A New Thought about Modeling of Bilevel Programming Problems

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Abstract

This paper presents a new mathematical model for bilevel linear programming problems (BLPPs). A new kind of constraint is proposed to emphasis the pre-determined hierarchy in the bilevel decision-making process. The solution to the BLPPs based on this new model is defined. A number of simple examples of BLPPs have been solved using the new model. It has been shown that the new model is able to handle a wider range of BLPPs.

1. Introduction

The origin of the bilevel programming problems can be traced back to 1952 when they had been formulated by H.V. Strackelberg in a monograph on market economy [1]. Dempe [2] defined the BLPPs as mathematical optimization problems where the set of all variables is partitioned between two vectors X and Y, which are controlled by the upper level (leader) and the lower level (follower), respectively, and vector Y is to be chosen as an optimal solution of a second mathematical programming problem parameterized in vector X. According to Dempe, BLPPs turn to be complicated mathematical problems because (a) they are NP-hard; (b) their formulation has inherent difficulties even with respect to the notion of a solution and for many of its reformulations as one-level optimization problems regularity conditions cannot be satisfied at any feasible point. Bard [3] described the linear bilevel programming problems as a mathematical model as follows:

For $x \in X \subset R^n$, $y \in Y \subset R^m$, $F: X \times Y \to R^1$, and $f: X \times Y \to R^1$,

$$\min_{x \in X} F(x, y) = c_1 x + d_1 y \qquad (1.1a)$$

$$\text{subject to } A_1 x + B_1 y \le b_1 \qquad (1.1b)$$

$$\min_{y \in Y} f(x, y) = c_2 x + d_2 y \qquad (1.1c)$$

$$\text{subject to } A_2 x + B_2 y \le b_2 \qquad (1.1d)$$

where $c_1, c_2 \in R^n$, $d_1, d_2 \in R^m$, $b_1 \in R^p$, $b_2 \in R^q$, $A_1 \in R^{p\times n}$, $B_1 \in R^{p\times m}$, $A_2 \in R^{q\times n}$, $B_2 \in R^{q\times m}$.

The solution to the problem (1.1) was described by a set of definitions as follows [3, on P196]:

(a) Constraint region of the BLPP, denoted by S

$$S \overset{\Delta}{=} \{(x, y) : x \in X : y \in Y, A_1 x + B_1 y \le b_1, A_2 x + B_2 y \le b_2\} \qquad (1.2)$$

(b) Feasible set for the follower for each fixed $x \in X$

$$S(x) \overset{\Delta}{=} \{y \in Y, A_2 x + B_2 y \le b_2\} \qquad (1.3)$$

(c) Projection of S onto the leader's decision space

$$S(X) \overset{\Delta}{=} \{x \in X : \exists y \in Y, A_1 x + B_1 y \le b_1, A_2 x + B_2 y \le b_2\} \qquad (1.4)$$

(d) Follower's rational reaction set for $x \in S(X)$

$$P(x) \overset{\Delta}{=} \{y \in Y : y \in \arg\min[f(x, \hat{y}) : \hat{y} \in S(x)]\} \qquad (1.5)$$

(e) Inducible region (IR)

$$IR = \{(x, y) : (x, y) \in S, y \in P(x)\} \qquad (1.6)$$

In terms of the above notation, the BLPP can be written as

$$\min\{F(x, y) : (x, y) \in IR\} \qquad (1.7)$$

It can be seen from (1.7) that the solution to BLPP (1.1) can be found by solving a one-level linear programming problem in which the objective function is ***min F(x,y)*** and the constraint region is IR from pure mathematical point of view. According to Bard [3], the inducible region IR is the intersection of two sets, which are the leader's constraint region S and the region determined by the follower's optimal solution for all $x \in S(X)$, *i.e.*

$$\{(x, y) : x \in S(X), y \in P(x)\}$$

where P(x) is defined in (1.5). So it is obvious that if the leader's constraint region has no intersection with the follower's optimal solution set, i.e., the IR is empty, the problem (1.1) or (1.7) has no solution. However, this is not always the case. For example, one can expect the solution to BLPP (1.1) in a real world situation as the optimal solution for both objective functions over S or the optimal solution for the leader objective function over S. Further more, this model assumes that the information is only perfect for the leader, not the follower. This might lead to a situation in which the solution is not reasonable.

