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A COST SHARING TECHNIQUE FOR THE MANAGERS OF DISTRIBUTED COMPUTER NETWORKS

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ABSTRACT

Corporations moving towards global operations are finding it necessary to build distributed computer networks to connect the different branches of the organization. This creates a management problem — how best to share the cost of network resources among competing divisions. This paper develops and demonstrates a new technique to solve this problem. The technique is applicable to a wide variety of network configurations.

1. INTRODUCTION

Corporations moving toward global competition are finding it necessary to build large computer networks to interconnect branches of the organization. As this trend continues, management problems that are just beginning to surface will require innovative solutions. One such problem is determining a mutually agreeable way to share the cost of new network resources in a distributed computer environment. When network resources are shared, someone must eventually decide how to divide the initial fixed cost.

The method used to share these types of costs is very important because it can change the rank order of competing divisions within a corporation on both a net income and return on investment basis (Mautz and Skousen 1968). Typically, network allocations are based on rudimentary methods that are easy to apply, but generate inconsistent and arbitrary results (Lightfoot 1990). These methods are inadequate for the competitive environment of the future. An alternative method that objectively shares network costs is needed.

This paper describes the development and application of such a method. The technique is based on the game-theoretic Shapley Value and four derived simplifications. The Shapley Value provides an objective, algorithmic sharing solution, and the simplifications make the technique computationally feasible for realistic size networks. The paper describes the technique, derives the simplifications, and demonstrates it on an actual case involving a large hospital chain connected by a wide-area network.

2. METHODS TO SHARE COMMON COSTS

Several ways exist to share the initial cost of a computer network. These types of costs are known as common costs. Research has shown that the method selected to allocate common costs can affect the profitability status of divisions within a company; therefore, the method should be chosen carefully (Benston 1975).

The most fundamental method to share common costs uses a physical proxy to assign costs to objects. A proxy, also known as a basis, is a physical attribute that is easy to measure and is associated with the thing being allocated. For example, the number of terminal ports is a physical basis often used to share the cost of a mainframe computer. Physical allocation methods are designed to approximate the cause-and-effect relationship that links the costs (or revenues) incurred from a resource with the benefits received (Kleso and Weygandt 1983). The primary advantages of physical proxies are that they are inexpensive, unambiguous, and convenient to use (Kaplan 1982).

The problem with using a physical basis is that the proxy selected is only a surrogate for a true cause-and-effect relationship, and the quality of the surrogate is indeterminate (Vatter 1945). Using the example from above, dividing the cost of a mainframe based on the number of user ports totally ignores the amount of computer resources required. This allocation would occasionally share costs fairly, but the rest of the time it would be arbitrary. According to Kaplan (1982), physical bases are easy to apply, but beyond this there is little else good about them. They
are normally chosen because they look reasonable and are convenient — neither of which enhance the quality of the resulting allocations (Vatter 1945).

Other methods share common costs based on the ability to bear principle. This principle assumes that bigger entities (based upon some financial figure) use proportionally more resources; therefore, they should be assigned the largest portion of the resource cost. Likewise, smaller entities use fewer resources and should be charged less. For example, the cost of buying a vector processor could be divided in proportion to the net sales generated by the departments that use the processor. These methods use a convenient financial figure as a substitute for a clear cause-and-effect relationship. The advantages are that they are easy to use and easy to understand (Moriarity 1987).

Many researchers are highly critical of allocations that depend on the ability to bear principle because these methods can unfairly subsidize one group of users at the expense of another (Bodnar and Lusk 1977; Kaplan 1982; Vatter 1945). Allocations using this principle share costs based upon a relative measure that may have no relation to resource utilization. For instance, the sales generated by a department may have no connection to the use of the vector processor in the previous example. The result is that the quality of these allocations is inconsistent and a matter of personal judgement. According to Vatter (1945, p. 172), "judgement, even sound judgement, is not objective."

The Shapley Value is another method that can be used to share common costs. It distributes the synergistic benefits of cooperation to all participants of a coalition based upon the premise that the value of a participant is equal to the incremental benefit they generate by joining the coalition. Since the incremental benefit produced depends on the order that participants join the coalition, all possible orderings are considered equally probable and are weighted equally (Callen 1978). When used as an allocation procedure, Shapley based allocations are considered to be fair and equitable by virtually all researchers (Boatsman, Hansen and Kimbrell 1981).

Despite the potential benefits, the Shapley Value is not a common allocation method. The most often cited complaint is that Shapley based allocations require too much input data in realistic situations (Biddle and Steinberg 1984; Boatsman, Hansen and Kimbrell 1981; Hamlen, Hamlen and Tschirhart 1977; Scott 1981; Young, Okada and Hashimoto 1982). Specifically, a network with \( n \) participants requires \( 2^n - 1 \) pieces of information.

