ICEB 2004 Proceedings

International Conference on Electronic Business (ICEB)

Winter 12-5-2004

Decision Making and Analysis for Unexpected Road Blockages

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ABSTRACT

The unexpected road blockage problem (URBP for short) is considered for the case when some blockages occur on the road at certain times and such blockages would be revealed only upon reaching them. Papadimitriou and Yannakakis proved that devising a strategy that guarantees a given competitive ratio is PSPACE-complete if the number of edges that might be blocked is not fixed; Bar-noy and Schieber considered the deterministic and stochastic variations of the recoverable-URBP in the worst-case criterion. In this paper, we present an offline algorithm for the recoverable-URBP optimal solution and the complexity of the algorithm is $O(n^2)$ (n is the number of nodes). Two online strategies, waiting strategy and the Greedy strategy, are proposed and the competitive ratios of the two strategies are given. Furthermore, we compare the two strategies based on competitive ratio analysis.

Keywords: decision making, unexpected road blockage, offline algorithm, competitive ratio

1. INTRODUCTION

The unexpected road blockage problem (URBP for short) can be stated as follows, a traveler knows a graph (a map) *G(V, E)*, and the traveler has to go from s (origin) to t (destination). However, the map is unreliable, some blockages may occur on the road at certain times (blocked by unexpected events).

The unexpected road blockage problem is first introduced by Papadimitriou and Yannakakis [1], they proved that if the number of roads that might be blocked is not fixed, and then devising an online algorithm with a bounded competitive ratio is PSPACE-complete. Bar-Noy and Schieber [2] studied several variations of the problem. In the recoverable-URBP, each site is associated with a recovery time to reopen any blocked road that is adjacent to it, they presented a polynomial-time traveling strategy that guarantees the shortest worst-case travel time, in the case where an upper bound on the number of blockages is known in advance, and the recovery times are not long relative to the travel times. For the stochastic recoverable-URBP, they present an $O(m\log n)$ (n and m are the number of nodes and edges) time algorithm for designing a travel strategy from all sites to a fixed destination that guarantees the shortest expected travel time, under the assumption that the recovery time of the blocked edges are smaller than the edge's passing time.
For the deterministic recoverable-URBP, they gave an $O(k^2m+kn\log n)$ (k is the number of that road blockages) time algorithm for designing a travel strategy from all sites to a fixed destination guarantees the shortest worst-case travel time, under the assumption that the recovery time of the blocked edges are smaller than the edge's passing time. But, the assumption made in [2] is impractical for most transportation process. In the Irrecoverable URBP, once a road is blocked, it remains blocked forever. Y.Xu et al [3] proposed two strategies, greedy strategy and reposition strategy, and showed that $2k+1$ and $2^{k+1}-1$ are their tight competitive ratios [4] [5] respectively, and proved that no online strategy has competitive ratio less than $2k+1$ for Irrecoverable online URBP and in most cases, greedy strategy is better than reposition even though the competitive ratio of the former strategy is greater than the later ones.

In this paper, we focus on the recoverable URBP. We present an offline algorithm for the optimal solution based on analysis of the blocked edges cost (time) which is effected by the different starting time and end time of blockages, and the memory makes the problem be unfit for a converse solution of dynamic programming. The algorithm is a modification to the Dijkstra algorithm [6] and the complexity of the algorithm is $O(n^2)$. Then, according to the different assumptions on the recovery time, we present two different strategies, waiting strategy and the Greedy strategy; we show the competitive ratios of the two strategies. Furthermore, the comparison between these two strategies is discussed based on competitive ratio analysis.

2. PROBLEM STATEMENTS AND FORMULATION

Suppose the traveler has to go from s to t . He knows a map that consists of set of roads between two sites. The traveler will choose a shortest road (SR for short) from s to t . At certain times, the SR will be blocked by some events, such as traffic accidents. Assume that the blockages are recoverable. If all of the blockages' position and its recovery time are known in advance, the problem is considered as an offline problem. If the blocked edges occur one by one without any predictable information, the problem is considered as an online problem.