This paper presents a new mathematical model of BLPPs. The new model is developed by modifying the existing model of BLPPs defined by Bard [3] to release the above limitations. The first key point is to relax the problem's constraints by setting the follower's objective function as a reference-to constraint instead of a subject-to constraint. The second key point is to distinguish the two situations: one is when the information is perfect to the leader only and when the information is perfect to both leader and followers. A number of examples of linear BLPPs are depicted and the results show that the new model is able to handle a wider range of BLPPs and produce more reasonable solutions.

The rest of the paper is organized as follows. Section 2 describes development of the new model of BLPPs. Section 3 compares the defined solutions to a number of sample BLPPs using the new and existing models. Section 4 concludes the paper.

2. A New Mathematical Model of Bilevel Programming Problems

Refer to the model of BLPPs (1.1), we separate the problem into the leader's problem or the upper level problem

$$\min_{x \in X} F(x, y) = c_1 x + d_1 y \tag{2.1a}$$

$$\text{subject to} \quad A_1 x + B_1 y \le b_1 \tag{2.1b}$$

and the follower's problem or the lower level problem

$$\min_{y \in Y} f(x, y) = c_2 x + d_2 y \tag{2.2a}$$

$$\text{subject to} \quad A_2 x + B_2 y \le b_2 \tag{2.2b}$$

where (2.1b) and (2.2b) are referred as the leader's and the follower's subject-to constraint functions, respectively. Both problems are parametric linear programming problems because each level can only control one variable vector. When these two problems are combined to form a BLPP, there two possible cases: one is that the information is perfect for both levels, which means that both leader and follower know the objective functions and subject to constraints at the other level and the other case is that the information is perfect for the leader only, which means only the leader knows the objective functions and constraints of the follower. In the following sections, these two cases will be discussed separately, then a generalised model will be presented.

2.1 Case 1 -- A BLPP with perfect information for both levels

Given a BLPP with perfect information for both levels, for a possible $x \in S(X)$, the leader knows the follower's response y(x) which satisfies both the leader's and the follower's subject-to constraints and will select a x* so that the leader's objective function F(x*,y(x*)) is minimized.

The model can be written as

$$\min_{x \in X} F(x, y) = c_1 x + d_1 y \tag{2.3a}$$

$$\text{subject to} \quad A_1 x + B_1 y \le b_1 \tag{2.3b}$$

$$A_2 x + B_2 y \le b_2 \tag{2.3c}$$

$$\min_{y \in Y} f(x, y) = c_2 x + d_2 y \tag{2.3d}$$

$$\text{subject to} \quad A_2 x + B_2 y \le b_2 \tag{2.3e}$$

$$A_1 x + B_1 y \le b_1 \tag{2.3f}$$

This model can be simplified to a form that is the same as (1.1) by omitting (2.3c) and (2.3f) because of the perfect information for both sides. The solution to (2.3) is the same as the one to (1.1) except for the follower's feasible set, which should be re-defined as

$$S(x) \overset{\Delta}{=} \{ y \in Y,\ A_1 x + B_1 y \le b_1,\ A_2 x + B_2 y \le b_2 \} \tag{2.4}$$

for each fixed $x \in S(X)$.

2.2 Case 2 -- A BLPP with perfect information only for the upper level

Given a BLPP with perfect information only for the upper level, the follower does not know the leader's objective function and the subject-to constraints. So the follower can only response to a fixed $x \in S(X)$ over its own constraint region which is defined by (2.2b). This might lead to a situation in which the constraint region S is non-empty and compact, but the follower's response to a fixed $x \in S(X)$ does not satisfy the constraint (2.1b). The existing model of BLPPs (1.1) gives no solution in this situation. This is not always reasonable because the solution to a BLPP should be expectable in a real world situation provided that the intersection of the subject-to constraint regions from both levels is non-empty and compact. In order to release this limitation, we modify the existing model of BLPP (1) as follows:

$$\min_{x \in X} (c_1 x + d_1 y) \tag{2.5a}$$

subject to $A_1 x + B_1 y \le b_1$ (2.5b)

reference to $\min_{y \in Y} (c_2 x + d_2 y)$ (2.5c)

subject to $A_2 x + B_2 y \le b_2$ (2.5d)

where "reference to" is similar to "subject to" but used to combine the upper level problem and the lower level problem to reflect the pre-determined hierarchy in the bilevel decision marking process.

