December 2006

Weight-proportional Information Space Partitioning Using Adaptive Multiplicatively-Weighted Voronoi Diagrams

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ABSTRACT

We define the spatial constraints and objective function for weight-proportional partitioning of information spaces. We evaluate existing methods in light of these definitions and find that none performs as desired. We then formulate an alternative approach based on an adaptive version of the multiplicatively weighted Voronoi diagram; *i.e.*, the diagram's weights are computed based on a set of predefined area relationships. We test this adaptive solution using various sets of ideal-typical data and find that it behaves well. Next, we evaluate users' capabilities to properly estimate area relationships in this type of model. We compare the adaptively computed results with those from more traditional partitionings; *i.e.*, a rectangular partitioning and an ordinary, straight-lined Voronoi diagram. Surprisingly, we find that the adaptive multiplicatively weighted diagram outperforms the ordinary Voronoi solution.

Keywords

Information space, partitioning, Voronoi diagrams, weighted, algorithm, optimization, usability testing., trremaps

INTRODUCTION

As the volume and complexity of electronic information increase, interest in effective and efficient visualization of this information through the application of cartographic techniques increases accordingly; *e.g.*, Börner and Chen (2002); Börner *et al.* (2003), Chen, (1999), Couclelis (1998), Dodge (2001), Dodge and Kitchin (2001), and Peuquet and Kraak (2002). Fabrikant and Buttenfield (2001) and Skupin (2000) refer to these attempts as the use of spatial metaphor; *i.e.*, the use of spatial constructs such as distance, density and area, or topological constructs such as containment or adjacency to visualize the relationships between otherwise nonspatial phenomena. When applied to a set of informational items; *e.g.*, a set of journal articles, web pages, newspaper headlines or book titles, the result is considered a so-called 'information space.'

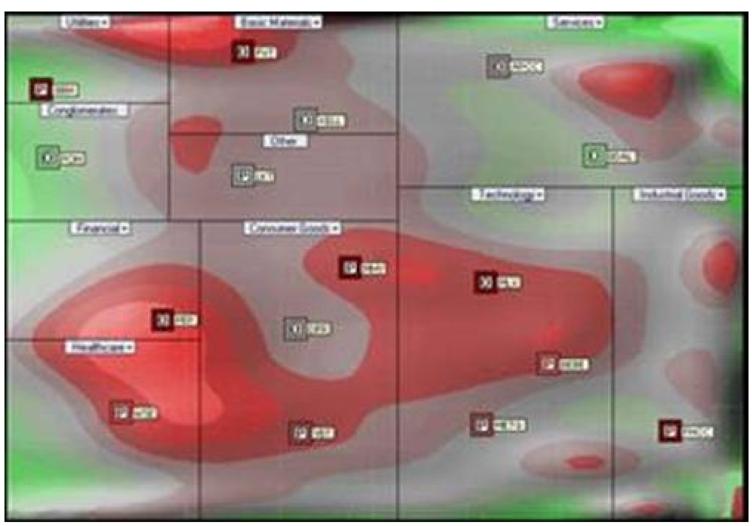

Figure 1: Rectangular information space partitioning by WebMap Technologies Inc.

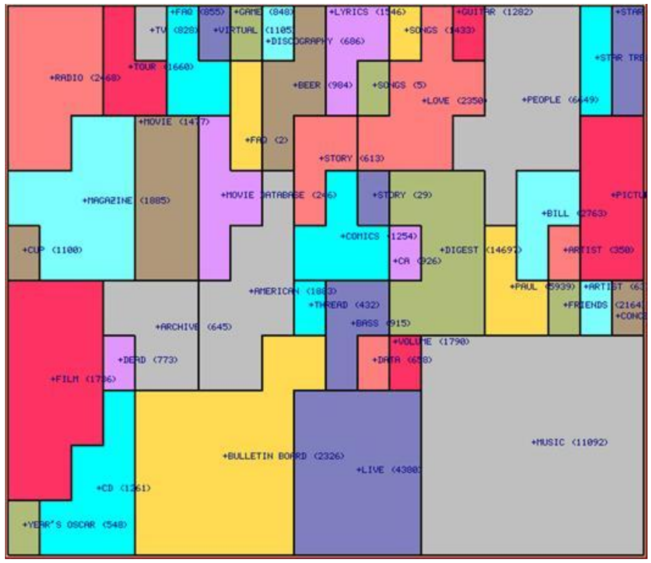

Figure 2: ET-Map by Chen *et al.* (1998).

Figure 3: InfoSky information space visualization by Andrews *et al.* (2002).

Figures 1 through 3 provide some examples of information spaces. Figures 1 and 2 show variations of so-called treemaps (Johnson and Shneiderman, 1991; Schvaneveldt *et al.*, 1989; Balzer *et al.* 2005). Figure 1 shows a treemap made commercially available by *WebMap Technologies*. Similar rectangular spaces can be found at *SmartMoney.com*. Figure 2 by Chen *et al.* (1998) represents a two-dimensional, rectangular space in which groups of Web pages are represented by colored, rectilinear regions. The space is hierarchical in that each of the regions themselves can be mapped again into a new, two-dimensional space revealing its inner composition. A region's area is meant to correspond to its weight or magnitude defined by the number of items it contains. Figure 3 shows an attempt at information space partitioning by Andrews *et al*. (2002). The space, again two dimensional, contains clusters of newspaper articles and is partitioned by means of a power Voronoi diagram (Okabe *et al.*, 2000).