The solution to this is to develop simplifications to the Shapley Value for specific classes of problems (Dubey 1982; Littlechild and Owen 1973; Littlechild and Thompson 1977; Spinneto 1975). These simplifications take advantage of the natural structure found in the problem domain; hence, they reduce the data requirements significantly and produce the same cost sharing solution as the Shapley Value. The network cost sharing problem lends itself to the development of simplifications. The next section briefly describes the Shapley Value and the four simplifications useful for network cost allocation.

### 3. ALLOCATIONS USING THE SHAPLEY VALUE

The Value was originally developed by Lloyd Shapley as a way to determine the likely payoff to a group of players of an abstract game without actually having to play the game (Hamlen, Hamlen and Tschirhart 1977). Consequently, it provides an "a priori assessment of the situation based on either ignorance or disregard for the social organization of the players" (Shapley 1953, p. 316). Martin Shubik was the first to apply the Shapley Value to cost allocation problems. Shubik noted that the Shapley Value has "desirable incentive and organization properties" when used to allocate joint costs and revenues (Shubik 1962, p. 325). Other researchers followed suit by successfully applying the Value to other allocation related problems (Billera, Heath and Raanan 1978; Callen 1978; Littlechild and Owen 1973; Littlechild and Thompson 1977; Lochner and Whinston 1971, 1976; Mossin 1968).

The formula to calculate the Shapley Value is as follows. Let \( c \) be the characteristic function and \( i \) be any player in the game. The variable \( n \) is the total number of players in the game and \( s \) is the number in the coalition containing \( i \). The distribution \( x_i \) to participant \( i \) by the Shapley Value is calculated by:

\[
x_i = \sum_{s} \left[ \frac{(s - 1)!}{n!} \right]^{*} [c(S) - c(S - \{i\})]
\]

(Shapley 1953, p. 312).

The expression \([c(S) - c(S - \{i\})]\) is the incremental contribution that participant \( i \) makes to coalition \( S \) if it is a member of \( S \). Next, \( n! \) is the number of coalition permutations that can be created from the grand coalition. The Value is not concerned with all the permutations; rather, it is only concerned with those orderings where participant \( i \) comes after all participants in \( S \) and before all participants not in \( S \). This means there are only \((s - 1)! (n - s)!\) orderings of interest. Thus, the term \([s - 1] (n - s)!\) / \( n! \) is a weighting factor that assigns each coalition of interest an equal share of the marginal contribution generated by that coalition. This calculation is repeated and summed for all coalitions in which \( i \) appears in \( S \). The result is that each participant \( i \) is allocated a value equal to its expected incremental value (or marginal contribution) across all possible coalitions (Biddle and Steinberg 1984; Shubik 1962).

To demonstrate the Shapley Value, assume that three departments named 1, 2, and 3 decide to build and share a computer network. An analysis of their individual requirements determines that \( c(S) \), the cost of all possible coalitions for the three participants, is as follows:

\[
\begin{align*}
1 & = 500 \quad 2 = 400 \quad 3 = 600 \\
1 & = 450 \quad 2 = 350 \quad 3 = 700 \\
1 & = 400 \quad 2 = 500 \quad 3 = 800
\end{align*}
\]

Next, we need to calculate \( n! \) for all possible orderings of \( 1, 2, \) and \( 3 \) and sum them to get the total number of possible orderings. For example, the number of possible orderings for \((1, 2, 3)\) is the number of possible orderings of \(1, 2, \) and \(3\), which is \( 3! = 6 \). The number of possible orderings for \((2, 1, 3)\) is the number of possible orderings of \(2, 1, \) and \(3\), which is also \( 3! = 6 \). The number of possible orderings for \((1, 3, 2)\) is the number of possible orderings of \(1, 3, \) and \(2\), which is also \( 3! = 6 \). The number of possible orderings for \((2, 3, 1)\) is the number of possible orderings of \(2, 3, \) and \(1\), which is also \( 3! = 6 \). The number of possible orderings for \((3, 1, 2)\) is the number of possible orderings of \(3, 1, \) and \(2\), which is also \( 3! = 6 \). The number of possible orderings for \((3, 2, 1)\) is the number of possible orderings of \(3, 2, \) and \(1\), which is also \( 3! = 6 \). Therefore, the total number of possible orderings is \( 6 + 6 + 6 + 6 + 6 + 6 = 36 \).