Let S_f denote a set formed by a pair of x and y where $x \in S(X)$ and $y \in P(x)$, where S(X) and P(x) are defined in (1.4) and (1.5). If S_f does not belong to S, the leader will ignores the follower's responses and seeks for its own optimal solution over S. Otherwise, the leader will seeks for its own optimal solution over S_f. Consequently, the notation of solution to (2.5) is similar to the one to (1.1) and listed as follows:

(a) Constraint region S

$$S \overset{\Delta}{=} \{(x, y) : x \in X : y \in Y, A_1 x + B_1 y \le b_1, A_2 x + B_2 y \le b_2\} \tag{2.6}$$

(b) Projection of S onto the leader's decision space

$$S(X) \overset{\Delta}{=} \{x \in X : \exists y \in Y, A_1 x + B_1 y \le b_1, A_2 x + B_2 y \le b_2\} \tag{2.7}$$

(c) Feasible set for the follower for each fixed $x \in S(X)$

$$S(x) \overset{\Delta}{=} \{y \in Y, A_2 x + B_2 y \le b_2\} \tag{2.8}$$

(d) Follower's rational reaction set for $x \in S(X)$

$$P(x) \overset{\Delta}{=} \{y \in Y : y \in \arg\min[\ f(x, \hat{y}) : \hat{y} \in S(x)]\} \tag{2.9}$$

(e) Follower's optimal solution constraint region S_f

$$S_f \overset{\Delta}{=} \{(x, y) : x \ge 0, x \in S(X), y \ge 0, y \in P(x)\} \tag{2.10}$$

(f) Inducible region (IR)

$$IR \overset{\Delta}{=} \begin{cases} \{(x, y) : (x, y) \in S, y \in P(x)\} & S_f \subseteq S \\ \{(x, y) : (x, y) \in S\} & otherwise \end{cases} \tag{2.11}$$

2.3 A generalised model of BLPPs

The "reference to" can also be used in the model (2.3), which is for a BLPP with perfect information for both levels, to emphasis the two levels in the problem. Since S_f is always in S in this situation, "reference to" is the same as the "subject to". Under the assumption that the leader's and the follower's constraints implicitly contains the other's subject-to constraints, the notation of solution by (2.6)-(2.11) also can be used for the case 1. Therefore, both the cases presented in section 2.1 and 2.2 can be generalised by the model (2.5) with the notation of solution definition (2.6)-(2.11), this is regarded as the new model in the paper.

3. Comparison of the Existing and New Models by Sample BLPPs

3.1 Sample BLPP 1

Given $x \in R^1$, $y \in R^1$ and X = {x ≥ 0}, Y = {y ≥ 0}, the leader's problem is

$$\min_{x \in X} (x - 4y) \tag{3.1.1}$$

and the follower's problem is

$$\min_{y \in Y} (y) \tag{3.1.2a}$$

subject to $-x - y \le -3$ (3.1.2b)

$-2x + y \le 0$ (3.1.2c)

$2x + y \le 12$ (3.1.2d)

$-3x + 2y \le -4$ (3.1.2e)

i) *Case 1*

Form a BLPP with perfect information for both sides as

$$\min_{x \in X} (x - 4y) \tag{3.1.3a}$$

reference to $\min_{y \in Y}(y)$ (3.1.3b)

subject to $-x - y \leq -3$ (3.1.3c)

$-2x + y \leq 0$ (3.1.3d)

$2x + y \leq 12$ (3.1.3e)

$3x + 2y \leq -4$ (3.1.3f)

The constraint region S can be determined by

$$S \overset{\Delta}{=} \{(x, y) : x \geq 0, y \geq 0, -x - y \leq -3, -2x + y \leq 0, 2x + y \leq 12, 3x + 2y \leq -4\} \quad (3.1.4)$$

The projection of S onto the leader's decision space S(X) can be determined to be

$$S(X) \overset{\Delta}{=} \{x : 1 \leq x \leq 4\} \quad (3.1.5)$$

Feasible set for the follower for each fixed $x \in S(X)$

$$S(x) \overset{\Delta}{=} \{y : y \geq 0, -x - y \leq -3, -2x + y \leq 0, 2x + y \leq 12, 3x + 2y \leq -4\} \quad (3.1.6)$$

