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# PSEUDO-CRITERION BASED GROUP CONSENSUS SEEKING

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## Abstract

*The procedure for the iterative unification of decision-makers' opinions is proposed. Since the adaptability of a quantitative model and comparability of individual group members' results are required, it is based on the alternative sorting analysis restricted by the localization principle. It is capable of the automatic adjustment of preferential parameters, which are in accordance with the pseudo-criterion concept. A mathematical optimization program is applied so that robust conclusions are obtained. To ensure convergence, the consensus and agreement measures are defined.*

**Keywords:** Interactive decision support, multi-criteria decision analysis, pseudo-criterion, sorting, group work, consensus

## Introduction

The application of the pseudo-criterion and the outranking relation concepts represents one of the fundamental approaches to decision analysis (Roy 1996). The use of indifference and preference thresholds deals in an effective and practical way with imprecision, indetermination and uncertainty of the data. Real-life applications of outranking methods show that a threshold model is easily accepted by decision-makers, while in contrast, capturing inaccuracies with probability distributions has been found to be somewhat difficult for people to understand (Miettinen and Salminen 1999). However, existing methods for group decision analysis, which are based on the concepts of pseudo-criterion and outranking relation, have several serious drawbacks: a substantial cognitive load is put on a person because of many required input parameters, a poor insight into the derivation of results from input data is given, a quantitative decision model is insufficiently adaptable, and most important of all, a credible and just group agreement is not assured.

In group decision-making, methods belonging to the ELECTRE (*ELimination Et Choix Traduisant la Réalité*) (Roy 1996, Mousseau et al. 1999) and PROMETHEE (*Preference Ranking Organisation METHod for Enrichment Evaluations*) (Brans et al. 1997) families perform a final evaluation of alternatives by compensating values of preferential parameters, set by individual group members. A decision thus results from aggregated values. However, these aggregations do not necessarily represent the opinion of any decision-maker. So, a chosen alternative might not be preferred by the majority of involved people; it could merely be a consequence of considerable disharmony within the group. Moreover, in the case of the ELECTRE methods, the credibility of a decision is also hindered by the fact that the coalition is limited to criteria weights, which denote just a subset of input data. For the above-listed reasons, a procedure is needed that takes into account the following facts:

- All preferential parameters are important in group decision-making.
- A consensus, or at least a compromise, should be reached. An alternative that is chosen according to average parameter values is neither a consensus nor a compromise.
- The level of consensus should be known.
- Equality among the involved people should be guaranteed.

It is a quite reasonable assumption that there exist considerable discrepancies between initial preferential specifications of individual group members. These differences can even increase during the course of discussion and mathematical analysis. Reaching an agreement about a subset of acceptable alternatives is therefore a hard task, which cannot be solved instantly, but rather requires a progressive, iterative, unremitting deepening of problem understanding, and also calls for the adapting of personal opinions in order to harmonize with beliefs of other involved decision-makers. To reach uniformity on individuals' views, an active mechanism for convergent group consensus seeking is needed. It should be able to tell a decision-maker how he can modify his preferential parameters so that they will – to as high degree as possible – correspond to the preferences of the whole group (Herrera-Viedma et al. 2002).

## Two-Categorical Alternative Sorting

To enable high adaptability of a quantitative model and high comparability of individual group members' results, which are required for the sake of reaching a consensus, the alternative sorting analysis is implemented. Sorting refers to the absolute assignment of a set of alternatives into pre-existing ordinally defined categories or classes (Zopounidis and Doumpos 2002). In contrast to the more usual ranking approach, where  $m \cdot (m - 1)$  relative pairwise comparisons between alternatives have to be considered, only  $m$  pieces of information about category memberships are needed. But sorting by itself does not guarantee a fulfillment of both specified conditions. For this reason, the localization principle is introduced. The global problem of assigning alternatives to  $p + 1$  ordered classes is reduced to the two-categorical partition of a set of feasible alternatives; all acceptable choices belong to the positive category  $C^+$ , while unsatisfactory ones are members of the negative category  $C^-$ . As a consequence, many advantages appear:

- Since any two categories have to be delimited by a reference vector, which is also termed a profile, and since for each profile and for each criterion at least four parameters have to be considered – a referential value on the criterion domain as well as indifference, preference and veto thresholds, instead of  $4 \cdot p \cdot n$  only  $4 \cdot n$  input values are necessary. The cognitive load is thus considerably decreased.
- Because of mental and time constraints, a decision-maker is rarely capable of altering many reference vectors at once. Therefore, it is difficult for him to figure out how different profiles affect alternative evaluation. But when he concentrates on only one profile instead, it is easy for him to modify referential values. By doing so, he can tighten or loosen demands and see what effect this has on alternative selection. Consequently, learning about a given problem situation, a decision model and advantages or weaknesses of alternatives is greatly improved.
- The dispersion of alternatives across classes is reduced. Comparability of individuals' results is therefore increased.
- Because fewer input parameters are required, the unification of opinions becomes an easier task.
- The problem localization principle enables semi-automatic derivation of criteria weights according to selective strengths of veto thresholds (Bregar et al. 2003).

An essential presumption is that two categories suffice for a correct choice. It is justified by the fact that the most interesting alternatives are the ones which belong to the best available class. In the end, these are the only considered alternatives, as the decision model has to be a sieve with the purpose of reducing the number of desired choices and assigning them to a single subset with as few elements as feasible. A delimitation between the best category and other categories therefore has far greater significance than delimitations between less favourable classes.

### ***Preferential Information***

In order to implement the alternative sorting analysis, some basic notions of the ELECTRE TRI method (Mousseau et al. 1999) are used. However, the concepts of ELECTRE TRI must be modified to enable group consensus seeking.

The set of alternatives is partitioned into two exclusive categories –  $C^+$  and  $C^-$ . The categories are delimited by the profile  $b$ , which is defined as a vector of  $n$  referential values on criteria domains. Let this vector be denoted as  $(g_1(b), g_2(b), \dots, g_n(b))$ . Let similarly  $g_j(a_i)$  denote the value of an alternative  $a_i \in A$  that is measured with regard to a criterion  $x_j \in X$ . The assignment of each alternative to either the positive or the negative class then results from comparisons of values  $g_j(a_i)$  with values  $g_j(b)$ , where  $j = 1, \dots, n$ . Because numerical evaluations are subject to imprecision, indetermination and uncertainty, and because people are unable to perceive small differences in data, it is essential that an alternative does not have to outperform the profile on all criteria to be sorted into the positive class  $C^+$ . Weaknesses on some criteria are therefore admissible and can be compensated with advantages on other criteria. Two intra-criterion parameters are needed to allow for compensation – the indifference threshold  $q_j$  and the preference threshold  $p_j$ . These thresholds form the basis for computing the indices  $c_j(a_i, b)$  and  $c_j(b, a_i)$ , which express the degree of concordance with the assertions “the alternative  $a_i$  is at least as good as the profile  $b$ ” and “the profile  $b$  is at least as good as the alternative  $a_i$ ” respectively. Each partial index considers a single criterion  $x_j$ . Its contribution to the aggregation is determined by the weighting coefficient  $w_j$ .

In real-life problems, alternatives having very poor values are not taken into consideration or they are modified in order to improve these values. This means that certain criterion weaknesses are not accepted to be compensated by good values on some other criteria. To model partial incompensation between criteria, the discordance concept is applied. It is based on the veto threshold  $v_j$ .

### ***Implementation of the Localization Principle***

The threshold model generally leads to three different types of binary relations: preference, indifference and incomparability. The incomparability relation occurs when there exist at least two conflicting criteria. In this case, the differences  $g_j(b) - g_j(a_i)$  and  $g_k(a_i) - g_k(b)$  exceed the veto thresholds  $v_j$  and  $v_k$  respectively (it is presupposed that both criteria  $x_j$  and  $x_k$  are maximized). Then neither the alternative  $a_i$  is treated to be at least as good as the profile  $b$  nor the profile  $b$  is treated to be at least as good as the alternative  $a_i$ . Since the profile  $b$  represents the delimitation of the categories  $C^+$  and  $C^-$ , it cannot be clearly stated whether the alternative should be assigned to  $C^+$  or to  $C^-$ . Consequently, the membership of  $a_i$  is undetermined.