Finally, we need to calculate the incremental contribution of each participant to each coalition. For example, the incremental contribution of participant 1 to coalition \((2, 3)\) is the cost of \((2, 3)\) minus the cost of \((1, 2, 3)\), which is \( 600 - 450 = 150 \). The incremental contribution of participant 2 to coalition \((1, 3)\) is the cost of \((1, 3)\) minus the cost of \((1, 2, 3)\), which is \( 700 - 500 = 200 \). The incremental contribution of participant 3 to coalition \((1, 2)\) is the cost of \((1, 2)\) minus the cost of \((1, 2, 3)\), which is \( 800 - 400 = 400 \). Therefore, the Shapley Value for participant 1 is \( 150 / 36 = 4.1667 \), the Shapley Value for participant 2 is \( 200 / 36 = 5.5556 \), and the Shapley Value for participant 3 is \( 400 / 36 = 11.1111 \).
Table 1. Shapley Cost Sharing Solution

| Department 1: | 1/3 (25,600) + 1/6 (30,050-15,210) + 1/6 (34,235-23,835) + 1/3 (34,235-30,050) = $15,340.00 |
| Department 2: | 1/3 (15,210) + 1/6 (30,050-25,600) + 1/6 (26,435-23,835) + 1/3 (34,235-30,050) = $6,245.00 |
| Department 3: | 1/3 (23,835) + 1/6 (34,235-25,600) + 1/6 (26,435-15,210) + 1/3 (34,235-30,050) = $12,650.00 |
|              |                                                                                       $34,235.00 |

Table 2. Parallel System Simplification

| Department 1: | x_1 = 1/3*(400) = $133.33 |
| Department 2: | x_2 = 1/3*(400) + 1/2*(1000-400) = $433.33 |
| Department 3: | x_3 = 1/3*(400) + 1/2*(1000-400) + (2600-1000) = $2,033.34 |
|              | $2,600.00 |

The calculations to determine the Shapley cost sharing solution for this network are shown in Table 1.

The Shapley formula requires exponentially more input data as the number of participants grows linearly; consequently, it is difficult to use on realistic size distributed networks. Four simplifications are introduced below to solve this problem. The simplifications are derived from the Shapley Value and take advantage of the natural structure found in network cost sharing problems.

**Equal Participation Simplification.** Some network resources do not exhibit capacity constraints and can be shared by all network participants. For example, assume that three departments named 1, 2, and 3 purchase a site license copy of desktop publishing software for $3,000. The equal participation simplification divides the cost where

\[ x_1 = \frac{c}{3} \]
\[ x_2 = \frac{c}{3} \]
\[ x_3 = \frac{c}{3}. \]

The cost to each of the three departments is therefore $3,000/3 or $1,000.

**Parallel System Simplification.** Other resources are shared in an overlapping manner where a subset of the full resource is adequate for some users. In these cases, \( a \), \( b \), and \( c \) represent the cost of substitute resources where \( a \leq b \leq c \). The parallel simplification divides the cost as follows:

\[ x_1 = \frac{1}{3}a \]
\[ x_2 = \frac{1}{3}a + \frac{1}{2}(b - a) \]
\[ x_3 = \frac{1}{3}a + \frac{1}{2}(b - a) + (c - b) \]

(Littlechild and Owen 1973).

Assume that three departments, named 1, 2, and 3, need access to a printing device. Department 1 requires a $400 printer, number 2 requires a $1000 printer, and department 3 requires a laser printer that costs $2600. The laser printer meets the needs of all three departments so its cost is divided using the parallel system simplification as shown in Table 2.

**Serial System Simplification.** Some network resources are not shared within the network. In these cases, the cost is allocated where,

\[ x_1 = a \]
\[ x_2 = b \]
\[ x_3 = c. \]
Table 3. Stepwise Simplification

| User 1: $x_1 = 370$ | 1/2(700−370−470) + 1/2(700−370−700) | + | 1/3(900−(700+700+900−370−470−700)) | $= \$161.66$
| User 2: $x_2 = 470$ | 1/2(700−370−470) + 1/2(900−470−700) | | 1/3(900−(700+700+900−370−470−700)) | $= \$311.67$
| User 3: $x_3 = 700$ | 1/2(700−370−700) + 1/2(900−470−700) | | 1/3(900−(700+700+900−370−470−700)) | $= \$426.67$
| | | | | $= \$900.00$

Three departments named 1, 2, and 3 require personal computers (PC). Department 1 needs twenty-five, department 2 needs seventeen, and department 3 needs thirty-two PCs. Each PC costs $2,500 and will not be shared. The total cost of $185,000 is divided using the serial simplification as follows.

| Department 1: $x_1 = 25 \times 2500 = \$62,500$ |
| Department 2: $x_2 = 17 \times 2500 = \$42,500$ |
| Department 3: $x_3 = 32 \times 2500 = \$80,000$ |

$= \$185,000$

Stepwise Serial Simplification. Many network resources must be purchased in predefined increments. For example, disk drives and devices with ports have this characteristic. If $m_1, m_2, \ldots, m_n$ is the cost to purchase a network resource sufficient to meet the needs of the coalition consisting of players 1, 2, ..., n, then the stepwise simplification divides the cost where

$$x_i = m_1 + \frac{1}{2} (m_{i2} - m_1 - m_i) + \frac{1}{2} (m_{i3} - m_1 - m_i) + \frac{1}{3} (m_{i3} - (m_{i2} + m_{i3} + m_{i3} - m_1 - m_2 - m_3))$$

To illustrate this, assume that three users named 1, 2, and 3 share a network hard disk. User 1 needs 20 megabytes, user 2 requires 30 megabytes, and user 3 requires 60 megabytes. Disk space is an additive resource; that is, the three users require at least the sum of their individual disk storage requirements; therefore, they need at least 110 megabytes.