3. AN OFFLINE ALGORITHM

Let $G(V,E)$ denote an edge weighted graph with |v| vertices. v_1 denotes the origin and v_n the destination, $E=\{e_{ij}/(v_i,v_j),v_i,v_j\in V;i\neq j\}$ denote the set of the edges between the two sites , $T(v_1)$ denotes the starting time of v_1 , $T(e_{ij})$ denotes the passing time of e_{ij} without the blockage , $T'(e_{ij})$ denotes the recovery time segment of the blockage on e_{ij} , $T'(e_{ij})=[t_i',t_i'']$, t_i' denotes starting time of the blockage , t_i'' denotes the end time of the blockage.

For convenience of the discussion, we do not distinguish the cost and the time for the traveler. In addition, all discussions are based on the following essential assumptions:

(1)The traveler only stays at the vertices of G .

(2)The blockage occurs on the e_{ij} only once.

(3) $T_1(v_1)=0$

3.1 The analysis of waiting time $T_1'(e_{ij})$

Let t_i denote the reaching time of the v_i who is the starting vertex of e_{ij} , $T_1'(e_{ij})$ denotes the waiting time of e_{ij} .The waiting time $T_1'(e_{ij})$ is different from the blockages' recovery time $T(e_{ij})$ .we consider the several cases of $T_1'(e_{ij})$ under the different starting time and end time of blockages as follows：

(1)If the starting time of the blockage occurs in $[0,t_i]$ $(t_i'<t_i''<t_i)$ then $T_1'(e_{ij})=0$

(2)If the starting time of the blockage occurs in $[t_i,t_i'']$ $(t_i\le t_i'\le t_i'')$, and $t_i'\le T(e_{ij})+t_i$ then $T_1'(e_{ij})=t_i''-t_i$

(3)If the starting time of the blockage occurs in $(t_i,t_i'']$ $(t_i<t_i'\le t_i'')$, and $t_i'>T(e_{ij})+t_i$ then $T_1'(e_{ij})=0$

(4)If the starting time of the blockage occurs in (t_i',t_i'') $(t_i'<t_i<t_i'')$ then $T_1'(e_{ij})=t_i''-t_i$

From the above analysis, we can obtain a lemma as follows：

Lemma 1 If the blockages are recoverable, then the passing time of the blocked edge e_{ij} is only relative with the t_i'' and $T(e_{ij})$, but irrelative with t_i' .

3.2 Algorithm analysis

Let $T_1(v_n)$ denote the shortest time from v_1 to v_n , $P(v_i)$ (the shortest time from v_1 to v_i)denote the permanence label of v_i ,, L (the upper bound for the passing time of the shortest paths from v_1 to v_i)denote the temperament label of v_i .

Step 1:

Let $S_0=\{v_1\}, i=1, \overline{S}=\{v_j\}(j=2,\cdots n)$

（ $\overline{S}$ denotes the adjacent vertices set of v_i ）

and

$$\begin{cases} T_1(v_1)=0 \\ L(v_j)=\infty \qquad v_j\in\overline{S} \end{cases}$$

Step 2

If $v_n\in S_i$, then end the computation, otherwise, turn to the next step.

Step 3

If $v_j\notin S_i$ and $e_{kj}=(v_k,v_j)\in E$, from the analysis in section 3.1, compute the $T_1'(e_{kj})$ and $T(e_{kj})+T_1'(e_{kj})$.

Step 4

If $L(v_j)\ge P(v_k)+(T(e_{kj})+T_1'(e_{kj}))$, then modify the $L(v_j)$ to $P(v_k)+(T(e_{kj})+T_1'(e_{kj}))$, Otherwise, turn to the next step.