A possible solution to the above problem is the introduction of the incomparability category (Jaszkievicz and Ferhat 1999). The approach gives an adequate insight into the characteristics of alternatives and thus enables the adaptiveness of personal preferences. However, an additional class hinders the comparison and the unification of individual group members' choices. Moreover, the incomparability category has to be an empty subset at the end of the performed analysis as it is meant to show alternatives that are neither acceptable nor unsatisfactory at a certain point in time. A decision-maker still hesitates over the status of these alternatives, but they eventually have to be unambiguously sorted.

It must be assured that each alternative is strictly better or worse than the single profile in order to enable two-categorical sorting. The localization principle thus calls for the prevention of the incomparability relation. To solve the “incomparability problem”, veto thresholds are treated asymmetrically. This is justified by the noncompensatory nature of the veto concept and originates from the explicitly regarded primary viewpoint of the logical evaluation of the truthfulness of the presupposed alternative assignment to the positive category  $C^+$ . This fixed point of view implicitly determines the complementary logical evaluation, which confirms or rejects the truthfulness of the assignment to the negative category  $C^-$ . The positive semantics can be mathematically denoted as:

$$a_i \in C^+ \Rightarrow a_i \notin C^-,$$

$$a_i \notin C^+ \Rightarrow a_i \in C^-.$$

In the sense of the consideration of a decision-maker's beliefs, it is only important whether the alternative  $a_i$  is good enough to be assigned to the positive category  $C^+$  and not whether  $a_i$  is convenient for  $C^-$ . This is utterly reasonable since alternatives belonging to  $C^+$  are solely chosen for further analysis or for implementation. The approach is compatible with Boolean logic, where two poles exist as well (Bisdorff 2000). It is necessary to evaluate one of them, while the second one is determined by the first one's negation.

In practice, asymmetry means that an alternative  $a_i$  with very poor values on some criteria is excluded from the positive class. It is not important though, if the profile  $b$  does not reach one or more veto thresholds when compared with  $a_i$ , because this information does not confirm that  $a_i$  is a member of the  $C^+$  class nor does it prevent the assignment of  $a_i$  to the  $C^-$  class. But small weaknesses of an alternative should be compensated. For this reason, indifference and preference thresholds are treated symmetrically. The interpretation of preferential information is thus symmetrically-asymmetrical and leads to the assignment rule. The alternative  $a_i$  is good enough to be sorted into the positive category  $C^+$ , when all its weaknesses that are measured according to the  $q_j$  and  $p_j$  thresholds are compensated with advantages and when no difference  $g_j(b) - g_j(a_i)$  exceeds the veto threshold  $v_j$  (it is again presupposed that criteria are maximized).

In contrast to the ELECTRE type methods, instead of two discordance indices, only one index  $d_j(a_i)$  is defined for each criterion  $x_j, j = 1, \dots, n$ . It is evident from the above explanation that the second index does not contribute to the classification in the context of the proposed semantics. It merely points to discrepancies in the characteristics of alternatives. However, even in this sense it cannot provide enough information to effectively guide a decision-maker in the process of preference elicitation. This originates from the concepts of the ELECTRE methods. Because of the discordance indices, the values of the credibility indices decrease. They can, however, fall under the specified  $\lambda$ -level independently of the number of criteria, which oppose a veto. Consequently, essential differences in the quality of alternatives, which are all labeled incomparable to the profile, may exist. So, in addition to the fact that the second discordance index says nothing about the classification, it does not offer a decision-maker much help in performing the model analysis either.

Since the incomparability relation no longer exists, another mechanism is introduced to indicate conflicting alternatives and to help a decision-maker express sensible and robust values of preferential parameters. It is founded on two concepts (Bregar et al. 2003):

- The noncompensatory influence of the asymmetrical veto thresholds is dealt with independently of the compensatory effect that the indifference and preference thresholds have in the selection process.
- The appropriate distance metrics are defined that indicate which alternatives are (un)robustly sorted.