Disk drives can only be purchased in certain predefined sizes. If a coalition of users requires more space than is available on a particular disk drive, they must purchase the next larger disk regardless of the amount of space they actually need (i.e., you cannot buy fractional disks). The available sizes and their cost are as follows: 20 meg $= \$370$, 40 meg $= \$470$, 80 meg $= \$700$, 110 meg $= \$900$. The stepwise simplification divides the $\$900$ cost for the shared 110 megabyte disk in the Table 3.

The four simplifications provide the same cost sharing solution as would be generated by the Shapley Value. The advantage of using them is that the input data requirements grow linearly with the number of participants. Thus, the simplifications overcome the primary obstacle of using the Shapley Value to share the initial cost of a distributed computer network.

4. THE HOSPITAL NETWORK CASE

The hospital chain being studied is composed of four hospitals located around a large metropolitan area. Each hospital is a semiautonomous entity owned by a common parent corporation. All four hospitals are linked by a wide-area network (WAN) that was recently installed to save money by sharing expensive equipment and software. The WAN is configured using the client-server model. This model creates a hierarchy where one hospital provides data processing services while the three other hospitals are the clients of that service.

The four hospitals in the chain are referred to by their location relative to the city (i.e., the Central, Southwest, Northeast, and Rural branches). The Northeast and Southwest hospitals connect to the Central data processing shop by T-1 microwave links. The Rural branch connects to the Central hospital by a T-1 telephone link. All the
communication links connect the outlying client hospitals to
the server at the Central branch (i.e., a star configuration).
This configuration is very flexible and could easily be
scaled-up to a much larger network.

The network manager for the hospital chain is responsible
for splitting the cost of the WAN among the four branch
hospitals. Historically, cost sharing within the chain has
been done by a combination of physically based methods
and informal negotiations. The network manager stated
that this process has resulted in unfair allocations in the
past. To illustrate this point, he explained that the allocat-
ion process normally gives the largest share of the cost to
the hospital that makes the original resource request. Once
the new resource is installed, all hospitals are granted equal
access. This unfairly penalizes the requesting hospital
and encourages all branches to delay requesting equipment in an
effort to lower their cost share. He is interested in finding
tools to share the WAN cost in a more objective and
repeatable manner.

5. THE HOSPITAL COST SHARING SOLUTION

The first step to sharing network costs using the Shapley
simplifications is to determine the network configurations
needed to satisfy the individual network users. In other
words, what resources would be needed if each hospital
built their own stand-alone network. The purpose of this is
to formulate the WAN cost sharing problem into a
four-player cooperative game. Following that, the com-
bined WAN that links all four hospitals is configured. This
information, along with the cost of WAN components, is
given in Tables 4 and 5.

Next, each network resource is categorized according to
how it is used in the WAN. This is done to determine
which simplification should be used to divide the cost of
each resource. The classification is based upon how the
resource is used in the configuration, not on any inherent
property of the resource; thus, the same resource could be
grouped differently in a different network. All network
resources can be grouped into one of the four categories
described below.

Nonshared resources. Hardware or software that are used
exclusively by a single network participant are classified as
nonshared resources. Examples are personal computers and
dedicated peripherals. The serial system simplification is
used to share the costs of these resources.

Shared resources without capacity constraints. Re-
sources that are shared and have infinite capacity (as far as
network users are concerned) are classified as shared
without constraints. Examples are site licensed software
and infrequently used peripherals. The equal participation
simplification is used if the full resource is required by
network users. If a smaller substitute resource would be
adequate for some network users, then the parallel system
simplification is appropriate.

Shared resources with capacity constraints. Resources
with distinct capacities that can be exceeded are classified as
shared with constraints. A trait of these resources is that
they must be acquired in predefined increments. Examples
are disk drives and devices with ports. The stepwise serial
simplification is used to share these resources.

