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THE EFFECT OF IMPRECISION IN SPECIFICATION OF PAIR-WISE COMPARISONS ON RANKING OF ALTERNATIVES USING FUZZY ANALYTIC HIERARCHY PROCESS

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Abstract

Fuzzy Analytic Hierarchy Process enables decision makers to provide subjective judgments using imprecise pair-wise comparisons of various factors and alternatives. This paper investigates the effect of imprecision in specification of pair-wise comparisons on the final ranking of alternatives. Results obtained from an experiment using a Fuzzy AHP prototype system are presented.

Introduction

Saaty's (1980) Analytic Hierarchy Process (AHP) is a very popular and practical technique used for modeling and solving multi-criteria decision problems. In the classical AHP, decision makers supply precise pair wise comparisons of various factors and alternatives on a ratio scale [1/9-9]. These pair wise comparisons capture decision maker's subjective judgments and help in ranking a set of alternatives.

One of the difficulties encountered by decision makers, following this approach, is related to the specification of subjective judgments using precise pair-wise comparisons (e.g., 1/5, 4, 1). Extensions to AHP using fuzzy set theory are proposed to address this difficulty (Buckley, 1984; Chang 1996). Fuzzy AHP techniques enable decision makers to specify their subjective judgments using imprecise numbers (e.g., about 2, between 3 and 4) for pair-wise comparisons. Although such approaches have inherent advantages with respect to the specification, the effect of imprecision on the final ranking of alternatives has not been investigated (Leung and Cao, 2000).

This paper presents the results of an experiment designed to investigate the changes in ranking of alternatives when the level of imprecision is varied. In the next section, we briefly describe the design of the experiment and the Fuzzy AHP prototype system developed for the experiment. In Section 3, we present the results of the experiment.

Experiment Design

In order to investigate the effect of imprecise specification of pair-wise comparisons on the ranking of alternatives, we selected two example decision problems (car purchase and nuclear waste alternative prioritization) provided with the demo version of Expert Choice 2000 (www.expertchoice.com). Figure 1 shows the car purchase decision model with various factors and the final ranking of alternatives.

All the pair-wise comparisons defined in these examples are mapped to imprecise values using triangular membership functions. Figure 2 illustrates the mapping process where a precise value, p , is mapped to an imprecise value defined using the 3-tuple $\langle p-\delta, p, p+\delta \rangle$ where δ indicates the level of imprecision. This approach was used to map all the precise pair-wise comparisons between 1 and 9 and the corresponding reciprocals (e.g., 1/3) are computed using fuzzy division.

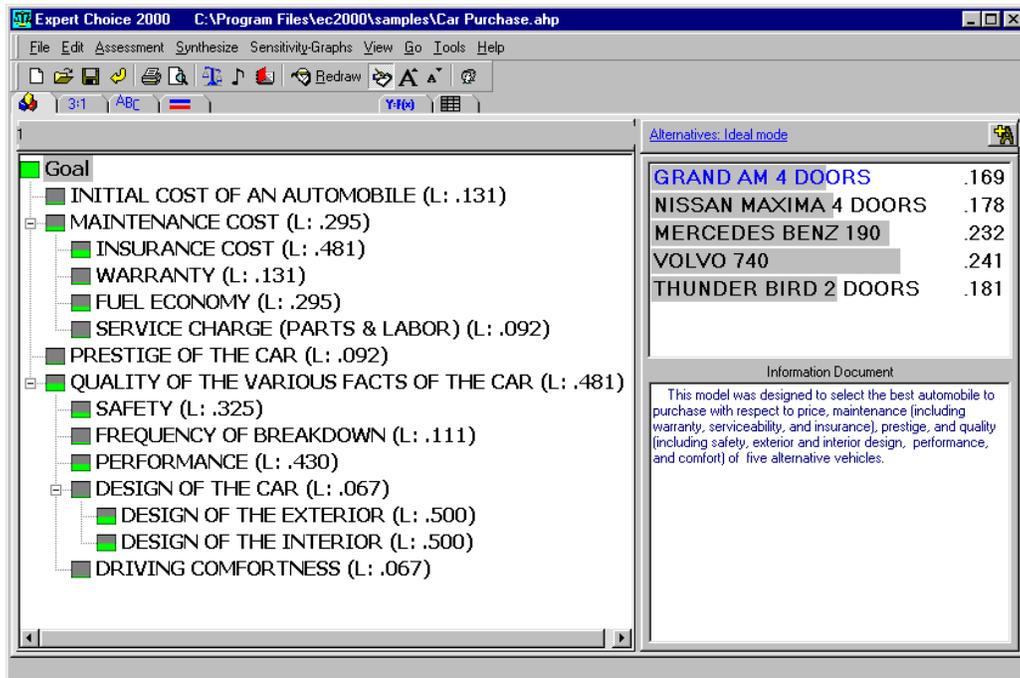
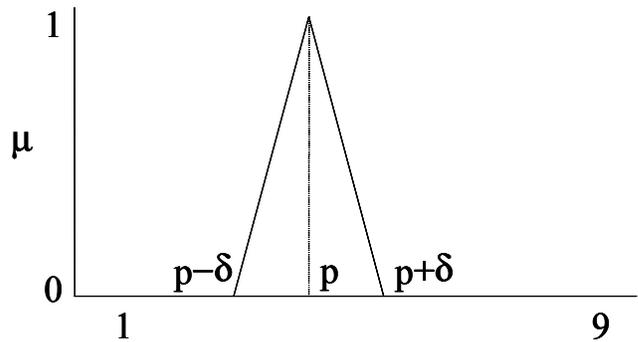


Figure 1. Car purchase Decision Model in Expert Choice 2000

A prototype Fuzzy AHP system was implemented in Visual C++ 6.0 for the experiment. This prototype provides facilities for defining AHP models with fuzzy pair-wise comparisons and evaluates precise ranking of alternatives. Figure 3 depicts the car purchase decision model defined using this prototype. Alternatives A1, A2, ..., A5 correspond to the five car models shown in Figure 1. Pair-wise comparisons and specific values of δ (e.g., 0.1, 0.2) from the original model are entered into the prototype system. For each value of δ fuzzy rankings were computed using the algorithms proposed by Buckley (1984) and Chang (1995) and then the rankings were defuzzified using centre of gravity and extent analysis methods to determine precise ranking of the alternatives.



