Reputation in Repeated Pay-to-Bid Auctions

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Abstract

This study primarily focuses on pay-to-bid auctions in which bidders pay a fixed fee for each bid to increase the price and explores the reputation of bidders within the auctions. The reputation effects can be discovered from sample observations in pay-to-bid auction websites. Pay-to-bid auctions are highly susceptible to manipulative behaviors by an aggressive bidder. To explain the phenomenon, a basic model in which two bidders take part in a series of pay-to-bid auctions is developed and an extension of a multi-player model builds on the basic model. The question of an optimal auction from the auctioneer’s standpoint, in an asymmetric setting, is addressed. It is expected to theoretically show that the results from previous symmetric pay-to-bid auction models do not carry over to repeated auctions when one of the bidders is endowed with a reputation for bidding aggressively.

Keywords

Pay-to-bid auctions, reputation, aggressive bidding behaviors, incomplete information, game theory.

Introduction

A pay-to-bid auction, also known as a penny auction, is a relatively new auction mechanism run by a number of online companies such as QuiBids, OrangeBidz, and HappyBidDay. In the simplest form of this format, each pay-to-bid auction begins at a price of zero with a timer that limits the time to bid. A bidder is charged a bidding fee (e.g., $0.50) for each bid. Bids increase the current price by a fixed increment ($0.01) determined by the auctioneer. After each bid, the auction timer is extended by a set period of time (e.g., 10 seconds). Once the time expires before another bid is placed, the last bidder wins the auction and pays the current price. The winner will have paid the cost of all his bids and the final price.

At first glance, the pay-to-bid auction does not sound attractive because bidders encounter the risk of having to pay bidding fees without winning the auction. However, the compelling part of this auction is that the bidder who wins an auction can sometimes save more than 90% off the current retail price. For example, on Quibids.com, a $50 Walmart gift card is often sold for $0.18, and similarly, a new 32GB iPhone 7 is sold for $7.05 rather than its retail price of $649. Although blogs and newspapers (Cobb, 2013; Reklaitis, 2009; Gimein, 2009) have questioned the virtue of the auction format, referring to pay-to-bid auctions as gambling, pay-to-bid auctions have important differences (no exogenous source of randomness and skill playing a role in the expected outcome) and are growing in popularities.

Discovered from sample observations in pay-to-bid auction websites, bidders in a pay-to-bid auction take part in a series of similar but independent actions over a period of time. In such a situation, each bidder can draw inferences about others from their past behavior. In this setting, aggressive bidders use a strategy to establish the reputation that they will win at any cost. The other bidders, when computing optimal strategies, take into account not only the history but also the effects of the aggressive behaviors in subsequent auctions. In particular, the following questions can be raised. Does it cost any of the bidders to establish or maintain a reputation for bidding aggressively? Does the reputation help increase the aggressive bidders’ likelihood of winning? Finally, how does the reputation impact the profit of the
auctioneers? To answer these questions, we develop a robust generalized equilibrium model to investigate individual reputation and incomplete information of bidders in online pay-to-bid auctions.

Literature Review

This study is related to two streams of research - reputation and pay-to-bid auctions.

Literature on Reputation

The earliest formalization of reputation effects in games with incomplete information has been developed by Kreps and Wilson (1982); Milgrom and Roberts (1982a, b). The authors of these papers use reputation to resolve Selten (1978)’s chain store paradox and conclude that firms adopt predatory or limit pricing behavior to eliminate current rivals and signal a reaction to future entry attempts. Considering the class of all repeated games where a long-run player faces a sequence of short-run opponents, Fudenberg and Levine (1989) generalize and extend the results of Kreps and Wilson (1982). Schmidt (1993) provides a generalization and qualification of Fudenberg and Levine’s results for the case of two long-run players.

Regarding equal discount factors, Chan (2000) obtains a folk theorem in perfect equilibrium strategies for all perturbed repeated games except those where the commitment action is a dominant action in the state game or those with strictly conflicting interests. Based on Chan’s conclusion, Cripps et al. (2005) show that the perturbed version of folk theorem fails.

An early paper about reputation in auctions is from Bikhchandani (1988). He demonstrates that having a small reputation advantage can allow a bidder to almost always win a pure-common-value auction, and that this reputation advantage may be very easy to sustain in a repeated context. Although Kwiek (2002) provides a very similar conclusion as Bikhchandani, he deals with one/two-sided reputation in infinitely repeated games for a more general environment where the auction does not have to be in purely common values.