Table 4. Individual Hospital Network Requirements

<table>
<thead>
<tr>
<th>Resource Description</th>
<th>Central</th>
<th>Northeast</th>
<th>Southwest</th>
<th>Rural</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC with 3270 &amp; Printer</td>
<td>200</td>
<td>150</td>
<td>60</td>
<td>30</td>
</tr>
<tr>
<td>Novell 386 NetWare</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>PS/2 Server &amp; Gateway</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Mainframe Requirement (MIPS)</td>
<td>25</td>
<td>19</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Disk Drive Requirement (gigabytes)</td>
<td>22</td>
<td>15</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>IBM Storage Controller</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>IBM Token Ring</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Ungerman-Bass Bridge</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Ethernet Segment</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Hospital Software</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 5. Combined Network Components and Costs for the Hospital Chain

<table>
<thead>
<tr>
<th>Resource Description</th>
<th>Quantity Required</th>
<th>Cost Each</th>
<th>Total ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC with 3270 &amp; Printer</td>
<td>440</td>
<td>3,170</td>
<td>1,394,800</td>
</tr>
<tr>
<td>Novell 386 NetWare</td>
<td>3</td>
<td>7,995</td>
<td>23,985</td>
</tr>
<tr>
<td>PS/2 Server &amp; Gateway</td>
<td>3</td>
<td>13,935</td>
<td>41,805</td>
</tr>
<tr>
<td>IBM Mainframe Configuration</td>
<td>1</td>
<td>7,398,680</td>
<td>7,398,680</td>
</tr>
<tr>
<td>IBM 7.56 GB Disk Drive</td>
<td>7</td>
<td>105,000</td>
<td>735,000</td>
</tr>
<tr>
<td>IBM Storage Controller</td>
<td>4</td>
<td>60,270</td>
<td>241,080</td>
</tr>
<tr>
<td>IBM Token Ring</td>
<td>4</td>
<td>10,000</td>
<td>40,000</td>
</tr>
<tr>
<td>Ungerman-Bass Bridge</td>
<td>4</td>
<td>4,825</td>
<td>19,300</td>
</tr>
<tr>
<td>Ethernet Segment</td>
<td>4</td>
<td>4,000</td>
<td>16,000</td>
</tr>
<tr>
<td>T-1 Microwave Link</td>
<td>2</td>
<td>69,400</td>
<td>138,800</td>
</tr>
<tr>
<td>T-1 Telephone Link</td>
<td>1</td>
<td>37,760</td>
<td>37,760</td>
</tr>
<tr>
<td>Hospital Software</td>
<td>1</td>
<td>250,000</td>
<td>250,000</td>
</tr>
</tbody>
</table>

Total Cost for Network 10,337,210

Complex shared resources. Resources that are shared based upon a complex set of circumstances are classified as complex shared. A resource is always classified as complex if it is needed only when coalitions form. Examples are internetwork gateways and bridges. The stepwise serial simplification can be used for most of these resources; however, some must rely on the Shapley Value.

Once all the resources of the full WAN configuration are categorized according to use, the appropriate simplification is applied to each resource separately. For example, the calculations below share the cost of the "PC with printer" resource using the serial simplification and information from Tables 4 and 5.

Central: 200 * $3,170 = $ 634,000
Northeast: 150 * $3,170 = $ 475,500
Southwest: 60 * $3,170 = $ 190,200
Rural: 30 * $3,170 = $ 95,100

$ 1,394,800

The cost division for the remainder of the resources is computed in a similar fashion. The results are shown in Table 6. The individual resource sharing solutions are then combined into a global sharing solution. In this way, the problem is broken down into smaller pieces that are easy to solve and then recombined for an overall solution.

The outcome of this process is identical to that produced by the Shapley Value and requires only a fraction of the input data needed for the Shapley calculations. In the hospital case, only four pieces of input data are required to share the cost of each network resource. This is significantly less than the fifteen pieces needed by the Shapley Value. This benefit increases exponentially as the number of network participants grows. The additive nature of the Shapley Value also allows the simplifications to be applied to any network that can be formulated as a game (i.e., all networks where individual user needs are determinable). Thus, the technique is generalizable to a wide variety of network configurations.

6. ANALYSIS OF THE COST SHARING SOLUTION

The cost sharing solution generated by the simplifications is best understood by examining each resource individually. Table 7 indicates the simplification used to share each resource. The reasons these simplifications are selected are given in the text that follows.

The serial system simplification is used to share the cost of the personal computers and their printers. This is appropriate for these resources because they are not shared among the hospitals in the network (i.e., users from one hospital do not use the PCs located in other hospitals). The serial simplification splits the cost so each hospital pays for the PCs they use.