**Aggregation of Partial Indices**

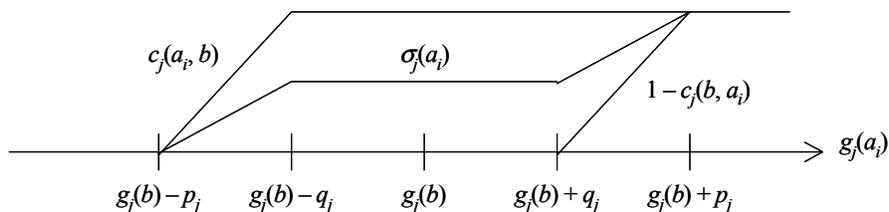
To express the degree of concordance with the assertion “the alternative  $a_i$  belongs to the  $C^+$  class”, the indices  $c_j(a_i, b)$  and  $c_j(b, a_i)$  are aggregated:

$$\sigma_j(a_i) = \frac{c_j(a_i, b) + (1 - c_j(b, a_i))}{2} = \frac{c_j(a_i, b) + \tilde{c}_j(a_i, b)}{2}$$

Since the indices  $c_j(a_i, b)$  and  $c_j(b, a_i)$  are defined on the  $[0, 1]$  interval and since the inequality

$$\min(c_j(a_i, b), \tilde{c}_j(a_i, b)) \leq \sigma_j(a_i) \leq \max(c_j(a_i, b), \tilde{c}_j(a_i, b))$$

holds, it is assured that the index  $\sigma_j(a_i)$  is regarded as a fuzzy averaging operator. Figure 1 shows its graphical interpretation for the case of a maximized criterion.



**Figure 1. The Degree of Concordance with the Assertion “ $a_i$  belongs to  $C^+$ ” with Respect to a Maximized Criterion**

On the interval  $[g_j(b) - q_j, g_j(b) + q_j]$ , the alternative and the profile are indifferent and the index  $\sigma_j(a_i)$  has the value of  $\tau$ . When the evaluation of the alternative exceeds the indifference threshold, the index  $\sigma_j(a_i)$  increases. It reaches the upper level of 1 at the preference threshold. Similarly, the degree of concordance decreases as the difference  $g_j(b) - g_j(a_i)$  becomes greater than the threshold  $q_j$ . For the sake of the compensation, the indices  $\sigma_j(a_i)$  are combined:

$$\sigma(a_i) = \frac{\sum_{j=1..n} w_j \cdot \sigma_j(a_i)}{\sum_{j=1..n} w_j}$$

As  $\sigma(a_i) = \tau$  denotes strict equality among the alternative and the profile, the classical  $\lambda$ -cut may be used to determine the “crisp” membership of the alternative:

$$a_i \in C^+ \Leftrightarrow \sigma(a_i) \geq \lambda, \text{ where } \lambda \in [\tau, 1]$$

Because of the introduced positive semantics and because the index  $\sigma_j(a_i)$  combines the indices  $c_j(a_i, b)$  and  $c_j(b, a_i)$ , there is no need to explicitly verify whether the alternative is a member of the negative category. This prevents logical nonsense, which can – according to Bisdorff (2000) – occur when applying standard outranking methods.

The fuzzy union operator is used to compute the degree of discordance with the assertion “the alternative  $a_i$  belongs to the  $C^+$  class”:

$$d(a_i) = \max_{j=1..n} d_j(a_i)$$

An alternative cannot be excluded from the positive class with greater certainty than it is excluded according to a criterion on which its performance is the poorest. The noncompensatory absolute influence of veto thresholds is the reason why the indices  $\sigma(a_i)$  and  $d(a_i)$  need not, and should not, be joint together.

## Compromise

The proposed two-categorical sorting ensures a compromise in a very simple way. An acceptable alternative is assigned to the positive class. It thereby receives one vote. As all group members operate on the same alternative set, votes are plainly added. Let  $o$  be the number of decision-makers and  $C_k^+$  the subset of alternatives that are approved by the  $k$ -th individual. Then the sum of votes for the  $i$ -th alternative is:

$$v_i = \text{card}(a_i \in C_k^+, k = 1, \dots, o)$$

Alternatives can now be ranked from the most preferable ones, for which  $v = \max_{i=1..m} v_i$  holds true, to those that receive the least votes. It is thus clear how many participants in the decision-making process agree upon a given choice and it can never happen that a decision is made, which is not in accordance with the opinion of the majority of people involved.