Pair-wise comparison

Figure 2. Mapping Precise Pair-Wise Comparisons to Imprecise Values

Results

The rankings obtained by varying the level of imprecision in the pair-wise comparisons matched (100% for level of imprecision below 0.7 for car purchase decision problem and 100% for nuclear waste prioritization) with those generated by Expert Choice 2000 for both the decision problems. However, the relative weights of the final rankings have changed significantly as the level of imprecision increased. Figure 4 shows this effect for the car purchase decision problem.

Conclusion

The results from our experiment show that imprecise specification of pair-wise comparisons has very little effect on the final ranking of alternatives. This implies that replacing precise pair-wise comparisons with imprecise comparisons does not compromise the benefits of AHP. These results also suggest the possibility of applying consistency determination methods of classical AHP to verify consistency of imprecise pair-wise comparisons after mapping to precise values using methods such as centre of gravity. We are conducting further investigation and experimentation to determine the extent to which these results can be generalized.

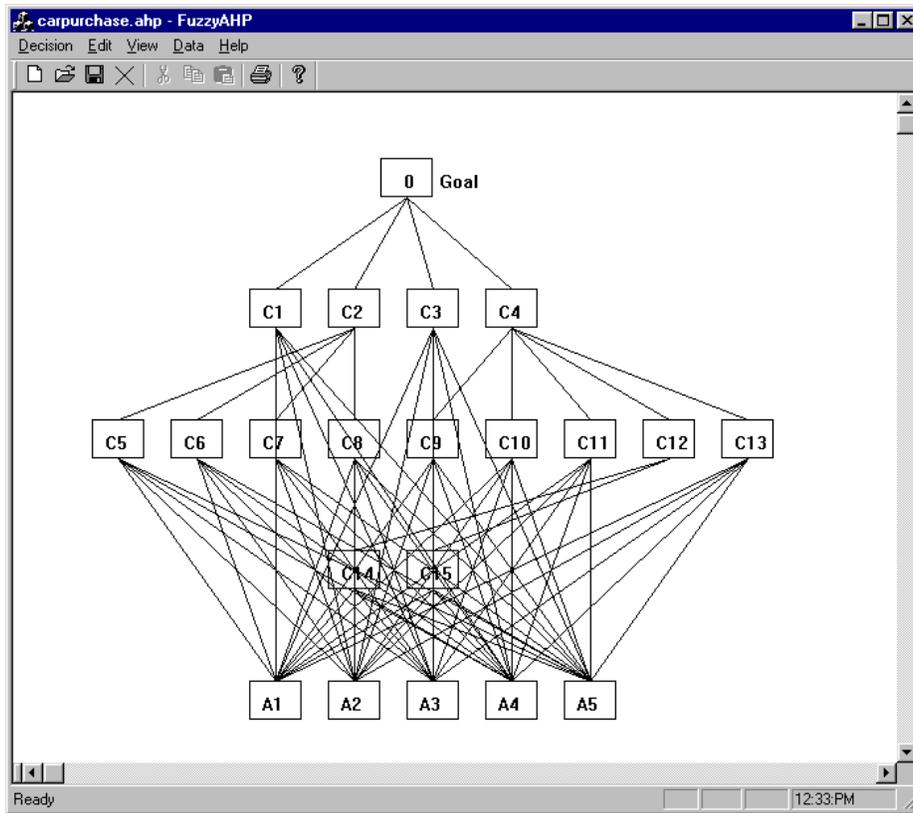


Figure 3. Car Purchase Decision Model in Fuzzy AHP Prototype

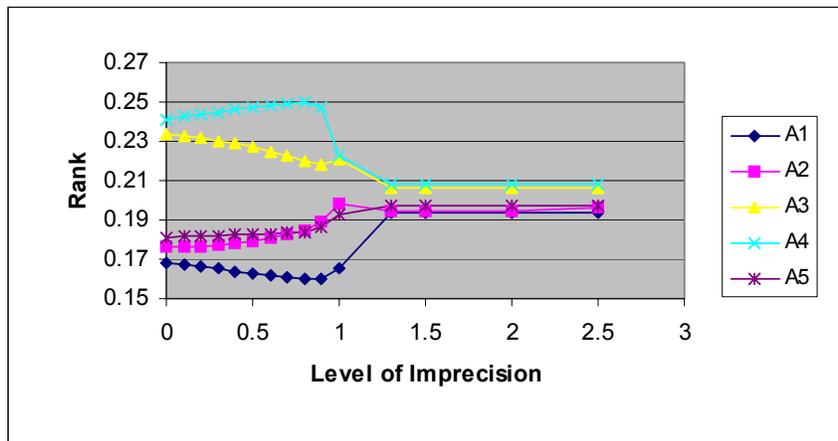


Figure 4. Effect of Imprecision on the Ranking of Alternatives

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