The follower's rational reaction set for $x \in S(X)$

$$P(x) \overset{\Delta}{=} \{y \in Y : y \in \arg\min[\, f(x, \hat{y}) : \hat{y} \in S(x)]\} = \begin{cases} 3 - x & 1 \leq x \leq 2 \\ 3x/2 - 2 & 2 \leq x \leq 4 \end{cases} \quad (3.1.7)$$

Graphically, S, S(X), S(x) and P(x) can be depicted in Fig.1. It can be seen that the set $S_f = \{(x, y) : x \in S(X), y \in P(x)\}$ depicted as a piece of line AB and BC that belongs to S. Therefore, the inducible region IR should be

$$IR = \{(x, y) : (x, y) \in S, y \in P(x)\} = \begin{cases} 3 - x & 1 \leq x \leq 2 \\ 3x/2 - 2 & 2 \leq x \leq 4 \end{cases} \quad (3.1.8)$$

The problem can then be written as

$$\min\{\, F(x, y) = x - 4y : (x, y) \in IR\} \quad (3.1.9)$$

According to the corollary 5.2.3 [1 on P.200], the solution to the problem can be found by comparing the F(x,y) values over the vertexes of IR as shown in Table 1.

Table 1 Finding the solution to (3.1.3)

x	y	F(x,y)=x-4y
1	2	-7
2	1	-2
4	4	-12

It can be seen that the optimal solution to (3.1.3) occurs at x=4. The leader selects x=4 and the

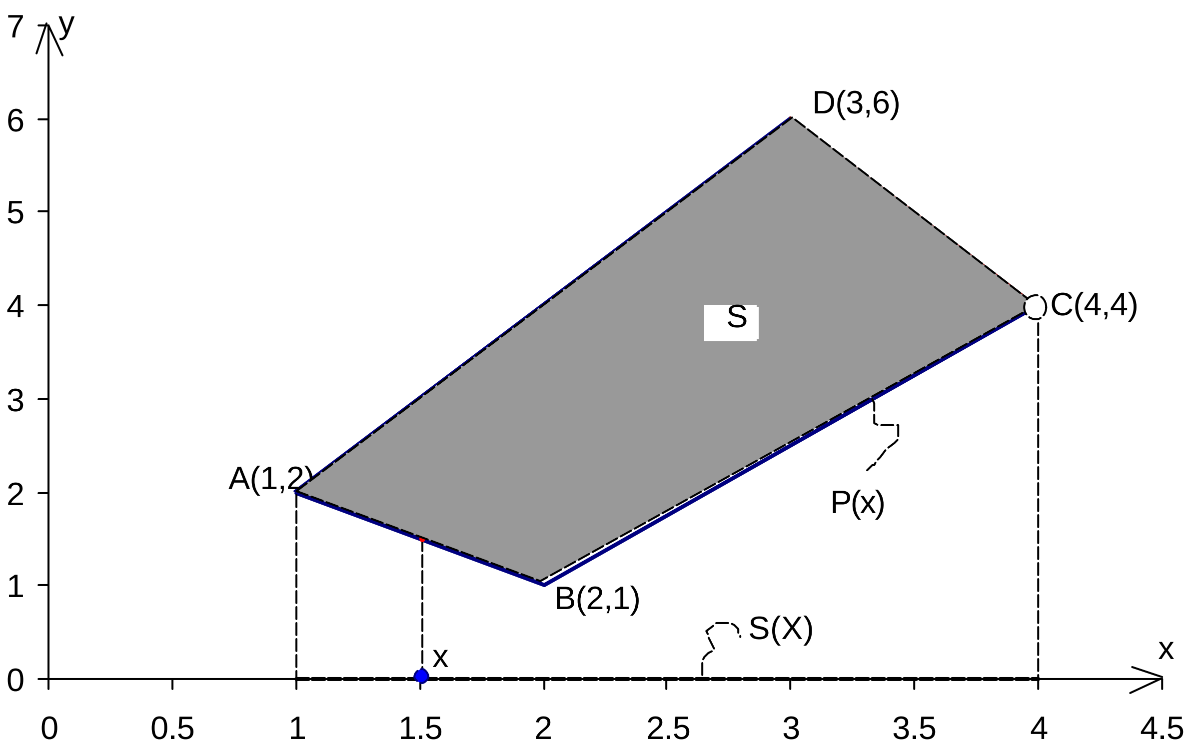

Fig.1 Illustration of S, S(X), S(x) and P(x) for the sample BLPP 1 (case 1)

follower responses y=4 to make F(x,y) = x-4y= -12.

ii) *Case 2*

Forming a BLPP with perfect information only for the leader side will result in an identical problem as (3.1.3) because the leader has no explicit constraints.