Step 5

Let $L(v_{j_i})=\min_{v_j\notin S}\{L(v_j)\}$,

if $L(v_{j_i})\le+\infty$, then update the L of V_{j_i} to P , and let $P(v_{j_i})=L(v_{j_i})$ and $S_{i+1}=S_i\cup\{v_{j_i}\}$ ($k=j_i$), then, update i to $i+1$ and turn to step 2, otherwise, end the computation. At this time , if $v\in S_i$ then $T_1(v)=P(v)$, if $v\notin S_i$ then $T_1(v)=L(v)$ 。

3.3 The complexity of algorithm analysis

Let n and m be the number of vertices and edges in $G(V,E)$. In the worst-case criterion, when the traveler reaches the vertex v_i , and find all of edge which starting vertex is v_i have the blockages, we know that the computation times of $T_1'(e_{ij})$ is m ,the computation times of step3 is $2m$ and the computation times of step3 is $O(n^2)$ at most. From the above analysis , we know that the complexity of the algorithm is $O(n^2)$.

Theorem 1 The algorithm can compute the passing time of shortest path from v_1 to v_n within $O(n^2)$ time.

4. THE ONLINGE STRATEGY

Since the blockages are recoverable, when the traveler reaches the blockage, he may choose such strategies: one is that he stays at the blockage and waits for reopening of the road, then go on along the SR . It is reasonable for the traveler to wait until the road is reopened if the recoverable time is not too long. Another strategy that he chooses an efficient path goes along a new path or waits until the road reopening.

Let $G(V,E)$ denote an edge weighted graph with |*v*| vertices, s denote the origin and t denote the destination, $\delta(e_1,e_2,\cdots e_i,\cdots e_k)$ $e_i=(x_i,y_i)$ $(i=1,2,\cdots k)$ denote the blocked edges sequences, $x_i\,(i=1,2,\cdots k)$ denote the beginning vertex of e_i , $y_i\,(i=1,2,\cdots k)$ denote the ending vertex of e_i , $t'(e_i)$ denote the recovery time for the blockage happened at e_i , $t(e_i)$ denote the passing time of the e_i , $S(s,t)$ denote the cost (time) from s to t without any blockage, x_i-t denote the road from x_i to t on SR .

For convenience of the discussion, we do not distinguish the cost and the time for the traveler. In addition, all discussions are based on the following essential assumptions:

(1)G is connected even when the blocked edges are removed.
(2)The blockage information is available to the traveler only when he reaches it, and the passed path cannot be blocked.
(3)When the traveler meets a blockage, he may obtain the information about the recovery time.

For the classical online problem, the competitive ratio is a constant irrelative with the sequences of events. Let $C_{opt}(R)$ denote the cost of the optimal offline problem, $C_A(R)$ denote the cost of the strategy *A* for the online problem. Strategy *A* is called α *-competitive*[7][8] if the following inequality holds $C_A(R)\le\alpha\cdot C_{opt}(R)+\beta$, where α and β are constants irrelative with R . Papadimitriou and Yannakakis proved that if the number of roads that might be blocked is not fixed, and then the online algorithm with a constant competitive ratio does not exist. For the URBP, we give a new definition for the function competitive ratio. Let $C_{opt}(\delta)$ denote the cost of the optimal solution for the offline case for the traveler to go from s to t, $C_A(\delta)$ denote the cost of the online strategy *A* for the traveler to go from s to t .

Strategy *A* is called $f(k)$ *-competitive*, if the following inequality holds

$$C_A(\delta)\le f(k)\cdot C_{opt}(\delta)$$

where $f(k)$ is relative with k .

We will analyze the performance of the different strategies and show the competitive ratio in the following sections.

We use the following reasonable fact to bind the optimal solution.

FACT: $C_{opt}(\delta)$ is no less than $S(s,t)$.