## Consensus and Agreement Measures

Since a high level of comparability is attained as a consequence of the applied localization principle, it is an uncomplicated task to define a consensus measure. Let  $z_i$  be the consensus degree reached for the  $i$ -th alternative. The equality  $z_i = 0$  holds true, if exactly half of individuals in the group assign the alternative  $a_i$  to the positive category  $C^+$  and the other half to the negative category  $C^-$ . In this case, the greatest possible separateness between decision-makers occurs. Consequently, it is totally undetermined whether  $a_i$  is an appropriate choice. On the contrary,  $z_i$  equals to 1 when all participants classify  $a_i$  into the same category. Then the group is perfectly uniform. For an odd number of decision-makers, the  $i$ -th partial consensus degree always exceeds the zero value as in the most inconvenient case,  $\rho = \lfloor o/2 \rfloor$  members assign the alternative to the first and  $\rho + 1$  members to the second, opposite subset. Let

$$v_i^+ = v_i \text{ and } v_i^- = o - v_i$$

denote how many participants assign the alternative  $a_i$  to the  $C^+$  class and to the  $C^-$  class respectively. Then:

$$z_i = \frac{v_i - \rho}{o - \rho}, \text{ where } v_i = \max(v_i^+, v_i^-)$$

An operator, which aggregates the partial consensus indices, should not only ensure compensation but has to consider the weakest alternative as well:

$$Z = \gamma \cdot \min_{i=1..m} z_i + (1 - \gamma) \cdot \frac{\sum_{i=1..m} z_i}{m}, \gamma \in [0, 1]$$

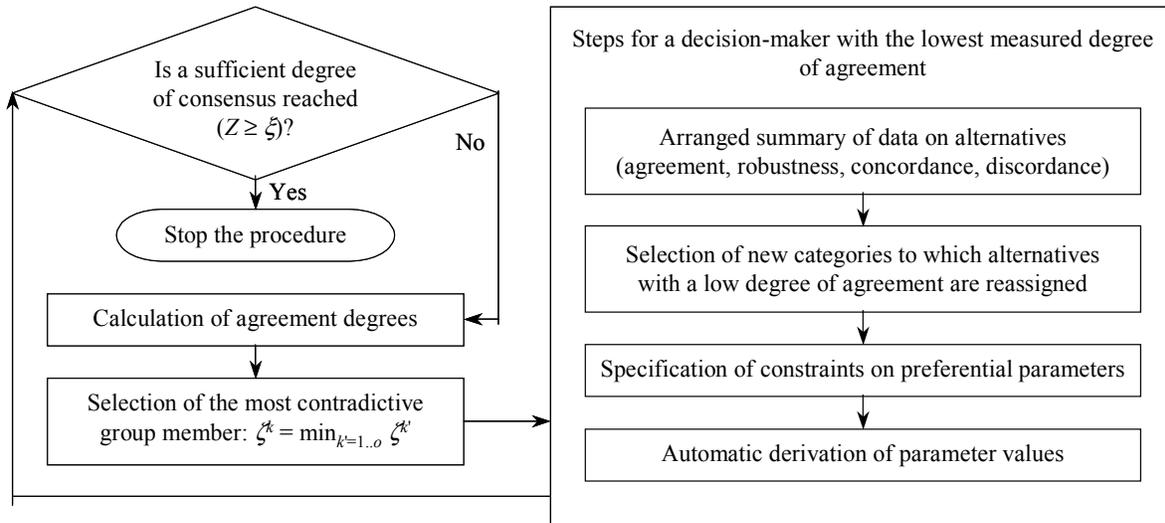
Another measure is important for the sake of active preference unification in the process of group consensus seeking. It is called the degree of agreement. If the  $k$ -th decision-maker assigns the  $i$ -th alternative to the same category as all the other group members, then he completely agrees with the majority opinion. Thus,  $\zeta_i^k = 1$ . On the contrary,  $\zeta_i^k = 0$ , if considering values of preferential parameters of the  $k$ -th individual,  $a_i$  belongs to the category that is in contradiction to preferences of all the other group members. Then this participant alone opposes the collective choice. So, the more people that assign an alternative to the same category as an individual does, the higher the level of agreement that is reached from the perspective of this person:

$$\zeta_i^k = \begin{cases} \frac{v_i^+ - 1}{o - 1} & , a_i \in C_k^+ \\ \frac{v_i^- - 1}{o - 1} & , a_i \in C_k^- \end{cases}$$