In each of these cases the locations of regions are such that neighboring regions represent similar information items; *i.e.*, the distance between regions represents dissimilarity of their contents. Furthermore, the regions' areas are meant to represent the weight or magnitude of the information they contain; *e.g.*, the number of web links or newspaper articles or the relative amounts of stocks traded. In formal terms: If i and j represent regions and A_i represents the area of region i and W_i represents that region's weight or magnitude, then A_i and W_i would be proportional (1).

$$\frac{A_i}{A_j} = \frac{W_i}{W_j} \tag{1}$$

In the examples above, this common objective of weight-area proportional partitioning is operationalized using different methods, with different degrees of success. Although these methods seem to work from a user perspective –they have been empirically evaluated with respect to usability– they often only partly solve the partitioning problem. For instance, one partitioning may preserve weight but changes generator location whereas another maintains location but sacrifices weight-area proportionality.

We first present an objective (error) function that operationalizes (1); *i.e.*, weight-area proportionality and the constraints under which we want to minimize the error function. We follow this by a brief taxonomy of existing information space partitioning schemes and their capacity to minimize the error function while simultaneously satisfying these constraints. We then suggest the use of an adaptive multiplicatively weighted Voronoi diagram (AMWVD) as an alternative. An AMWVD is a multiplicatively weighted Voronoi diagram (MWVD) (Okabe *et al.*, 2000; Lan, 2004) that is computed backwards; *i.e.*, the generators' weights are considered dependent rather than independent variables. We provide an algorithm for computing the IWWVD and a set of idealtypical cases that test the capabilities of this AMWVD and find that it works quite well. However, since these AMWVDs are meant to be used as information visualizations, we must assess people's ability to 'take advantage' of their superior weight-area proportionality. Hence, we present the results of an experiment in which an AMWVD is tested against two other, well-known partitionings. Results are mixed, but by and large, AMWVDs perform quite nicely.

DEFINITIONS

Whereas (1) represents a general notion of area-weight proportionality, in order to assess partitioning methods we must choose a specific operationalization. Although many are possible −refer to Trubin (2006) for a discussion− we will use (2) in the remainder of the paper:

$$E = \sum_{i=1}^{n} \frac{\frac{|W'_i - A'_i|}{A'_i}}{n} \tag{2}$$

where W'_i and A'_i are proportions; *i.e.*, $W'_i = \frac{W_i}{\sum_{i=1}^{n} W_i}$ and $A'_i = \frac{A_i}{\sum_{i=1}^{n} A_i}$, and n is the number of regions.

Hence, E is the mean proportional error across all regions.

We specify the following constraints for minimizing (2) given a finite, Euclidean, orthogonal information space S_D with dimensionality D containing n points or generators g_i with magnitudes or weights W_i into n regions r_i each with area A_i.

- inclusiveness: $g_i \in r_i$ (3)
- exclusiveness: $\sum A_i = S_D$ (4)
- locality: $\Delta g_i = 0$ (5)

Constraint (3) states that each generator g_i is located inside its region r_i while constraint (4) states that all of the available space must be partitioned; *i.e.*, no unallocated space must remain. Constraint (5) states that the location of a generator cannot change; *i.e.*, generators cannot be moved in an attempt to satisfy the other constraints.

Andrews *et al.* (2002) additionally suggest that all regions must be convex. Whereas in their solution this enforces straight-lined region boundaries, we do not favor such a constraint because it prohibits some intuitive arrangements such as regions contained inside other regions.

Partitioning method	***Constraints***			***Objective Function***
	(3)	(4)	(5)	(2)
ET-Map (Chen *et al.*, 1998)	ü	ü		
Rectangular (*WebMap Technologies* & *SmartMoney.com*)	ü	ü		ü
Standard Voronoi diagram	ü	ü	ü	
Multiplicatively weighted Voronoi diagram	ü	ü	ü	
Power Voronoi diagram		ü	ü	
InfoSky (Andrews *et al., 2002*)	ü	ü		

Table 1: Partitioning methods and compliance (ü) with constraints (3),(4) and (5) and objective function (2).

Table 1 lists several well-known partitioning schemes, their compliance with the above constraints and their ability to minimize (2) (Trubin, 2006). Of these, the partitioning by Andrews *et al.* (2002) most closely approaches the ideal case. However, it too cannot successfully minimize (2) without violating at least one of the constraints because it only ordinally satisfies proportionality; *i.e.*, if $W_i > W_j$ then $A_i > A_j$ but $A_i/A_j \neq W_i/W_j$. Furthermore, Andrews *et al.*'s solution does not satisfy the locality constraint (5). Because power Voronoi diagrams often violate constraint (3); *i.e.*, a region must contain its own generator (Okabe *et al.*, 2000), Andrews *et al.* modified their generator placement methods to displace generators with the minimum amount necessary to comply with (3). Such displacement, however, violates the locality constraint (5).