4.1 The Waiting Strategy

WS (Waiting Strategy): when the traveler reaches e_i (along the SR) and finds that e_i is blocked and cannot pass through within $t(e_i)$, he choose to stay at x_i and wait for reopening of the road, then go on along the x_i . Denote this strategy as *W.*

The total cost of the traveler from s to t with *W* is

$$C_W(\delta)=S(s,t)+\sum_{i=1}^{k}t'(e_i) \qquad (4.1)$$

From (4.1) the following equality holds

$$C_W(\delta)=\left(1+\frac{\sum_{i=1}^{k}t'(e_i)}{S(s,t)}\right)\cdot S(s,t) \qquad (4.2)$$

In order to get clearer, we make the following assumptions and all the assumptions are in accordance with the reality.

Assumption 1 $t'(e_i) \le \alpha \cdot t(e_i)$ for any e_i. ($\alpha_i(i=1,2,\cdots k)$ denotes the ratio of $t'(e_i)$ to $t(e_i)$, $\alpha = \max\{\alpha_1, \alpha_2, \cdots \alpha_k\}$)

As

$$\sum_{i=1}^{k} t(e_i) \le S(s,t)$$

we have

$$\sum_{i=1}^{k} t'(e_i) \le \alpha S(s,t)$$

Substitute into (4.1), the following inequality holds:

$$C_W(\delta) \le (1+\alpha) \cdot S(s,t) \qquad (4.3)$$

From the above analysis, we can obtain the following theorem:

Theorem 2 If $t'(e_i) \le \alpha \cdot t(e_i)$ holds, then the competitive ratio of W is $1+\alpha$.

According to the definition of the competitive strategy and competitive ratio, we could conclude that W is a competitive strategy with competitive ratio $(1+\alpha)$ for online recoverable URBP.

Similar, if $\alpha = 1$, from theorem 2, we have

Corollary 1 If $t'(e_i) \le t(e_i)$ holds, then the competitive ratio of W is 2.

From the above discussion, it can be found that the competitive ratio of Waiting strategy only related to α (the ratio between recovery time $t'(e_i)$ and passing time $t(e_i)$),and it has no relation with the blockages numbers and their positions. When $\alpha = 1$, the travel time would not exceed the two times of the $S(s,t)$.

For waiting strategy, let $S(x_i, t)$ denote the passing time from x_i to t when the $i-th$ blocked edge occurs on $S(s,t)$. Because the blocked edges always happen at the SR, so we could further relax our assumptions.

Assumption 2 $t'(e_i) \le \alpha \cdot S(x_i, t)$ for any e_i. ($\alpha_i(i=1,2,\cdots k)$ denotes the ratio of $t'(e_i)$ to $S(x_i,t)$, $\alpha = \max\{\alpha_1, \alpha_2, \cdots \alpha_k\}$)

For $\quad S(x_i,t) \le S(s,t)$

then $\quad \sum_{i=1}^{k} t'(e_i) \le k \cdot \alpha \cdot S(s,t)$

Substitute into (4.1), the following inequality holds:

$$C_W(\delta) \le (1+k\cdot\alpha) \cdot S(s,t)$$

Theorem 3 If $t'(e_i) \le \alpha \cdot S(x_i,t)$ holds, then the competitive ratio of W is $1+k\cdot\alpha$.

Similarly, if $\alpha = 1$, from theorem 1, then ,we have

Corollary 2 If $t'(e_i) \le \alpha \cdot S(x_i,t)$ holds, then the competitive ratio of W is $k+1$.

From the proof of theorem 3, we would find that when the assumption is further relaxed, if $t'(e_i) \le \alpha \cdot S(s,t)$, then the theorem 3 and corollary 2 always hold.

4.2 The Greedy Strategy

GS(Greedy Strategy): when the traveler reaches e_i (along the SR) and finds that e_i is blocked and can not pass through within $t(e_i)$, he choose an efficient path, staying at x_i and waiting for reopening of the road and go on along the $x_i - t$ or choose a new shortest path from x_i to t without the blockages, whichever is better. Denote this strategy as G.