## Mechanism of Consensus Seeking

The active mechanism of directing group members toward unified opinions is founded on the progressive increasing of the consensus degree  $Z$ , that is on the convergence of aggregated values  $z_i$  toward the specified threshold  $\xi$ . At first sight, it seems that  $z_i$  has to be primarily raised for those alternatives which have reached a low degree of consensus ( $z_i \approx 0$ ). Their status is namely absolutely undetermined. Neither can they be with certainty assigned to the positive nor to the negative category since there does not exist a prevailing majority of decision-makers that would have enough strength to either approve or disprove the suitability of their selection. But (almost) total indetermination disables the decision support system to effectively advise a person about the adjustment of input preferential parameters. The adjustment is done for the purpose of excluding one or more alternatives from the current category and classifying them anew into the other category. In the case when exactly half of decision-makers assign  $a_i$  to  $C^+$  and another half to  $C^-$ , it is not evident in which direction the category change should be carried out. For this reason, the problem of reaching a consensus is approached from the other side. It is presumed that it is the most simple and credible task to increase the degree of consensus for all alternatives that already have a high value of  $z_i$ . If this value is close to 1 ( $z_i \approx 1$ , but compulsorily  $z_i < 1$ ), then two facts may be taken into account:

- It is clear toward which category the group opinion leans when evaluating an alternative with a high level of consensus. Thus, it is righteous to demand the category change from those individuals that oppose this opinion as they are in the absolute, uninfluential minority.
- At first, the group concentrates on alternatives that are rather uniformly judged by its members. A full agreement about very good alternatives can therefore quickly be found. As these alternatives generally suffice for the right decision, other alternatives need not be dealt with at all or can be left for later consideration.



**Figure 2. The Group Consensus Seeking Procedure**

A decision-maker with the lowest degree of agreement is selected. Since this participant is in the strongest opposition to the collective choice, his preferential attitude is the principal reason why the value of  $Z$  is not high enough. He has to adjust the values of input parameters to such an extent that someone else becomes the most contradictive group member. Because it is always “the turn” of the participant with the lowest computed agreement level, two important gains arise:

- the values of  $\zeta^k$  incessantly increase and ensure the convergence of  $Z$  toward the threshold  $\xi$ ,
- equality among involved decision-makers is guaranteed as the only measure of the required conformation to opinions of colleagues is the deviation from the collective choice, which is independent of the person's rank.

It is reasonable that a decision-maker reassigns only alternatives with a low value of the  $\zeta_i^k$  index or with a low robustness level. In the opposite case, either a satisfactory degree of agreement is reached from this person's perspective, or his opinion, which is expressed through the values of input parameters, is so firmly stated that the conformation to the group is not sensible in spite of a considerable contradiction with it. Therefore, the decision support system has to show the  $k$ -th group member all partial agreement indices ordered from the lowest to the highest. For each alternative  $a_i$ , data on its sensitivity have to be additionally interpreted. The obtained information enables the manual selection of alternatives, which are subject to reassignment. This is essential because a decision-maker must be able to reject the proposed category changes. When he is convinced that his judgment is right, he may insist on his own choice. Other participants are thereby stimulated to rethink about the decision, enlighten their understanding of the problem situation from another possible point of view, and consider important facts that they have perhaps overlooked.

Suppose a decision-maker specifies which non-robustly classified alternatives with a low agreement level he is prepared to reassign to the other category:

$$a_i \in C_k^+ \rightarrow a_i \in \tilde{C}_k^-$$

or

$$a_i \in C_k^- \rightarrow a_i \in \tilde{C}_k^+$$

With  $C_k^+$  and  $C_k^-$  original and with  $\tilde{C}_k^+$  and  $\tilde{C}_k^-$  new categories are denoted. New values of parameters of the decision model – referential values of the profile  $g_j(b)$ , thresholds  $q_j, p_j$  and  $v_j$ , and weights  $w_j$  – can now be automatically derived for each of  $n$  criteria so that the required changes  $C_k^+ \rightarrow \tilde{C}_k^-$  and/or  $C_k^- \rightarrow \tilde{C}_k^+$  are attained for the chosen alternatives  $a_i \in A$  and so that the