ADAPTIVE MULTIPLICATIVELY WEIGHTED VORONOI DIAGRAMS

Voronoi diagrams are attractive partitioning tools because they inherently satisfy constraints (3) and (4). One type in particular −the multiplicatively weighted Voronoi diagram (MWVD)− offers some attractive opportunities. The MWVD is formally defined as in (6):

$$r_i = \left\{ x \mid \frac{\left\| x - x_{g_i} \right\|}{w_i} \leq \frac{\left\| x - x_{g_j} \right\|}{w_j} \right\} \qquad (6)$$

where x, x_{g_i} and x_{g_j} are location vectors and w_i and w_j are magnitudes determining how distances $\left\| x - x_{g_i} \right\|$ are weighted (Okabe *et al.*, 2000; Lan, 2004).

Figure 4 shows an MWVD using ten randomly located generators and a random set of weights. Notice how region boundaries have become arcs of Apollonius circles: sets of all points whose distances from two fixed points are in a constant ratio (Durell, 1928; Lan, 2004; Ogilvy, 1990; Okabe *et al.*, 2000). Also, note that a generator's region is no longer confined to those of its surrounding generators as generators with large weights can get space assigned that is beyond the far extent of some of their light-weighted neighbors.

The MWVD is attractive for weight-proportional partitioning because by manipulating the weights we indirectly manipulate the area of the resultant regions. In addition, MWVDs inherently satisfy constraint (5). The problem, however, is that the weights of an MWVD scale distance rather than area. Therefore, if we could find a weight set such that the resulting partitioning minimizes (2), we would simultaneously satisfy all constraints. The problem thus becomes an optimization problem: find a set of weights w_i such that (2) is minimized, subject to constraints (3), (4), and (5).

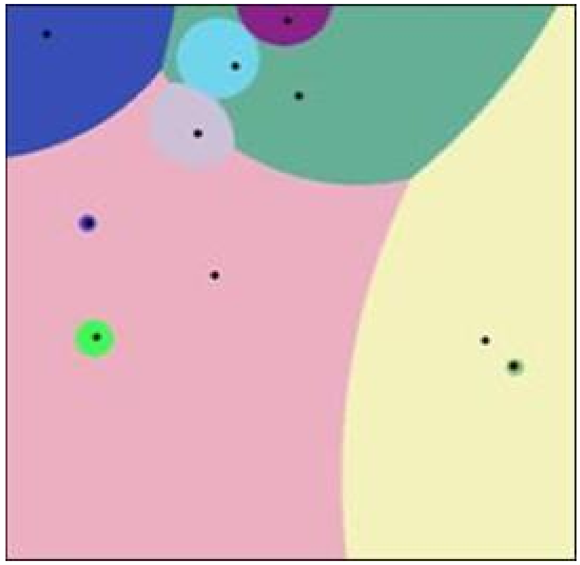

Figure 4: Multiplicatively Weighted Voronoi Diagram (MWVD).

ADAPTIVE ALGORITHM

As we are unaware of any analytical solutions to this problem, we suggest a method through which weights get iteratively adapted based on how well (2) is met at each iteration:

$$w_{i+1,j} = w_{i,j} + \Delta w_j \tag{7}$$

where $w_{i,j}$ is the weight of generator j at iteration i.

We define $a_{i,j}$ as the area allocated to generator g_j after iteration i. We define A_j as the target area of that generator. Both A_j and $a_{i,j}$ are proportions. Δw_j becomes positive if the generator receives less area than represented by $w_{i,j}$; Δw_j becomes negative if too much area is allocated. Weights at iteration $i=0$ represent the area targets; $w_{1,j} = A_j$.

We suggest making Δw_j proportional to three quantities: $w_{i,j}$, $A_j - a_{i,j}$, and a positive, nonzero scaling constant k *(8)*.

$$w_{i+1,j} = w_{i,j} + w_{i,j}\, k(A_j - a_{i,j}) \tag{8}$$

After regrouping of terms:

$$w_{i+1,j} = w_{i,j}\, (1 + k(A_j - a_{i,j})) \tag{9}$$

Choosing $k=1$ is a safe default as it will not result in negative weights. If $k=1$ we obtain:

$$w_{i+1,j} = w_{i,j}\, (1 + A_j - a_{i,j}) \tag{10}$$

ALGORITHM TEST RESULTS

We present several degenerate or pathological cases which test the algorithm for its abilities to minimize (2) subject to the constraints above (for a more complete overview of testing results refer to Reitsma and Trubin (2005) and Trubin (2006)).

Figures 5, 6 and 7 show results for the standard Voronoi diagram (A), the MWVD (B) and the AMWVD (C) respectively. All solutions were computed using a 1000x1000 grid.

- Figure 5 represents the problem of nine uniformly placed generators with a randomly selected weights $1 \leq W_i \leq 9$.

- Figure 6 shows the case of ten low-weight ($W_i = 1$) generators surrounding a single high-weight ($W_i = 10$) one. The central generator must be able to 'escape' its low-weight neighbors in order to claim a proportional amount of area.
- Figure 7 shows the degenerate case of ten in-line generators with weights linearly increasing from 1 on the left to 10 on the right.