Both cases produce an identical solution to the one obtained by the existing model [3].

3.2 Sample BLPP 2

Given $x \in R^1$, $y \in R^1$ and $X = \{x \geq 0\}$, $Y = \{y \geq 0\}$, find the solution for a BLPP formed by the following two problems: the leader's problem is

$$\min_{x \in X} (x - 2y) \tag{3.2.1a}$$

subject to $x - y \leq 0$ (3.2.1b)

and the follower's problem is

$$\min_{y \in Y} (x + y) \tag{3.2.2a}$$

subject to $-x + 3y \leq 4$ (3.2.2b)

i) *Case 1*

Form a BLPP with perfect information for both sides as

$$\min_{x \in X} (x - 2y) \tag{3.2.3a}$$

subject to $x - y \leq 0$ (3.2.3b)

reference to $\min_{y \in Y} (x + y)$ (3.2.3c)

subject to $-x + 3y \leq 4$ (3.2.3d)

The constraint region S can be determined by

$$S \overset{\Delta}{=} \{(x, y) : x \geq 0, y \geq 0, y \leq \frac{1}{3}x + \frac{4}{3}, y \geq x\} \tag{3.2.4}$$

The projection of S onto the leader's decision space S(X) can be determined to be

$$S(X) \overset{\Delta}{=} \{0 \leq x \leq 2\} \tag{3.2.5}$$

Feasible set for the follower for each fixed $x \in X$

$$S(x) \overset{\Delta}{=} \{y : y \geq 0, y \geq x, -x + 3y \leq 4\} \tag{3.2.6}$$

The follower's rational reaction set for $x \in S(X)$

$$\begin{aligned} P(x) &\overset{\Delta}{=} \{y \in Y : y \in \arg\min[\, f(x, \hat{y}) : \hat{y} \in S(x)]\} \\ &= \{y : y = x\} \end{aligned} \tag{3.2.7}$$

Graphically, S, S(X), S(x) and P(x) can be depicted in Fig.2. It can be seen that S_f is a piece of line AB that is on S. Therefore, the inducible region IR should be

$$\begin{aligned} IR &= \{(x, y) : (x, y) \in S, y \in P(x)\} \\ &= \{(x, y) : 0 \leq x \leq 2, y = x\} \end{aligned} \tag{3.2.8}$$

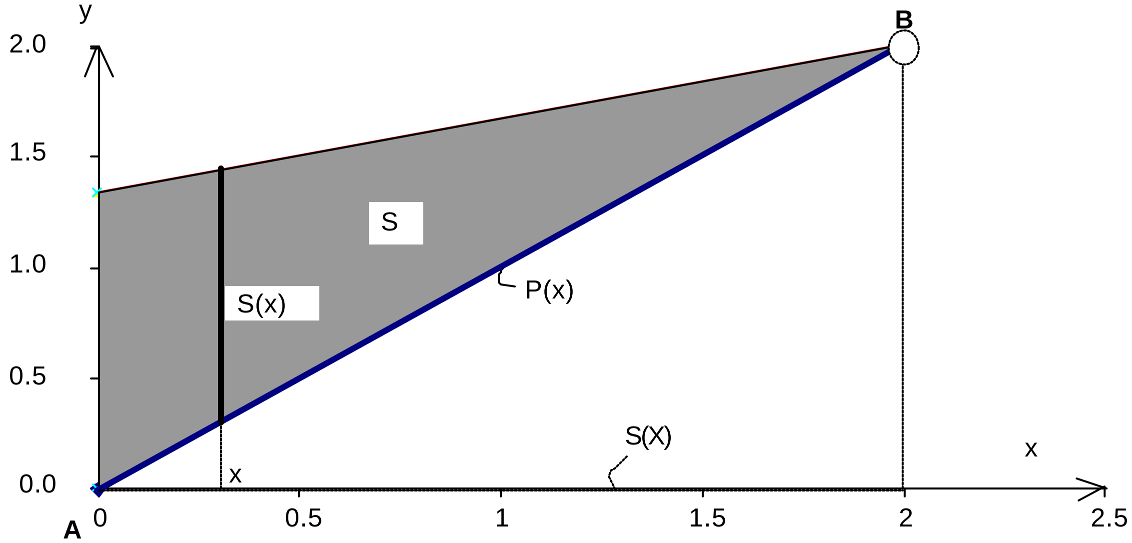

Fig.2 Illustration of S, S(X), S(x) and P(x) for the sample BLPP 2 (case 1)