Let $L'(x_i,t)$ denote the cost of the road that the traveler hasn't finished. If the traveler waits at the x_i and goes through when the e_i is recovered, then the passing time would be denoted as $L'(x_i,t) + t'(e_i)$ .If the traveler always finds a new shortest path from x_i to t without the blockages and denotes it as $L''(x_i,t)$, then the greedy strategy should choose the path with

$$\min\{L(x_i,t) + t'(e_i), L''(x_i,t)\}.$$

Denote $L(s, x_i)$ as the travel time before arriving e_i, $S(v_1, x_1)$ as the travel time from v_1 to x_1 on SR, $S(x_1, v_n)$ as the travel time from x_1 to v_n on SR, $C_G(\delta)$ as the total travel time from s to t by taking the greedy strategy. For simplicity, we suppose that every time the traveler chooses a new optimal path without waiting when he encounters the blocked edges, we can obtain:

$$L''(x_i,t) \le L'(x_i,t) + t'(e_i) \quad (i=1,2,\cdots k)$$

In the worst case, all the blocked edges happen at the chosen optimal path and we can obtain:

$$C_G(\delta) = L(s,x_i) + L''(x_i,t) \quad (i=1,2,\cdots k)$$

Assumption 1 $t'(e_i) \le \alpha \cdot t(e_i)$ for any

e_i .($\alpha_i\left(i=1,2,\cdots k\right)$ denotes the ratio of $t'\left(e_i\right)$ to $t\left(e_i\right)$, $\alpha=\max\left\{\alpha_1,\alpha_2,\cdots\alpha_k\right\}$)

If there is only one blocked edge, we have

$$C_G\left(e_1\right)=L(s,x_1)+L''(x_1,t)$$

Since

$$L''(x_1,t)\le L'(x_1,t)+t'(e_1)$$

$$t'\left(e_1\right)\le\alpha\cdot t\left(e_1\right)$$

and

$$t\left(e_1\right)\le L'(x_1,t)$$

$$L(s,x_1)=S(s,x_1)$$

$$L'(x_1,t)=S(x_1,t)$$

We obtain

$$L''(x_1,t)\le(1+\alpha)S(x_1,t)$$

$$C_G\left(e_1\right)=S(s,x_1)+(1+\alpha)S(x_1,t)$$

$$=S(s,t)+\alpha S(x_1,t)$$

$$S(x_1,t)\le S(s,t)$$

$$C_G\left(e_1\right)\le(1+\alpha)S(s,t)$$

When encountering the second blocked edge, we also have

$$C_G\left(e_1,e_2\right)=L(s,x_2)+L''(x_2,t)$$

$$\le L(s,x_2)+L'(x_2,t)+\ t'(e_2)$$

Since

$$L'(x_2,t)=L''(x_1,t)-L''(x_1,x_2)$$

$$L(s,x_2)]=L(s,x_1)]+L''(x_1,x_2)$$

we have

$$C_G\left(e_1,e_2\right)\le L(s,x_1)+L''(x_1,t)+t'(e_2)$$

Since $t'(e_2)\le\alpha\cdot t(e_2)$

$$t(e_2)\le L''(x_1,t)$$

we have

$$C_G\left(e_1,e_2\right)\le L(s,x_1)+(1+\alpha)L''(x_1,t)$$

Since

$$L''(x_1,t)]\le(1+\alpha)L'(x_1,t)$$

$$L(s,x_1)=S(s,x_1)$$

$$L'(x_1,t)]=S(x_1,t)$$

we have

$$C_G\left(e_1,e_2\right)\le(1+\alpha)^2S(s,t)$$

Similarly, it could be concluded that

$$C_G\left(e_1,e_2,\cdots,e_k\right)\le(1+\alpha)^kS(s,t)$$

Therefore the following theorem holds：

Theorem 4 If $t'\left(e_i\right)\le\alpha\cdot t\left(e_i\right)$ holds, then the competitive ratio of *G is* $(1+\alpha)^k$.