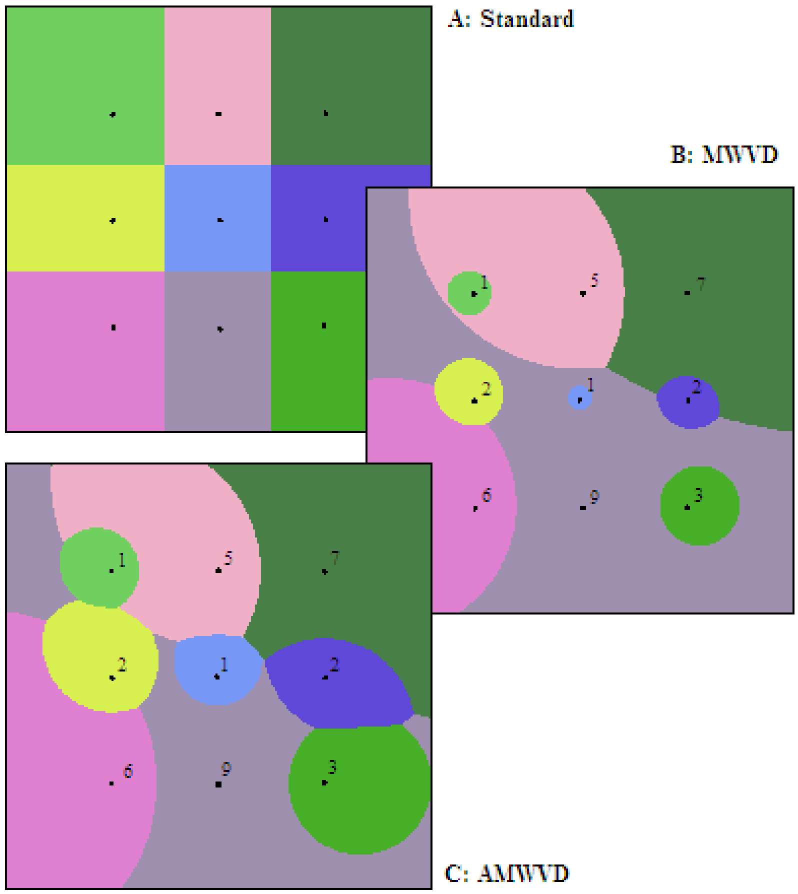

Figure 5: Standard (A), MWVD (B), and AMWVD (C) for nine evenly distributed points. (Integer) weights randomly selected between 1 and 9.

Note that whereas standard Voronoi diagrams ignore area-weight proportionality, in MWVDs, light-weight generators tend to be underallocated whereas heavy-weight ones become overallocated. The AMWVD algorithm, however, nicely minimizes (2) in each of these cases. Tests for scaled up versions of these cases under different spatial resolutions give similar results. For a full overview of the trajectories of the objective function for the various test cases, we refer to Reitsma and Trubin (2005) and Trubin (2006).

Finally, Figures 8 and 9 illustrate the importance of selecting appropriate spatial resolutions. The figures show the AMWVD solutions for the data displayed in Chen *et al.*'s ET-Map of Figure 1. Whereas Figure 8 uses the same spatial resolution (20x10) as Figure 1, Figure 9 uses a 1200x1200 resolution. Both AMWVD solutions are significant improvements over those in Figure 1: ($E_{\text{Figure 1}}$ = .825; $E_{\text{Figure 8}}$ = .51; $E_{\text{Figure 9}}$ = .002) but the higher resolution solution vastly outperforms the lower resolution one.

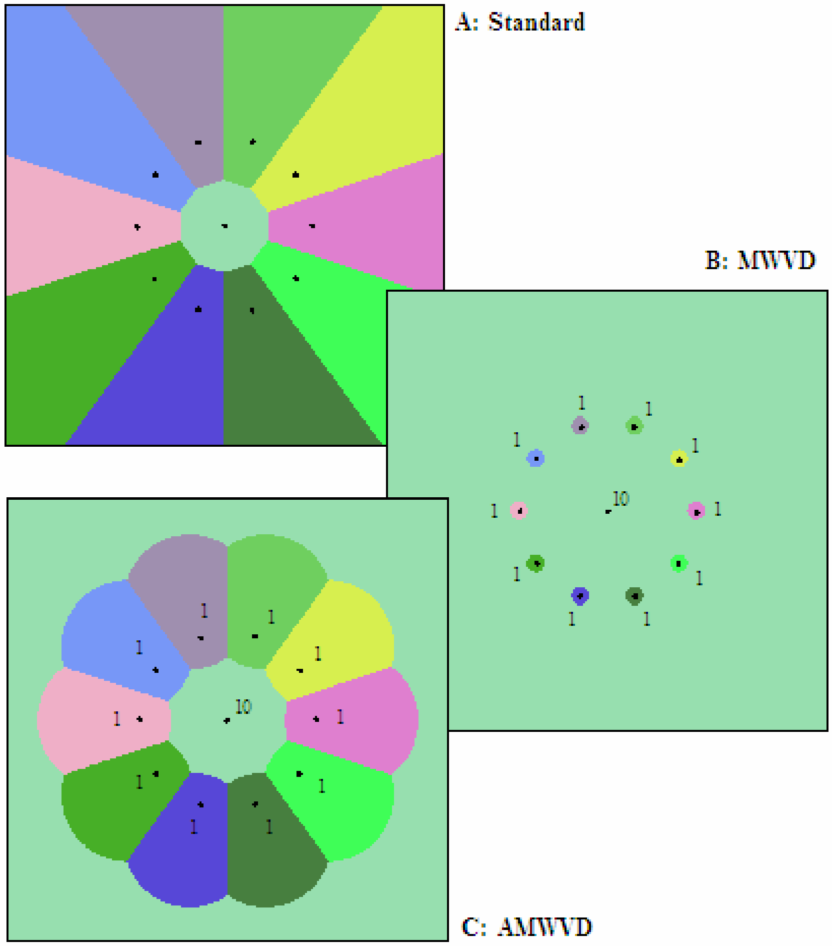

Figure 6: Standard (A), MWVD (B), and AMWVD (C) for a heavy-weight central generator surrounded by light-weight ones.