The problem can then be written as

$$\min\{ F(x,y) = x - 2y : (x,y) \in IR\} \tag{3.2.9}$$

The solution to the problem can be found by comparing the F(x,y) values over the vertexes of IR as shown in Table 2.

Table 2 Finding the solution to (3.2.3)

x	y	F(x,y)=x-2y
0	0	0
2	2	-2

It can be seen that the optimal solution to (3.2.3) occurs at x=2. The leader selects x=2 and the follower responses y=2 to make F(x,y) = -2.

ii) *Case 2*

Form a BLPP with perfect information only for the leader side as

$$\min_{x\in X} (x-2y) \tag{3.2.10a}$$

subject to $x - y \le 0$ (3.2.10b)

reference to $\min_{y\in Y} (x+y)$ (3.2.10c)

subject to $-x+3y \le 4$ (3.2.10d)

The feasible set for the follower for each fixed $x \in X$

$$S(x) \stackrel{\Delta}{=} \{y : y \ge 0, -x + 3y \le 4\} \tag{3.2.11}$$

The follower's rational reaction set for $x \in S(X)$

$$P(x) \stackrel{\Delta}{=} \{y \in Y : y \in \arg\min[f(x,\hat{y}) : \hat{y} \in S(x)]\}$$
$$= \{y : y = 0\} \tag{3.2.12}$$

Graphically, S, S(X), S(x) and P(x) can be depicted in Fig.3. It can be seen that S_f is a piece of line AB that is not entirely in or on S. Therefore, the inducible region IR should be

$$IR = \{(x,y) : (x,y) \in S\} = S \tag{3.2.13}$$

The problem can then be written as

$$\min\{ F(x,y) = x - 2y : (x,y) \in IR\} \tag{3.2.14}$$

The solution to the problem can be found by comparing the F(x,y) values over the vertexes of IR as shown in Table 3.

Table 3 Finding the solution to (3.2.10)

x	y	F(x,y)=x-2y
0	0	0
2	2	-2
0	4/3	-8/3

It can be seen that the optimal solution to (3.2.10) occurs at x=0. The leader selects x=0 and the follower responses y=4/3 to make F(x,y) = x-2y = -8/3.

Table 4 tabulates the solutions to the sample BLPP 2 using the new model for the cases 1 and 2 and the existing model. It can be seen that the