Similar, if $\alpha=1$, from Theorem 4, we have

Corollary 3 If $t'\left(e_i\right)\le t\left(e_i\right)$ holds, then the competitive ratio of G is 2^k **.**

From the proof of theorem 4, we would find that when the assumption is further relaxed, if $t'\left(e_i\right)\le\alpha\cdot S'(x_i,t)$ **,** $t'\left(e_i\right)\le\alpha\cdot S(s,t)$, theorem 4 and corollary 3 always hold.

4.3 Comparisons between two strategies

Table 1 Competitive ratio between Two Strategies

Recovery time assumption	Competitive ratio	
	Waiting strategy	Greedy strategy
$t'\left(e_i\right)\le t\left(e_i\right)$	2	2^k
$t'\left(e_i\right)\le\alpha\cdot t\left(e_i\right)$	$1+\alpha$	$(1+\alpha)^k$
$t'\left(e_i\right)\le S(s,t)$	$k+1$	2^k
$t'\left(e_i\right)\le\alpha\cdot S(s,t)$	$1+k\cdot\alpha$	$(1+\alpha)^k$

From the above analysis, within the assumption $t'\left(e_i\right)\le\alpha\cdot t\left(e_i\right)$ **,** we know that the competitive ratio of *W* does not relate itself to the number of the blockages , if $0<\alpha\ll 1$, then $S(s,t)\le C_W\left(\delta\right)\approx 2S(s,t)$. In this case, the total waiting (cost) time is very little comparing with the moving cost on SR (just as a red light for the traffic control), this means the *W* strategy is more efficient and the total cost (time) taken by using strategy *W* approaches the cost (time) taken by an offline optimal. If $\alpha\gg 1$, then $C_W\left(\delta\right)\le(1+\alpha)S(s,t)$. In this case, the total (cost) time of waiting may be very long(such as a traffic accident happens)，but it needs to consider in more details to evaluate the performance of the *W* strategy. From (4.3), we see that the competitive ratio is mainly dependent on the ration between the total waiting time and the $S(s,t)$, not only on α, this means that in some cases, even if α is very larger, but the performance of *W* may still be very efficient. For the worst case (all α_i are the same and α is a large), *W* is not as efficient as the traveler expected.

We know that the competitive ratio of greedy strategy increases exponentially in anyone assumption, so, even in the worst case, waiting strategy is better than greedy strategy when the blocked edges always happen on the newly chosen path. However, people always would like to choose the greedy strategy when encounter the jam. They think that the greedy strategy is efficient and the cost of moving is less than waiting. In fact, this choice reflects that people do not have the correct expectation on unexpected crisis. Since greedy is widely acquired, so in what situation is greedy strategy superior to waiting strategy would be the future direction of the research.

5. CONCLUSION

As discussed in the paper, we present an offline algorithm for the Recoverable URBP based on that the blocked edges cost (time) is effected by the different starting time and end time of blockages, and the memory makes the problem be unfit for a converse solution of dynamic programming. The algorithm is a modification to the Dijkstra algorithm, and the complexity of the algorithm is $O(n^2)$. The optimization theory always treats the problem from perspectives of stand-by and assumes that the constraints do not vary, which is always non-realistic. The competitive algorithm may shed some light on the optimization of the problem taking the variations of critical factors. We study the online strategies when the blockages that occur one by one without any predictable information except whose recovery time are known. By making different assumptions on the recovery time of blocked edges, two strategies, waiting strategy and the greedy strategy are proposed and the competitive ratios of the two strategies are given. Furthermore, by comparing these two strategies, it could be found that waiting strategy is superior to greedy strategy in the same assumption. For the recoverable URBP, there have some further direction to work, such as how to deal with the problem when blocked happen at the vertices.

ACKNOWLEDGEMENT

The authors would like to acknowledge the support of research grant No.10371094, 70121001 from the NSF of China.

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