USER TESTING

Even if AMWVDs conform well to area-weight proportionality, the resulting maps may not be easy to interpret. Especially the curvilinear and sometimes discontinuous nature of this type of diagram may prevent proper estimation of area relationships. For instance, the Gestalt principles of *continuity*, *closure* and *area* (Wertheimer, 1923a, 1923b, 1923c) might prevent people from properly estimating the relative sizes of areas in (A)MWVDs. Hence, we might have to tradeoff the AMWVD's superior area-weight proportionality for a lack in human interpretability. To assess this potential trade-off we devised an experiment.

Of the many variables related to area cognition, our experiment concentrates on only two: the accuracy of the comparison and the time required to conduct that comparison. We consider both ordinal and ratio scale comparisons.
We selected three partitioning schemes for human subjects testing: the ordinary Voronoi diagram, the AMWVD, and a rectangular partitioning. The ordinary Voronoi diagram was selected because of its visual similarity with the power diagram suggested by Andrews *et al*. The rectangular partitioning was selected as it is used regularly in commercial information space products.

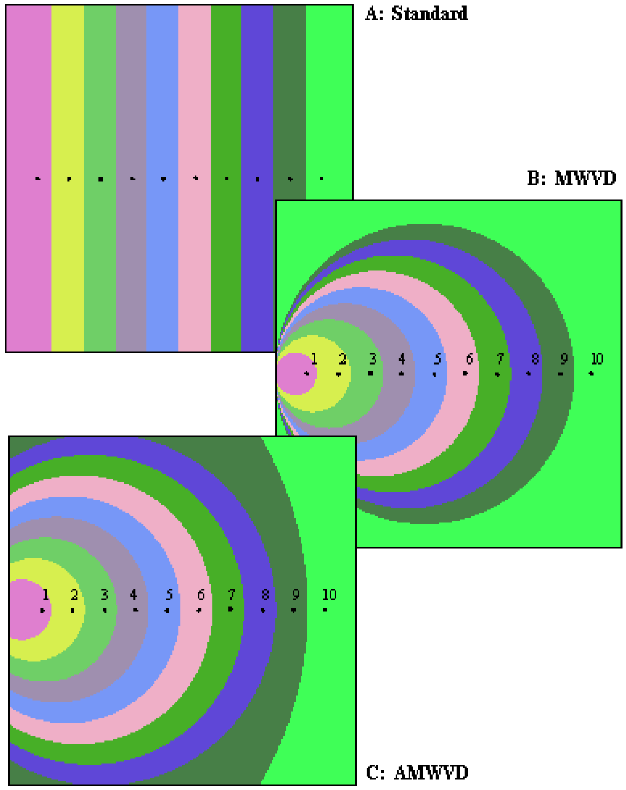

Figure 7: Standard (A), MWVD (B), and AMWVD (C) for nine in-line points. (Integer) weights increase linearly from left to right.

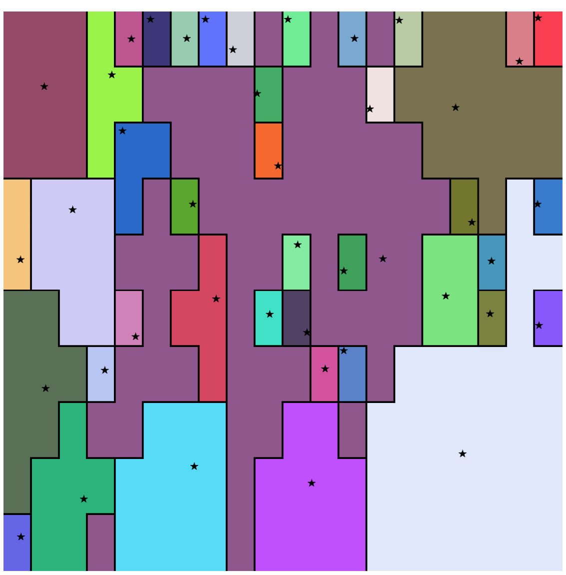

Figure 8: AMWVD of Chen *et al.*'s (1998) data; resolution: 20 x10.

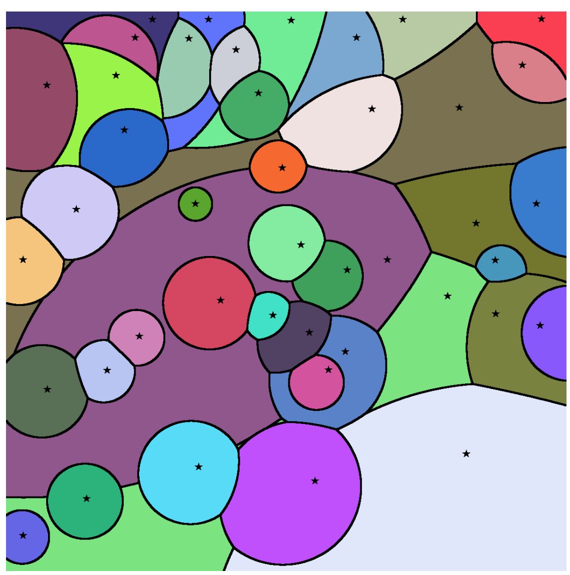

Figure 9: AMWVD of Chen *et al.*'s (1998) data; resolution: 1200 x1200.