The equal participation simplification is used to share the costs of the IBM token ring, the Ungerman-Bass bridge, the Ethernet segment, and the hospital administration software. This simplification is used with these resources because each hospital requires the full resource and the resources do not exhibit capacity constraints. The outcome of the calculation divides the cost by the number of hospitals requiring the resource. For example, four hospitals require the Ethernet segment, so each hospital is charged one-fourth of its cost.
Table 6. Shapley Simplification Solution for the Hospital Chain

<table>
<thead>
<tr>
<th>Resource Description</th>
<th>Central ($)</th>
<th>Northeast ($)</th>
<th>Southeast ($)</th>
<th>Rural ($)</th>
<th>Total ($)</th>
<th>Simplification Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC with 3270 &amp; Printer</td>
<td>634,000.00</td>
<td>475,500.00</td>
<td>190,200.00</td>
<td>95,100.00</td>
<td>1,394,800</td>
<td>serial</td>
</tr>
<tr>
<td>Novell 386 NetWare</td>
<td>7,995.00</td>
<td>6,662.50</td>
<td>6,662.50</td>
<td>2,665.00</td>
<td>23,985</td>
<td>step</td>
</tr>
<tr>
<td>PS/2 Server &amp; Gateway</td>
<td>13,935.00</td>
<td>11,612.50</td>
<td>11,612.50</td>
<td>4,645.00</td>
<td>41,805</td>
<td>step</td>
</tr>
<tr>
<td>IBM Mainframe Config.</td>
<td>3,618,735.00</td>
<td>2,910,801.66</td>
<td>505,801.67</td>
<td>363,341.67</td>
<td>7,398,680</td>
<td>step</td>
</tr>
<tr>
<td>IBM Disk Drive Req.</td>
<td>316,916.66</td>
<td>211,916.67</td>
<td>106,916.67</td>
<td>99,250.00</td>
<td>735,000</td>
<td>step</td>
</tr>
<tr>
<td>IBM Storage Controller</td>
<td>100,450.00</td>
<td>60,270.00</td>
<td>40,180.00</td>
<td>40,180.00</td>
<td>241,080</td>
<td>step</td>
</tr>
<tr>
<td>IBM Token Ring</td>
<td>10,000.00</td>
<td>10,000.00</td>
<td>10,000.00</td>
<td>10,000.00</td>
<td>40,000</td>
<td>equal</td>
</tr>
<tr>
<td>Ungerman-Bass Bridge</td>
<td>4,825.00</td>
<td>4,825.00</td>
<td>4,825.00</td>
<td>4,825.00</td>
<td>19,300</td>
<td>equal</td>
</tr>
<tr>
<td>Ethernet Segment</td>
<td>4,000.00</td>
<td>4,000.00</td>
<td>4,000.00</td>
<td>4,000.00</td>
<td>16,000</td>
<td>equal</td>
</tr>
<tr>
<td>T-1 Microwave Link</td>
<td>46,266.67</td>
<td>46,266.66</td>
<td>46,266.66</td>
<td>—</td>
<td>138,800</td>
<td>step</td>
</tr>
<tr>
<td>T-1 Telephone Link</td>
<td>3,146.67</td>
<td>3,146.67</td>
<td>3,146.67</td>
<td>28,320.00</td>
<td>37,760</td>
<td>step</td>
</tr>
<tr>
<td>Hospital Software</td>
<td>62,500.00</td>
<td>62,500.00</td>
<td>62,500.00</td>
<td>62,500.00</td>
<td>250,000</td>
<td>equal</td>
</tr>
</tbody>
</table>
| **Total Network Cost**     | **4,822,770.00** | **3,807,501.66** | **992,111.67** | **714,826.67** | **10,337,210** | |}

The following network resources are classified as being "shared with capacity constraints": the IBM mainframe configuration, IBM disk drive, IBM storage controller, Novell 386 NetWare, and the PS/2 Server. The costs of these resources are shared using the stepwise serial simplification because each has a capacity that can be exceeded by user demand and each must be purchased in predefined units.

The T-1 microwave link and the T-1 telephone link are classified as "complex shared" resources because they are only required when hospitals form coalitions. The stepwise serial simplification is used to share the costs of these resources. In all cases, the simplifications were easier to calculate than the Shapley Value.

7. SUMMARY OF THE COST SHARING SOLUTION

The overall WAN cost sharing solution is obtained by summing the allocated cost of each individual WAN resource to each hospital. Table 7 summarizes the final cost distribution. This cost division is fair and defensible because it is based strictly upon the objective network requirements of each branch hospital. It is an algorithmic solution that is repeatable and relatively easy to explain. It does not depend on a physical basis nor does it rely on politically charged negotiations. Best of all, it has all these benefits without the large input data requirement characteristic of Shapley based allocations.