Literature on Pay-to-Bid Auctions

Pay-to-bid auctions are relatively new but have quickly generated academic scrutiny. Augenblick (2015) and Hinnosaar (2016) theoretically analyze the auctions for risk-neutral bidders. Platt et al. (2013) introduce risk-loving behavior to explain away the negative average returns. Byers et al. (2010) consider the impact of particular asymmetries on outcomes by making a crucial distinction between the true values of the game’s parameters – valuation, bidding cost and the number of participants – and the way bidders perceive them.

Several papers have followed this first wave of analysis. From the bidders’ perspective, Goodman (2012) uses bid-level data to explore bidder reputation using aggressive bidding strategies. Wang and Xu (2012) use bid-level data to further explore bidder learning and strategic sophistication in a game from the marketplace. From the auctioneer’s perspective, Zheng et al. (2011) use a small field experiment to explore the effect of restricting the participation of consistent winners. Caldara (2012) uses an experiment and finds that timing does not matter but that more participants lead to higher auctioneer profits. Kim et al. (2014) theoretically and empirically compare economic effects of ascending versus descending pay-per-bid auctions. Ødegaard and Anderson (2014) theoretically analyze a penny auctioneer’s strategy when there is another fixed price sales channel. Reiner et al. (2014) empirically investigate the role of the Buy-Now feature in pay-to-bid auctions.

Thus far, we have not found research that studies pay-to-bid auctions as a repeated game - the existing literature on pay-to-bid auctions is all focusing on a one-stage game where bidders bid for a single item. Although several researchers (Byers et al., 2010; Goodman, 2012; Caldara, 2012) have studied reputation effects or aggressive bidding behaviors in pay-to-bid auctions, we propose to examine the reputation effects in repeated pay-to-bid auctions.

Theory Development

This section presents a theoretical model of reputation in the repeated pay-to-bid auctions and provides preliminary analyses.
The Model

The basic model of two bidders is given in this subsection. We characterize the expected equilibria in the stage game as well as the repeated game. It is speculated that his opponent never wins an auction if the aggressive bidder is of a strong type, and that the aggressive bidder can deter his opponent from entering the game by establishing a reputation that he will always win no matter what type he is.

Setup

First we consider two risk-neutral bidders, indexed by \( I = \{1, 2\} \), taking part in a sequence of \( n \) auctions. The auctions are indexed backwards, \( t = n - 1, n - 2, \ldots, 0 \), the first auctions called stage \( n - 1 \), since there are \( n - 1 \) auctions after this one and the last one is called stage 0. In each stage, a single item is auctioned. Assume that the game is started randomly with either bidder and that each bidder

There are two types of bidder 2, denoted \( \theta_2 = \{0, S\} \): an ordinary type \( \theta_2 = 0 \) or a strong type \( \theta_2 = S \), whereas there is one type - an ordinary type of bidder 1, i.e. \( \theta_1 = \{0\} \). The value of the \( n \) items to bidder 1 are independent and, for simplicity, identically distributed random variables denoted \( v^n_t = v_t, t = n - 1, n - 2, \ldots, 0 \). Type 20’s valuation is \( v^n_{20} = v^n_t = v_t \), whereas type 2S’s valuation is \( v^n_{2S} = \beta v_t \), where \( \beta \) is a constant strictly greater than one and is common knowledge. Bidder 2 knows his own type but bidder 1 does not. In the beginning, bidder 1 assesses a small probability \( \delta_n - 1 \) that bidder 2 is of type \( S \). At the end of each stage, bidder 1 updates \( \delta_t \) to \( \delta_{n-1} \) based on Bayesian rule. Each bidder observes the identity of the bidder who announced higher number of bids, \( i_w \in \{1, 2\} \), and his effective bids, \( \tilde{a}_{i_w} = a_{i_w} + 1 \). These constitute a public outcome, \( y_t = \{i_w, \tilde{a}_{i_w}\} \). Let \( H_t \) denote the set of all paths of public observable outcomes and let \( h \) be a typical element of this set. Let \( h_t = \{y_0, y_1, \ldots, y_t\} \) stand for the \( t \)-period public history. The discount factor for bidder \( i \) is denoted by \( q_i \).

A pure strategy of bidder \( i, s^i_t, i = 1, 2 \), is the total number of bids that bidder \( i \) places at stage \( t \), where

\[
{s^i_t : H_t \times \theta_i \rightarrow A_i}
\]

The utility for bidder \( i \) is

\[
\begin{align*}
&-s^i_t(h_t, \theta_i) \times c & \text{if } 0 \leq s^i_t < s^j_t, \\
&-s^i_t(h_t, \theta_i) \times c + v^i_t(\theta_i) - [2 \times s^j_t(h_t, \theta_j) + 1] \times k & \text{if } 0 \leq s^j_t < s^i_t, \forall i, j \in I
\end{align*}
\]

The two rows correspond to the situation in which at least one bidder enters. If \( i \) bids more times than \( j \), then \( i \) gets the object and pays the bid price as well as the bidding cost; if \( i \) bids fewer times than his opponent, then he loses and pays only the bidding cost only based on his number of bids - his opponent’s strategy is not observable. If neither bidder enters the game, then either gets a payoff of zero.