Theory and Hypotheses

A 1970s series of cartographic studies addressed size estimation of graduated mapping symbols such as circles, squares and wedges; *e.g.*, Chang (1977), Cox (1976), Crawford (1971, 1973), Flannery (1971), Groop and Cole (1978) and Williams (1956). All studies found that when such shapes are used to convey magnitude, perceived size differences are systematically underestimated following Steven's power law in psychometrics (Stevens, 1957) which relates the subject's response R to the size of the stimulus S (11):

$$R = kS^n \qquad (11)$$

where k and n are empirical constants.

When estimating the size of graduated circles the exponent n is normally smaller than 1.

From these previous and well-documented results we derived the following hypotheses:

- *Hypothesis I*: For all partitioning methods, subjects will systematically underestimate the differences between area sizes. The relationship between actual and estimated size follows Steven's rule.
- *Hypothesis II*: Underestimation of size differences in rectangular and standard Voronoi partitioning is less than in (I)MWVD partitioning.

In a study of the effects of circle overlapping on circle size estimation, Groop and Cole (1978) compare transparent superposition with opaque superposition. Groop and Cole found that the sizes of transparently overlapping circles are more accurately estimated than those of opaquely overlapped circles. They also found a positive correlation between the amount of overlap and estimation error. These results question some of the Gestalt principles mentioned earlier. For instance, from the *continuity* principle one might predict that no significant differences between size estimation of transparent *vs*. opaque circles should be observed. As (I)MWVDs very much resemble opaquely overlapping circles; *e.g.*, Figure 9, they are intuitively sensitive to the Gestalt principle of *continuity*. Groop and Cole's results however, indicate that they might not be. Hence,

- *Hypothesis III*: Size comparisons involving overlapping circle patterns will show the same amount of error as those not involving such patterns.

Although the 1970s studies do not address size estimation involving discontinuous areas, we know these are not uncommon in (I)MWVDs (*e.g.*, Figure 7). Although we know of no existing empirical evidence, it seems reasonable to expect that size estimation errors involving discontinuous areas will be larger than those involving continuous areas. Hence,

- *Hypothesis IV*: Size estimation error involving discontinuous areas is larger than for those not involving discontinuous areas.

Usability Experiment

To test the above hypotheses, a simple experiment was conducted. A rectangular space partitioned into ten areas is displayed on a computer screen. The space is partitioned according to one of the three partitioning methods described above. Pairs of regions are then highlighted. Test subjects are asked to select the larger of the two. Next, subjects are asked how many times bigger the selected (larger) region is than the smaller. Each subject first conducts a series of seven training comparisons followed by 30 actual ones. All training measurements are discarded. Regions involved in the comparison are selected at random. In addition to the comparison data, the time required by subjects to conduct the comparisons is measured.

Test subjects were undergraduate (junior) Management Information Systems (MIS) students that volunteered to take part in the experiment in exchange for a small amount of extra class credit.

To test the hypotheses we ideally need data such that across partitioning schemes, generators all have the same locations and corresponding regions have the same areas. Clearly, this is not possible since only AMWVDs preserve weight-area proportionality and locality. A workaround was to reconstruct the positional and area information from *WebMapTechnologies*' published map (Figure 3) and use these data as input for an AMWVD. To be able to test *Hypothesis*

IV, we ran a series of AMWVD solutions using these coordinate and area information, randomly selecting generator positions from within the generators' regions. From the results, we selected an AMWVD solution containing a significant amount of discontinuity in two of its regions (Figure 10,C). We used these same coordinates for the standard Voronoi solution (Figure 10B).

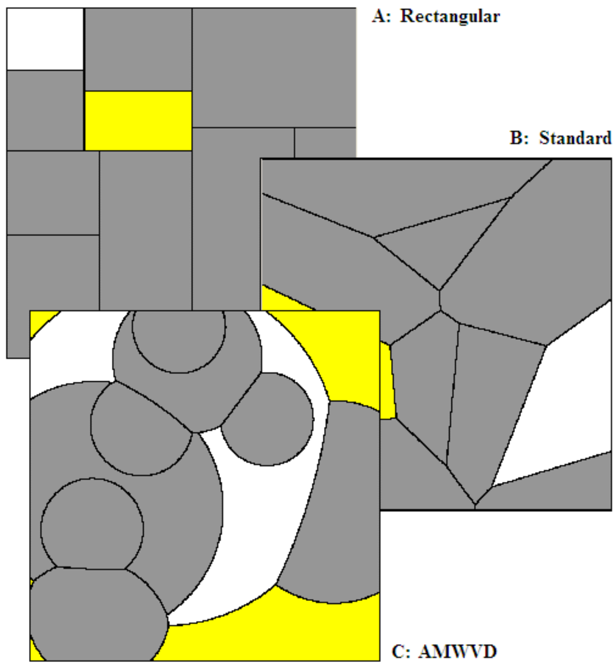

Figure 10: Experimental partitionings: rectangular (A), standard Voronoi diagram (B), and AMWVD (C).