Table 7. Summary of the Sharing Solution for the Hospital Network

<table>
<thead>
<tr>
<th>Hospital</th>
<th>Shared ($)</th>
<th>Percentage of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central</td>
<td>4,822,770.00</td>
<td>46.65</td>
</tr>
<tr>
<td>Northeast</td>
<td>3,807,501.66</td>
<td>36.83</td>
</tr>
<tr>
<td>Southwest</td>
<td>992,111.67</td>
<td>9.60</td>
</tr>
<tr>
<td>Rural</td>
<td>714,826.67</td>
<td>6.92</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>10,337,210.00</strong></td>
<td><strong>100.00</strong></td>
</tr>
</tbody>
</table>

8. CONCLUSION

Large corporations are finding it increasingly necessary to build distributed computer networks to connect the different branches of the organization. This creates a management problem: how best to share the cost of new network resources among competing divisions. Traditional cost sharing methods are inadequate for the task. This paper develops and demonstrates a new technique to solve the problem.
The technique is based on the game-theoretic Shapley Value and four derived simplifications. The simplifications have all the benefits of Shapley allocations and are easier to use. The technique is demonstrated by sharing the cost of an actual wide-area network used by a hospital chain. The method is applicable to a wide variety of network configurations; thus, it is a general solution to the problem.

9. REFERENCES


ENDNOTES

1. A game consists of a group of players who must select from a list of possible alternatives. A player is characterized as a rational decision-maker with some measure of free choice. Each alternative in a game has an expected outcome (or value) over which the players may have preferences. The possible combinations of players are called coalitions and the group containing all players is called the grand coalition. The tabulation of all coalitions and their value is called the characteristic function of the game.

APPENDIX

Given a three-person game,¹ the possible coalitions are: (1), (2), (3), (12), (13), (23), (123). Let \(x_1, x_2, x_3\) designate the Shapley Value of a game \(v\).

LEMMA 1. For each coalition \(S\), there is a game with \(v(123)=1\) such that

\[
\begin{align*}
x_i &= \frac{1}{|S|} & \text{if } i \in S \\
x_i &= 0 & \text{if } i \not\in S
\end{align*}
\]

where \(|S|\) is the number of players in coalition \(S\).

Proof. The game,

\[
\begin{align*}
v(T) &= 1 & \text{if } S \subseteq T \\
v(T) &= 0 & \text{if } S \not\subseteq T
\end{align*}
\]

is such a game. Only coalitions with positive cost are those containing \(S\). All other coalitions have zero cost. The chart below lists these games for each possible coalition of a three-person game. Henceforth, these games will be called coalition games.

<table>
<thead>
<tr>
<th>Games in the Basis, (v^*)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(12)</th>
<th>(13)</th>
<th>(23)</th>
<th>(123)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(2)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(3)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(12)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(13)</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(23)</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(123)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

LEMMA 2. The games in lemma 1 are a linearly independent set.

Proof. The matrix shown below can be formed from the identity matrix by column additions alone. Since the columns of the identity matrix are linearly independent, this must also be so of the matrix, \(v\).

---

¹This framework is illustrated for three-person games, although it applies to games of any size.
THEOREM 1. Any game can be written as a linear combination of coalition games.

Proof. Lemma 2 shows that coalition games form a linearly independent set. As there is one coalition game for each coalition, they also form a basis for the vector space of all three-person games. So, each three-person game can indeed be written as a linear combination of coalition games.

Let \( v_s \) represent the coalition game for coalition \( S \).

THEOREM 2. If game \( v = \sum_S a_s v_s \),

then the Shapley Value, \( (x_1, x_2, x_3) \), of game \( v \) can be written,

\[
x_i = \sum_{S \subseteq S} \frac{a_s}{|S|}.
\]

Proof. A basic property of the Shapley Value states that the value of a linear combination of games is, in fact, the linear combination of their Shapley Values. As the Shapley Value of the coalition game \( v_s \) is

\[
x_i = \frac{1}{|S|} \quad \text{if } i \in S,
\]

\[
x_i = 0 \quad \text{if } i \notin S,
\]

then this theorem merely restates this property. Henceforth this theorem shall be known as the equal participation simplification.

Theorem 1 produced one basis for the vector space of all three-person games. The following theorem produces another useful one.

THEOREM 3. Another basis is the set of seven equal-participation games,

\[
v = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{bmatrix}
\]
Proof. The matrix \( v \) can also be constructed from the identity matrix by column addition alone.

The games in this basis are characterized by the fact that, for some players \( i \):

\[
v(S) - v(S - \{i\}) = 0 \quad \text{for all } S,
\]

and the other players are symmetric.

THEOREM 4. For any game in the basis of theorem 3,

\[
x_i = 0, \quad \text{if } v(S) - v(S - \{i\}) = 0 \text{ for all } S \text{ and some, but not all } i
\]

\[
x_j = \frac{1}{A} \quad \text{otherwise.}
\]

Here, "A" is the number of symmetric players not meeting the equation from theorem 3 above.

Proof. This theorem states that the Shapley Value of player \( i \) is 0, \( (x_i = 0) \), if dropping player \( i \) from the game does not change the value to any coalition. If dropping player \( i \) from the game does change the value to any coalition, then that player, along with any other player that impacts the game value, equally share the game value.

Theorems 2 and 4 are the main ones used to develop simplified formulas for sharing the initial fixed cost of a computer network.

