)}{t^2} \sum_{i=1}^{\text{int}(s-1)/2} \left( s - 2i \right) \ln D_s - \frac{(\theta - 1)(\bar{v} - v)}{t^2} \sum_{i=1}^{\text{int}(s-2)/2} \left( s - 1 - 2i \right) \ln D_s$, where $D_s$ and $D_t$ are in the same form as $D_2$ and $D_3$ but replacing $s$ with $s+1$. Because $i$ is up to \text{int}(s-1)/2 or \text{int}(s-2)/2, the values of $D_2$, $D_3$, $D_4$, $D_5$ are all greater than 1. For a given $i$, the values $\frac{s - 2i}{s}$, $D_s$ and $D_t$ increase in $s$, which implies that $\frac{s - 2i}{s} = \ln D_s \ln D_t > \frac{s - 1 - 2i}{s - 1} \ln D_s \ln D_t$. Hence the sign of the second-period expected quality difference depends on $\theta$. Specifically, $\hat{v}_1(p_i, s+1) > \hat{v}_2(p_i, s)$ for $\theta > 1$; $\hat{v}_1(p_i, s+1) < \hat{v}_2(p_i, s)$ for $\theta < 1$; and $s$ has no impact on profit for $\theta = 1$.

Under the parameter condition $\bar{p}_i + \ln \left( \frac{1 - e}{e} \right) < v < t + \bar{p}_i - \ln \left( \frac{1 - e}{e} \right)$, all rating levels are rated. This condition implies that $\frac{\partial \hat{v}_2}{\partial \theta} = \frac{\bar{v} - v}{t^2} \left[ (v - p_i)(t - v + p_i) + C_2 - C_1 \right] > 0$. Thus the second-period expected quality increases in the product popularity parameter $\theta$.

We next evaluate the impact of the unit misfit cost on the second-period expected quality. Since $\frac{\partial \hat{v}_2}{\partial t} = -\left( \bar{v} - v \right) \frac{2(1-\theta)(v - p_i)^2 + C_2 - C_1}{t^2} + \theta t(v - p_i)$, we know that $\frac{\partial \hat{v}_2}{\partial t} < 0$ for niche and neutral products ($\theta \leq 1$). At $\theta = 2$, we have $\frac{\partial \hat{v}_2}{\partial t} \bigg|_{\theta = 2} < 0$. Since the numerator of $\frac{\partial \hat{v}_2}{\partial t}$ is monotone in $\theta$, $\frac{\partial \hat{v}_2}{\partial t} < 0$ for popular products as well.

**Proof of Proposition 1**

To determine the firm’s optimal choice on the rating scale $s$, we just need to compare its profit level at the rating scale $s$ with that at $s+1$ for a given first-period price $p_i$. The firm’s profit function can be simplified as $\pi(p_i, s) = p_i + \hat{v}_2(p_i, s) - t\hat{v}_2$, and the profit difference is then given by $\pi(p_i, s+1) - \pi(p_i, s) = \hat{v}_1(p_i, s+1) - \hat{v}_2(p_i, s)$. As proved in Lemma 2, the sign of the profit difference depends on $\theta$. Specifically, $\pi(p_i, s+1) > \pi(p_i, s)$ for $\theta > 1$; $\pi(p_i, s+1) < \pi(p_i, s)$ for $\theta < 1$; and $s$ has no impact on profit for $\theta = 1$. As a result, the firm selects the maximum rating scale $s^* = \bar{s}$ for popular products and the minimum rating scale $s^* = 2$ for niche products.

**Proof of Proposition 2**

The first derivative of profit over $p_i$ is given by $\frac{\partial \pi}{\partial p_i} = 1 + \left( \bar{v} - v \right) \frac{\partial \hat{m}(p_i, s)}{\partial p_i} - \frac{\bar{v} - v}{2} \frac{\partial \hat{n}(p_i, s)}{\partial p_i}$, which can be simplified to $\frac{\partial \pi}{\partial p_i} = 1 - \left( \frac{\bar{v} - v}{t^2} \right) \left[ 2(1-\theta)(v - p_i) + t\theta \right]$. Thus $\frac{\partial^2 \pi}{\partial p_i^2} = \frac{2(1-\theta)(\bar{v} - v)}{t^2}$. For popular products ($\theta > 1$), $\frac{\partial^2 \pi}{\partial p_i^2} < 0$. Thus the profit function is concave in $p_i$. Solving the first-order condition
yields \( p_i = v - \frac{t^2 - t \theta (\bar{v} - v) - t}{2(\theta - 1)(\bar{v} - v)} \). This interior solution is feasible if \( 0 \leq v - \frac{t^2 - t \theta (\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)} \leq \bar{p}_i \). Therefore if the true quality is high, i.e., \( v > \bar{p}_i + \frac{t^2 - t \theta (\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)} \), then \( p_i = \bar{p}_i \); if the true quality is medium, i.e., \( \frac{t^2 - t \theta (\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)} \leq v \leq \bar{p}_i + \frac{t^2 - t \theta (\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)} \), then \( p_i = v - \frac{t \theta (\bar{v} - v) - t^2}{2(\theta - 1)(\bar{v} - v)} \); if the true quality is low, i.e., \( v < \frac{t^2 - t \theta (\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)} \), then \( p_i = 0 \). For neutral products (\( \theta = 1 \)), \( \frac{\partial \pi}{\partial p_i} = 1 - \frac{\bar{v} - v}{t} \), i.e., the profit function is linear in \( p_i \). If customers’ quality uncertainty is high relative to the misfit cost, i.e., \( \bar{v} - v > t \), then the profit decreases in \( p_i \) and thus \( p_i^* = 0 \). If customers’ quality uncertainty is low relative to the misfit cost, i.e., \( \bar{v} - v < t \), then the profit increases in \( p_i \) and thus \( p_i^* = \bar{p}_i \). For niche products (\( \theta < 1 \)), \( \frac{\partial^2 \pi}{\partial p_i^2} > 0 \). Thus the optimal \( p_i \) will take a boundary solution and we need to compare \( \pi(0, s) \) and \( \pi(\bar{p}_i, s) \). The profit difference \( \pi(\bar{p}_i, s) - \pi(0, s) \) can be simplified to

\[
\pi(\bar{p}_i, s) - \pi(0, s) = \bar{p}_i \left\{ \frac{(\bar{v} - v)(1 - \theta)}{t^2} \bar{p}_i + 1 - \frac{(\bar{v} - v)(2(1 - \theta)v + t\theta)}{t^2} \right\}.
\]

Therefore if \( v \leq \frac{\bar{p}_i}{2} + \frac{t^2 - t \theta (\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)} \), then \( p_i^* = \bar{p}_i \); if \( v > \frac{\bar{p}_i}{2} + \frac{t^2 - t \theta (\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)} \), then \( p_i^* = 0 \).

### References


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