Stage Game

An asymmetric stage game refers to the stage game with \( \delta > 0 \). Let \( g^i_t(\cdot) \) be the expected payoff of bidder \( i \) at stage \( t \). The following is necessary condition for \( s^i_t = (s^i_{t1}, s^i_{t20}, s^i_{t2S}) \) to be a pure-strategy equilibrium in the stage game:

\[
s^i_t \in \arg \max_{s^i_t} g^i_t(s^i_t, s^i_t), \text{ for } i = 1, 20, 2S.
\]

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1 We assume that the stage game will not end up with a tie.
Reputation in Repeated Pay-to-Bid Auctions

The main expected result is that in any pure strategy equilibrium of the stage game, if bidder 2 is of the strong type, then bidder 1 will never win against bidder 2 ($s_1 < s_{2o}$). The result is intuitively reasonable considering a very aggressive bidder has a much higher reserve price than normal bidders.

Repeated Game

The total payoff in the repeated game is a discounted sum of the sequence of stage payoffs:

$$u_t(\delta_t) = E[\sum_{i=0}^{n-1} (\alpha_i)^t \times g_t]$$

Note that bidder 2’s discount factor is close to one, i.e. bidder 2 is more patient than bidder 1. In this equilibrium, the bidders play their symmetric stage game best responses in each stage.

A perfect Bayesian equilibrium of the repeated game is a strategy profile $s_t = (s_1^t, s_{2o}^t, s_{2s}^t)$ such that

$$\hat{s}_t^i \in \arg \max_s u_t^i, \text{ for } i = 1, 2, 0, 2S$$

Since bidder 1 is not directly informed about bidder 2’s type but can observe the historical outcomes, bidder 1 will attempt to make inferences about the actual type of bidder 2 from his observation of $h_t$ based on Bayesian rule:

$$\delta_{t-1} = p(\theta_2 = S|h_t) = \frac{\delta_t \times p(h_t|\theta_2 = S)}{\delta_t \times p(h_t|\theta_2 = S) + (1 - \delta_t) \times p(h_t|\theta_2 = 0)}$$

In order to update bidder 1’s belief that bidder 2 is of type S, the probability that bidder 2 of type 0 wins the auction should be calculated. If $p(h_t|\theta_2 = 0) \to 1$, which means the bidder 2 wins the auction even he is an ordinary bidder, then bidder 1’s belief will not change, i.e. $\delta_{t-1} \to \delta_t$; similarly, if $p(h_t|\theta_2 = 0) \to 0$, then $\delta_{t-1} \to 1$ and bidder will not enter the game ($s_1 = 0$) in any pure strategy equilibrium.

Extensions

The extended model generalizes the game with more than two bidders. Due to its distinct mechanism, the game cannot be modeled exactly the same as the benchmark framework. As a result, we extend the sequential model based on Augenblick’s and Hinnoasaa’s work and include types and beliefs. It turns out that if an aggressive bidder always bids to keep himself in the leading position whatever his type is, his opponents will stop bidding independently of the probability that they believe the aggressive bidder is of a strong type.

The optimization of the auctioneer’s profit is partially addressed in this section as well. From the auctioneer’s perspective, a higher short-term profit is expected to be achieved if aggressive bidders participate the game. If the strong bidders block out normal bidders, however, the auctioneer can not profit more in the long run because the strong bidder can win cheaply.

Discussion

A series of the pay-to-bid auctions reveal the existence of reputation effects as bidders realize that their bidding actions and histories in any auction convey information about their type. These reputation effects are taken into account when computing optimal strategies. This study is theoretically important as it contributes to the assortment literature in several key aspects. First, we study the pay-to-bid auction in a repeated circumstance while previous research mainly focuses on a single auction. Second, my proposed research is the first to theoretically model reputation in the repeated pay-to-bid auction while prior research merely empirically or experimentally examines reputation effects using practical data. Finally, different from previous studies on reputation in other auction formats such as a second-price auction, my framework assumes that the strategy of the opponent is unknown to the bidder so that he updates his belief only based on the result of last stage game. The expected results and conclusions drawn from the model also provide managerial implications and can practically help auctioneers increase their profit by optimizing the auction.
mechanism. Specifically, this research helps the auctioneers characterize the participants about their types or beliefs and make better decisions on the game parameters such as the bidding increment.

REFERENCES