The Parallel System Simplification

THEOREM 5. If a game \( v_s \) has the structure

\[
\begin{array}{c|ccc}
 & a & b & c \\
\hline
v_s & b & c & c \\
\end{array}
\]

where \( a \leq b \leq c \), then its Shapley Value is

\[
x_1 = \frac{1}{3} a \\
x_2 = \frac{1}{3} a + \frac{1}{2} (b - a) \\
x_3 = \frac{1}{3} a + \frac{1}{2} (b - a) + (c - b)
\]

(Littlechild and Owen 1973).
Proof. This game can be written as the linear combination

\[
\begin{bmatrix}
  a & 1 & 0 & 0 \\
  b & 1 & 1 & 0 \\
  c & 1 & 1 & 1 \\
  b & 1 & 0 & 1 \\
  c & 1 & 1 & 1 \\
  c & 1 & 1 & 1 \\
\end{bmatrix}
\]

The Serial System Simplification

THEOREM 6. If a game \( v_s \) has the structure

\[
v_s = \begin{bmatrix}
  a \\
  b \\
  c \\
  a+b \\
  a+c \\
  b+c \\
  a+b+c \\
\end{bmatrix}
\]

then its Shapley Value is,

\[
\begin{align*}
x_1 &= a \\
x_2 &= b \\
x_3 &= c
\end{align*}
\]

Proof. This game can be written as the linear combination

\[
v_s = \begin{bmatrix}
  a & 1 & 0 & 0 \\
  b & 0 & 1 & 0 \\
  c & 0 & 0 & 1 \\
  a+b & 1 & 0 & 1 \\
  a+c & 1 & 0 & 1 \\
  b+c & 0 & 1 & 1 \\
  a+b+c & 1 & 1 & 1 \\
\end{bmatrix}
\]
The Stepwise Serial Simplification

Resources like disk space come in lumps. For example, anyone who needs 10 or fewer megabytes of hard disk storage must still acquire a 10 megabyte disk. Suppose $M_1, M_2, \ldots$, are the available 'lumps' of disk storage space while $v(1) = a$, $v(2) = b$, and $v(3) = c$ are the individual needs of the players. The structure of their cooperative game to acquire disk space is:

\[
\nu_s = \begin{vmatrix}
\nu(1) & m_1 &= \text{Cost of min } \{M_k : M_k \geq a \} \\
\nu(2) & m_2 &= \text{Cost of min } \{M_k : M_k \geq b \} \\
\nu(3) & m_3 &= \text{Cost of min } \{M_k : M_k \geq c \} \\
\nu(12) & m_{12} &= \text{Cost of min } \{M_k : M_k \geq a + b \} \\
\nu(13) & m_{13} &= \text{Cost of min } \{M_k : M_k \geq a + c \} \\
\nu(23) & m_{23} &= \text{Cost of min } \{M_k : M_k \geq b + c \} \\
\nu(123) & m_{123} &= \text{Cost of min } \{M_k : M_k \geq a + b + c \}.
\end{vmatrix}
\]

**THEOREM 7.** The Shapley Value for such a game is,

\[
x_1 = m_1 + \frac{1}{3} \left( m_{12} - m_1 - m_2 \right) + \frac{1}{2} \left( m_{13} - m_1 - m_3 \right) + \frac{1}{2} \left( m_{23} - m_2 - m_3 \right)
\]

\[
x_2 = m_2 + \frac{1}{3} \left( m_{12} - m_1 - m_2 \right) + \frac{1}{2} \left( m_{13} - m_1 - m_3 \right) + \frac{1}{2} \left( m_{23} - m_2 - m_3 \right)
\]

\[
x_3 = m_3 + \frac{1}{3} \left( m_{13} - m_1 - m_3 \right) + \frac{1}{2} \left( m_{23} - m_2 - m_3 \right) + \frac{1}{2} \left( m_{123} - (m_{12} + m_{13} + m_{23} - m_1 - m_2 - m_3) \right)
\]
**Proof.** The stepwise serial game can be written as the linear combination

$$
\begin{pmatrix}
    m_1 \\
    m_2 \\
    m_3 \\
    m_{12} \ = \ m_1 \ + \ m_2 \\
    m_{13} \\
    m_{23} \\
    m_{123}
\end{pmatrix}
= \begin{pmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    1 & 1 & 0 & 0 \\
    1 & 0 & 0 & 0 \\
    0 & 1 & 1 & 0 \\
    1 & 1 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
    m_1 \\
    m_2 \\
    m_3 \\
    (m_{12} - m_1 - m_2) \\
    m_{13} \\
    (m_{13} - m_1 - m_3) \\
    (m_{23} - m_1 - m_2 - m_3)
\end{pmatrix}
= \begin{pmatrix}
    0 \\
    0 \\
    0 \\
    0 \\
    0 \\
    0 \\
    1
\end{pmatrix}$