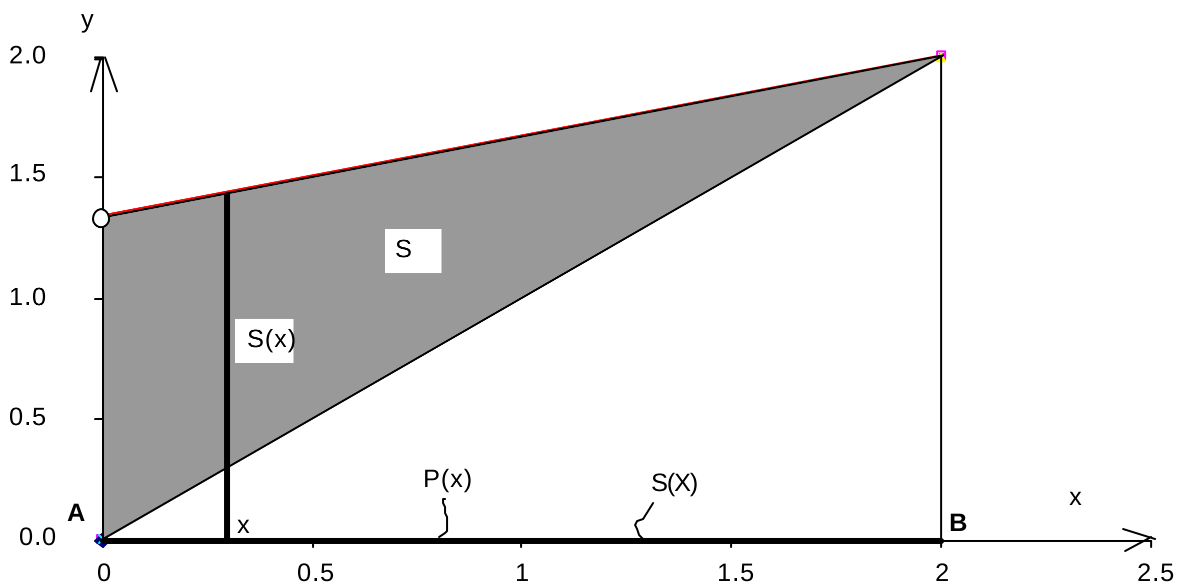

Fig.3 Illustration of S, S(X), S(x) and P(x) in the sample BLPP 2 (case 2)

solution using the new model with perfect information for the leader gives the most reasonable result.

Table 4 Comparison of solutions to sample problem2 using different models

	x	y	f(x,y)	F(x,y)
New model (case 1)	2	2	4	-2
New model (case 2)	0	4/3	4/3	-8/3
Existing model	0	0	0	0

3.3 Sample BLPP 3

Given $x \in R^1$, $y \in R^1$ and $X = \{x \geq 0\}$, $Y = \{y \geq 0\}$, find the solution for the a LBLPP formed with the following two problems: the leader's problem is

$$\min_{x \in X} F(x,y) = x - 4y \tag{3.3.1a}$$

subject to $-x - y \leq -3$ (3.3.1b)

$-3x + 2y \geq -4$ (3.3.1c)

and the follower's problem is

$$\min_{y \in Y} f(x,y) = x + y \tag{3.3.2a}$$

subject to $-2x + y \leq 0$ (3.3.2b)

$2x + y \leq 12$ (3.3.2c)

i) *Case 1*

Form a BLPP with perfect information for both sides as

$$\min_{x \in X} F(x,y) = x - 4y \tag{3.3.3a}$$

subject to $-x - y \leq -3$ (3.3.3b)

$-3x + 2y \geq -4$ (3.3.3c)

reference to $\min_{y \in Y} f(x,y) = x + y$

(3.3.3d)

subject to $-2x + y \leq 0$

(3.3.3e)

$2x + y \leq 12$

(3.3.3f)

The constraint region S can be determined by

$$S \overset{\Delta}{=} \{(x,y): x \geq 0, y \geq 0, -x - y \leq -3, -x + 2y \geq -4, \\ -2x + y \leq 0, 2x + y \leq 12\} \tag{3.3.4}$$

The projection of S onto the leader's decision space S(X) can be determined to be

$$S(X) \overset{\Delta}{=} \{1 \leq x \leq 4\} \tag{3.3.5}$$

Feasible set for the follower for each fixed $x \in S(X)$ is

$$S(x) \overset{\Delta}{=} \{y: y \geq 0, -2x + y \leq 0,\ 2x + y \leq 12, \\ -x - y \leq -3, -3x + 2y \geq -4\} \tag{3.3.6}$$