Thirty subjects −ten per partitioning scheme− generated 900 area comparisons. 13 comparisons were discarded because subjects indicated that their responses were mistaken, resulting in a dataset of 887 comparisons.

We are aware of at least one limitation to the described experimental design. Since all of the area comparisons offered to a subject are taken from only one type of partitioning, we cannot account for any systematic differences in subjects between partitionings. To eliminate this selection threat, each test subject would have to be offered comparisons from different partitioning schemes. However, we considered the cognitive burden of negotiating multiple partitionings to be too high and hence, decided to sacrifice some selection validity in exchange for intra-subject accuracy.

Experimental Results

Hypothesis I predicts underestimation of differences between region areas following Steven's power law. Table 2 lists the statistical comparisons of the estimated *vs.* actual area ratios across the three partitioning schemes (All tests reported here were conducted on both raw and log-transformed data. If no differences were obtained, reported test statistics refer to the untransformed data).

The results show statistically significant underestimation of mean differences ($estimated\ ratio_{\mu} < actual\ ratio_{\mu}$) in standard Voronoi partitioning only. However, in all three partitionings, the power coefficient n (16) is statistically significant and smaller than 1.0. Strong differences, however, appear in the explained variance (R^2). Whereas explained variance for both rectangular and AMWVD partitioning is quite high at 67.9% and 52% respectively, it drops to only 3% for standard Voronoi partitioning.

	Rectangular (WebMap)	***Standard Voronoi***	***AMWVD***
Estimation of area differences	estimated ratio$_\mu$: 2.215 actual ratio$_\mu$: 2.18 t: .42; DF: 576; p: .68	estimated ratio$_\mu$: 1.873 actual ratio$_\mu$: 2.229 t: -5.04; DF: 553; p < 0.01	estimated ratio$_\mu$: 2.054 actual ratio$_\mu$: 2.140 t: -1.06; DF: 577; p: .29
Steven's power law	k: 1.08; n: .89; R^2: .68; p < .01	k: 1.55; n: .17; R^2: .03; p < .01	k: 1.25; n: .63; R^2: .53; p: < .01

Table 2: Area estimation errors across partitioning schemes.

Hypothesis II specifies that estimation errors are expected to be smaller in rectangular and standard Voronoi partitionings than in AMWVDs. To test this, estimation error was defined as follows:

$$Estimation\ error = \frac{\left|\dfrac{Estimated\ area\ ratio}{Actual\ area\ ratio}\right|}{Actual\ area\ ratio} \tag{12}$$

Table 3 shows mean estimation errors and results for t-tests comparing the means across partitioning schemes.

	Rectangular (WebMap)	***Standard Voronoi***	***AMWVD***
Area estimation error	μ: .202	μ: .407	μ: .268
Rectangular (WebMap)		μ/μ: .51 t: -8.98; DF: 575; p < .01	μ/μ: .75 t: -3.42; DF: 565; p < .01
Standard Voronoi			μ/μ: 1.46 t: 6.54; DF: 535; p < .01

Table 3: Relative area estimation errors.

The results show that in accordance with *Hypothesis II*, rectangular partitioning performs better (≈ 25%) than AMWVD partitioning. However, standard Voronoi partitioning performs significantly worse than the other two.

	Selection of largest region		
	Incorrect	***Correct***	***Total***
Rectangular (WebMap)	6	285	291
Standard Voronoi	221	77	298
AMWVD	79	219	298
Total	306	581	887

Table 4: Ordinal area comparisons *vs.* partitioning scheme.

Table 4 tabulates correctness in selecting the largest region *vs.* partitioning scheme. Its data indicate that the above numerical comparison results are reflected in the ordinal ones. The associated χ^2s are 69.30 for the rectangular *vs.* the AMWVD and 133.44 for the AMWVD *vs.* the standard Voronoi partitioning. Both have *DF=1* and *p < 0.01*.

	Rectangular (WebMap)	***Standard Voronoi***	***AMWVD***
Actual area ratio	μ: 2.177	μ: 2.229	μ: 2.140
Rectangular (WebMap)		μ/μ: .977 t: -.64; DF: 584; p: .53	μ/μ: 1.04 t: .43; DF: 587; p:.66
Standard Voronoi			μ/μ: 1.02 t: 1.07; DF: 589; p: .28

Table 5: Mean area across partitionings.

These results show that whereas rectangular partitioning outperforms AMWVDs in subjects' ability to correctly estimate area, standard Voronoi partitioning performs poorly. Of course, standard Voronoi solutions different from the other two in that their regions' areas differ. Table 5 explores this issue by comparing the mean area of regions across the various partitionings. Note that although a standard Voronoi region's area is entirely a function of the relative location of its generator, the means of the actual area ratios in the test data do not significantly vary across the partitioning schemes. Although a t-test on the logged data suggest a near-to-significant (*p = .06*) difference between the standard Voronoi and AMWVD partitionings, the difference is so small that we do not consider it to be responsible for the anomalous behavior of the standard Voronoi partitioning.