memberships of all other alternatives are preserved. The adjustment of parameters consists of two phases. At first, alternatives are found and treated for which the conditions  $a_i \in C_k^- \rightarrow a_i \in \tilde{C}_k^+$  and  $d(a_i) > \alpha$  hold true. A prerequisite for their classification into the positive category is to loosen the veto thresholds  $v_j$  for all criteria according to which the measured performances are intolerably poor. However, the elimination of the veto effect might not suffice for the change of the class membership. If  $a_i$  still remains an element of the negative subset, the index  $\sigma(a_i)$  needs to be raised as well. This is achieved by either adjusting the weight vector  $(w_1, w_2, \dots, w_n)$  or by increasing values of the  $c_j(a_i, b)$  indices or by decreasing values of the  $c_j(b, a_i)$  indices. Analogous modifications in the opposite direction are applied to substitute  $C_k^+$  with  $\tilde{C}_k^-$ .

To derive new values of the parameters  $g_j(b)$ ,  $q_j$ ,  $p_j$  and  $w_j$ , an approach is used which was defined at the Lamsade laboratory (Dias et al. 2002). The desired categories of all alternatives are known. The decision support system is thus confronted with the problem of parameter determination on the ground of a sorted alternative set. The problem is solved by a mathematical optimization program:

$$\begin{aligned} & \text{maximize } \min \{x_i, y_i\}_{i=1..m} \\ & \text{subject to} \\ & \sigma(a_i) - x_i = \lambda, \forall a_i \in \tilde{C}_k^+, \\ & \sigma(a_i) + y_i = \lambda, \forall a_i \in \tilde{C}_k^-, \\ & g_j(b) - v_j \geq D_j^-, \forall j, \\ & g_j(b) + p_j \leq D_j^+, \forall j, \\ & 0 \leq q_j \leq p_j, \forall j, \\ & w_j \geq 0, \forall j. \end{aligned}$$

The program does not take into account the veto thresholds. It is nonlinear because of the  $c_j(a_i, b)$  and  $c_j(b, a_i)$  functions. When only the weight vector is derived, it becomes linear however. The category membership changes at the value of  $\sigma(a_i) = \lambda$ . The variables  $x_i$  and  $y_i$ , respectively, must be positive or at least equal to 0 to ensure the assignment of an alternative to the proper category. It is obvious that in the case of positively evaluated alternatives only the variables  $x_i$  are considered. On the contrary, the variables  $y_i$  are bound to unsatisfactory alternatives. The program maximizes the lowest of the relevant  $x_i$  and  $y_i$  values. The robustness of the assignments is thereby achieved. If it is not required that alternatives are robustly sorted, a different goal function may be used, for example, such that the discrepancy between original and derived parameter values is as small as possible. The problem is not solvable whenever an alternative  $a_i$  exists for which  $x_i < 0$  or  $y_i < 0$ . Then a different separation of the  $A$  set into the  $\tilde{C}_k^+$  and  $\tilde{C}_k^-$  classes is required.

To acquire credible results from the optimization program, it is sensible that a decision-maker specifies additional constraints on model parameters, for example, intervals of suitable criteria weights. In this way, the modification of parameters that are in accordance with an individual's preferences is prevented.

## Conclusion

The contributions of this paper are the following:

- The two-categorical alternative sorting analysis based on the pseudo-criterion concept is introduced. To enable this kind of analysis, the asymmetrical treatment of veto thresholds is grounded and applied, and an appropriate fuzzy aggregation operator for computing the degree of concordance is defined.

- The first method for group consensus seeking in the context of pseudo-criterion based decision analysis is introduced. To ensure convergence of the proposed procedure, the consensus and agreement measures are defined.

Although the decision support system actively indicates which alternatives are not correctly sorted by a decision-maker, it is an individual's task to manually approve the proposed category changes. For this reason, a heuristic will be defined within the scope of further work that will enable the system to determine the lowest levels of agreement and robustness that preserve alternative memberships. In addition, the procedure will be statistically evaluated as it is necessary to prove its convergence toward just group choices.

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