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DECISION SUPPORT MODELS FOR PORTFOLIO SELECTION

Heinz-Roland Weistroffer
Virginia Commonwealth University

Charles Smith
Virginia Commonwealth University

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Heinz Roland Weistroffer and Charles H. Smith
Virginia Commonwealth University
hrweistr@vcu.edu chsmith@vcu.edu

Abstract

Most decisions must consider multiple, generally conflicting, decision objectives. And many decisions require not a single solution alternative, but a set or portfolio of alternatives. A business will generally produce a portfolio of products. Government will raise revenue through a portfolio of taxes. A prospective college student is wise to apply to a mix of schools, including some with high likelihood of acceptance. A project team leader needs to select several team members, creating a balance of skills and taking into account the synergy of the team. While the single solution alternative decision problem has been widely studied and many solution approaches and decision support systems have been developed, the portfolio problem has been largely neglected. Specific problem environments have been discussed using heuristic, ad hoc methods, but no general methodology has been developed. This research focuses on developing theoretical concepts and formal models to be used in identifying problem types in real world decision environments, and on mapping possible solution techniques to these models.

Keywords: Portfolio selection, multi-criteria, multiple objectives, multiple attributes

Introduction

Most decisions involve satisfying multiple objectives. An alternative is chosen based on a multitude of often conflicting decision criteria. A solution is sought that provides the best compromise with respect to optimizing these various desired objectives. Multi-criteria decision making, in the last twenty years, has become an established field of research, with extensive theory, a wide choice of solution methods, and many available computer-based decision support packages. However, there are many decision settings that require a set of alternatives to be selected, rather than a single alternative. Examples include deciding on an investment portfolio, choosing colleges to apply to for graduating high school seniors, selecting members on a team, etc. Rather than selecting a best single investment option, a portfolio is desired that balances security with expected payoff. College applications should include schools that are most desirable and schools that are most likely to accept. Selecting members on a team requires a balance of players with various skills, as well as ensuring teamwork among the members. Thus in each of these situations, one cannot simply find the top x choices to constitute the portfolio or team, but rather the dependencies between the various selected individual alternatives must be taken into account. The utility attached to the alternatives is not additive, that is, the utility of the portfolio cannot, in general, be expressed as a sum of individual alternative utilities. The synergistic effects of the portfolio must be considered.

Literature on financial investment portfolio selection (e.g. Markowitz 1952, Elton and Gruber 1995, Ballestero and Romero 1996), as well as on R&D project selection (e.g. Golabi 1984, Stewart 1991, Henig and Katz 1996) is abundant. However, most of these publications do not address generalized mappings of solution methodologies to general portfolio problem types. Most published papers discuss very specific problem types or decision situations and very specific solutions, usually based on heuristics, and cannot easily be generalized. In addition, many specific techniques that have been proposed in the literature are not widely used because they tend to be too complex, do not address all relevant issues associated with portfolio selection, require too much input data, or may be too difficult to understand for decision makers to use (Ghasemzadeh and Archer 2000). According to Hess (1993) "management science has failed altogether to implement project selection models".

Problem Description

The general multi-criteria decision making (MCDM) problem (see for example Steuer 1986) involves a set of feasible decisions or possible actions, and a set of corresponding solutions or outcomes. The complexity arises from the reality that the outcomes are measured with respect to multiple criteria or objectives, and usually none of the outcomes will be optimum with respect to all of the criteria. Thus compromise solutions that are acceptable to the decision makers are sought. Mathematically, this problem can be expressed as follows:

$$\text{Maximize } \mathbf{F}(\mathbf{x}): D \rightarrow S,$$

where $D \subset \Re^k$ and $S \subset \Re^p$, and $\Re$ represents the real numbers, $\Re^k$ and $\Re^p$ are vector spaces. $\mathbf{F}(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_p(\mathbf{x})\}$ represents the p objectives, $\mathbf{x} = \{x_1, x_2, \ldots, x_k\}$ represents the k decision variables, D is the decision domain, and S is the solution domain. Since in general, the p objectives conflict with each other and cannot be maximized simultaneously, a compromise solution is sought that maximizes the overall utility, i.e.

$$\text{Maximize } U: \mathbf{F}(D) \rightarrow \Re.$$

An example of a MCDM problem might be designing a new auto engine. The decision domain would include all possible combinations of, say, quality of material used (represented by the value of x_1), engine capacity (represented by the value of x_2), etc. The solution domain would include corresponding decision outcomes with respect to fuel efficiency, perhaps represented by $f_1(\mathbf{x})$, engine power, represented by $f_1(\mathbf{x})$, construction cost, represented by $f_2(\mathbf{x})$, durability, represented by $f_3(\mathbf{x})$, etc. Ideally, we would like an engine to be fuel efficient (i.e. minimize fuel consumption), be powerful (i.e. maximize horse power), be reliable (i.e. maximize durability), and cheap to build (i.e. minimize cost). However, maximizing horse power and durability would likely result in less than optimum fuel consumption and cost.

An important special case of the MCDM problem is the multi-attribute decision making (MADM) problem, where the decision domain is a discrete set of solution alternatives. An example of a MADM problem would be selecting a candidate from a pool of new faculty applicants. The alternatives would be the set of applicants, each of whom would be evaluated based on multiple attributes, such as which courses the person would be able to teach, potential scholarship output, etc. The likelihood that one candidate would be optimum with respect to all attributes is slim.