	Rectangular (WebMap)	***Standard Voronoi***	***AMWVD***
Time (ms) used to select largest region	μ: 4,446	μ: 4,283	μ: 4,649
Rectangular (WebMap)		μ/μ: 1.04 t: .324; DF: 586; p: .66	μ/μ: .96 t: -.437; DF: 562; p: .66
Standard Voronoi			μ/μ: .92 t: .799; DF: 577; p: .42

Table 6: Ordinal area comparisons (time (ms)) across partitionings.

	Rectangular (WebMap)	***Standard Voronoi***	***AMWVD***
log(time (ms)) to select largest region)	μ: 3.648	μ: 3.632	μ: 3.667
Rectangular (WebMap)		μ/μ: 1.004 t: .571; DF: 586; p: .57	μ/μ: .995 t: -2.245; DF: 583; p: .03
Standard Voronoi			μ/μ: .9904 t: -2.862; DF: 592; p < .01

Table 7: Ordinal area comparisons (log(time (ms))) across partitionings.

Information on comparison timing, however, does provide some insight into why standard Voronoi partitioning performs so poorly. Tables 6 through 9 show data on the mean times required to conduct comparisons.

	Rectangular (WebMap)	***Standard Voronoi***	***AMWVD***
Time (ms) used to numerically estimate the size relationship	μ: 10,004	μ: 7,001	μ: 8,452
Rectangular (WebMap)		μ/μ: 1.429 t: 5.576; DF: 432; p < 0.1	μ/μ: 1.184 t: 2.772; DF: 477; p < .01
Standard Voronoi			μ/μ: .828 t: -3.830; DF: 578; p < .01

Table 8: Numerical area comparisons (time (ms)) across partitionings.

	Rectangular (WebMap)	***Standard Voronoi***	***AMWVD***
Time (ms) used to estimate both ordinal and numerical relationships	μ: 14,450	μ: 11,284	μ: 13,101
Rectangular (WebMap)		μ/μ: 1.281 t: 4.51; DF: 526; p < .01	μ/μ: 1.103 t: 1.924; DF: 524; p: .05
Standard Voronoi			μ/μ: .861 t: -3.17; DF: 594; p < .01

Table 9: Ordinal and numerical area comparisons (total time (ms)) across partitionings.

The data in these tables show that although area estimations from standard Voronoi partitionings have significantly greater error than those from the other partitionings, they required less time to conduct. Although for numerical estimations this effect only becomes manifest in the logged data (Table 7) its presence is very strong in the ordinal comparisons and when adding the times required to conduct both types of comparisons (Table 9). To interpret this effect, we offer that standard Voronoi partitioning either induces false confidence in being able to correctly gage relative area, or is so hard to gage that subjects essentially ‘give up’ quickly and make faulty guesses. Whatever the cause, its estimations are, on average, made more rapidly, yet have greater error.

Hypothesis III can be evaluated by comparing the results for AMWVDs with those of the other two. It appears that especially for ratio comparisons, error measures for the rectangular partitioning are significantly better than those for AMWVDs (Tables 2 and 3). Once again, however, AMWVDs outperform the standard Voronoi diagram.

Hypothesis IV addresses the issue of AMWVDs containing discontinuous regions and the difficulty of comparing the areas of those regions To test for this, we took care in designing the experiment to select an AMWVD solution which not only properly reflects the areas used in the rectangular partitioning, but which also contains some clearly discontinuous regions. To evaluate *Hypothesis IV* within the confines of our experimental data, we must compare the area estimation errors involving discontinuous regions with those not involving such regions. The mean estimation error for AMWVD comparisons not involving discontinuous areas (n=181) is .270 whereas for comparisons involving either one or both of the discontinuous regions (n=117) it is .266; a very small and nonsignificant difference. We find similar results when comparing the time it takes subjects to conduct the area comparisons. Hence, on the basis of our (limited) experimental data, *Hypothesis IV* must be rejected.

CONCLUSION

We presented AMWVDs as a method for partitioning continuous information spaces based on the information items' locations and their 'weights.' The objective of the method is to conserve location, while maximizing weight-area proportionality. We found that AMWVDs performed well in the context of various ideal-typical cases. Next, we subjected AMWVDs to comparative size estimation with two other partitioning schemes. We found that area comparisons for AMWVD partitioning, both ordinal and numerical, significantly outperform those of standard Voronoi diagrams, but are not quite as good as those observed with rectangular partitioning. However, to our knowledge, rectangular partitioning is incapable of conserving both location and area-weight proportionality. Hence, a 25% difference in mean estimation error, especially when compared with the 50% of mean error increase associated with standard Voronoi diagrams (Table 2) may be worth the significantly improved geometric capabilities of AMWVDs.

Although the usability testing involved three different partitionings, the partitioned space itself had a rectangular extent. This may or may not have impacted the testing results. This issue could be partly explored by comparing the results for comparisons involving peripheral regions with those not containing such regions. Alternatively, for standard Voronoi diagrams and AMWVDs this effect can be eliminated by the introduction of other, randomized spatial extents in the experiment.

Surprisingly, we did not find any evidence that the estimation of area of discontinuous regions was more erroneous than that of continuous regions. If this result holds in additional, more comprehensive tests, it would invalidate an important objection against MWVDs, namely their increased complexity due to discontinuous regions.

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