The MADM problem can be described as follows:

$$\text{Maximize } U(a): A \rightarrow \Re,$$

where A is the set of all solution alternatives. Each possible alternative is assessed based on its associated values for the multiple decision attributes. In the example above, each $a \in A$ represents one of the faculty candidates, whose utility is assessed based on attributes such as teaching ability, scholarship output, etc.

The portfolio selection problem adds another dimension of complexity. Instead of seeking a single alternative $a \in A$ that maximizes the overall utility, a set of alternatives, i.e. a portfolio $P = \{a_1, a_2, \ldots, a_n\}$ is required, where $P \subset A$.

Thus the problem becomes:

$$\text{Maximize } U: \{P \mid P \subset A\} \rightarrow \Re$$

i.e. find the subset P of A that provides maximum utility (usually subject to some constraints).

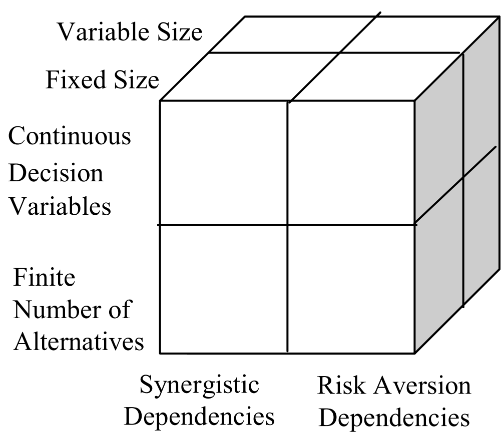
An example of such a portfolio selection problem may arise when a university department is hiring not one new faculty member, but rather needs to fill a fixed number (n) of positions. Here, A represents the total applicant pool, and P is a subset of n applicants. The intuitive approach is to rank all candidates and pick the n top ranked ones. However, this may not be a good strategy. For example, in order to cover all the various courses taught in the department, picking candidate **X** may very well make some other otherwise highly ranked candidate become less desirable, if that person's course specialties overlap with candidate **X**'s specialties. The value of each candidate is dependent on the other selected candidates. It may not even be best to include the highest ranked candidate out of the candidate pool. It is possible that two medium rated candidates have a higher combined value than picking the top candidate together with his/her best complimentary candidate. This eliminates sequentially selecting the top candidates as a general strategy.

To complicate matters further, n, the number of alternatives in the portfolio may be variable, and n equal constant constitutes an important special case.

Revisiting the engine design problem, suppose the company decides to build several new engines, to meet the different demands of its customer base. The problem now becomes one of finding a portfolio of different engines that are not only measured on the criteria mentioned earlier, but also on the extent to which customer needs or desires are fulfilled. This may be a case where n is variable. On the one hand, a large number of different engines will likely satisfy a greater contingent of customers. But on the other hand, having fewer products reduces production costs.

Decision Model Approaches

The above two examples of portfolio selection problems show one possible classification of such problems: fixed number of members in the portfolio versus variable number. Other factors that distinguish different types of portfolio problems include continuous decision variables (as in MCDM) resulting in possibly infinitely large decision alternative sets, versus a finite set of decision alternatives (as in MADM), resulting in a finite set (though possibly very large) set of potential portfolios. Further, the dependencies between alternatives may have different consequences. For example, when building a team (whether a sports team or perhaps an IS development team), it is not only the individual capabilities of team members that should be taken into account, but also the synergistic effects derived from people being able to work together, i.e. the total being greater than the sum of the parts. On the other hand, when constructing a portfolio of investments, there is no synergistic or team effect, but rather the goal is reduction of risk. The above described aspects can be represented by a three dimensional matrix resulting in eight different decision models:



Many special cases of the portfolio problem exist, often represented through constraints. For example, when hiring multiple new faculty members, constraints might be that one new hire must be at the assistant professor level and one at a senior level. Such constraints may limit the solution space of feasible portfolios considerably and thus make the selection easier. As alluded to earlier in this paper, many proposed solution techniques were designed for such special cases. Looking at all possible theoretical situations based on constraints is not really feasible; however, a classification with respect to such constraints might be useful if one can determine the most common types of real world decision situations. In other words, develop a number of representative model situations that cover many of the real world problems, such as:

- the financial investment portfolio problem
- the college selection problem
- the team selection problem
- the meal composition problem
- the R&D project selection problem

Many of these, though, will have multiple versions, complicating matters further.

Portfolio Selection Framework

In the above we focused on modeling the decision problems. Other researchers have maintained that a framework for portfolio selection should be considered a process that includes specific steps. The following framework is loosely based on that proposed by Ghasemzadeh and Archer (2000):

Preliminary activities	Pre-screening of alternatives	Check for compliance with general objectives
	Individual analysis	Determine individual criteria measures
	Screening	Check for compliance with individual constraints
Portfolio selection	Optimal portfolio selection	Use appropriate method to select a portfolio
	Portfolio adjustment	Reexamine general objectives and chosen portfolio, and if necessary make adjustments

Besides requiring appropriate decision models to put the five steps of this framework in a proper context, we also need specific selection methods (preferably implemented in DSS packages) to execute the fourth and fifth steps.

Future Work

The following tasks need to be performed in order to obtain a useful methodology for managers to use in selecting portfolios:

- Further classification of portfolio decision situations
- Further development of portfolio problem models to match these situations
- Development of effective techniques, mapped to these models
- Refinement of the framework for solving portfolio problems
- Development of user-friendly decision support software to support this framework

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