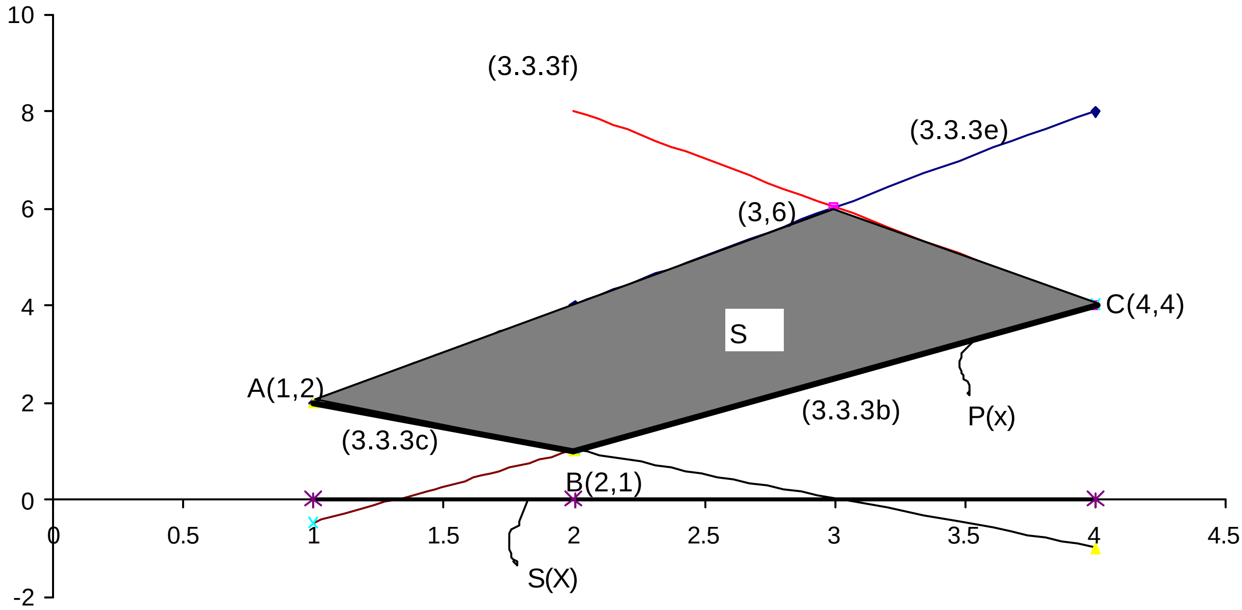

Fig.4 Illustration of S, S(x), S(X) and P(x) for sample BLPP 3 (case 1)

The follower's rational reaction set for $x \in S(X)$ is

$$P(x) \overset{\Delta}{=} \{y \in Y : y \in \arg\min[\, f(x, \hat{y}) : \hat{y} \in S(x)]\}$$

$$= \begin{cases} \{y : y = \frac{3x}{2} - 2, 1 \le x < 2\} \\ \{y : y = 3 - x, 2 \le x \le 4\} \end{cases} \quad (3.3.7)$$

$$S_f \overset{\Delta}{=} \{(x,y) : 1 \le x \le 2, -3x + 2y = -4\} \cap \{(x,y) : 2 \le x \le 4, -x - y = -3\} \quad (3.3.8)$$

Graphically, S, S(X), S(x) and P(x) can be depicted in Fig.4. It can be seen that S_f is the piece-wise line AB and BC which are on the S. Therefore, the inducible region IR should be S_f.

$$IR = \{(x, y) : (x, y) \in S, y \in P(x)\} = S_f \quad (3.3.9)$$

The problem can then be written as

$\min\{\, F(x, y) = x - 4y : (x, y) \in IR\}$

The solution to the problem (3.3.3) can be found by comparing the F(x,y) values over the vertexes of IR as shown in Table 5.

Table.5 Finding the solution to (3.3.3)

x	y	F(x,y)=x-4y
1	2	-7
2	1	-2
4	4	-12

It can be seen that the optimal solution for (3.3.3) occurs at x=4. The leader selects x=4 and the follower responses y=4 to make F(x,y) = -12.

ii) *Case 2*

Form a BLPP with perfect information for the leader only as

$$\min_{x \in X} F(x, y) = x - 4y \quad (3.3.10a)$$

subject to $-x - y \le -3$ (3.3.10b)

$-3x + 2y \ge -4$ (3.3.10c)

reference to $\min_{y \in Y} f(x, y) = x + y$ (3.3.10d)

subject to $-2x + y \le 0$ (3.3.10e)

$2x + y \le 12$ (3.3.10f)

Feasible set for the follower for each fixed $x \in S(X)$

$$S(x) \overset{\Delta}{=} \{y : y \ge 0, -2x + y \le 0, \quad 2x + y \le 12\} \quad (3.3.11)$$

The follower's rational reaction set for $x \in S(X)$

$$P(x) \overset{\Delta}{=} \{y \in Y : y \in \arg\min[\, f(x, \hat{y}) : \hat{y} \in S(x)]\}$$

$$= \{y : y = 0\} \quad (3.3.12)$$

$$S_f \overset{\Delta}{=} \{(x, y) : 1 \le x \le 4, y = 0\} \quad (3.3.13)$$

Graphically, S, S(X), S(x) and P(x) can be depicted in Fig.5. It can be seen that S_f is a piece of line AB that is out of S. Therefore, the inducible

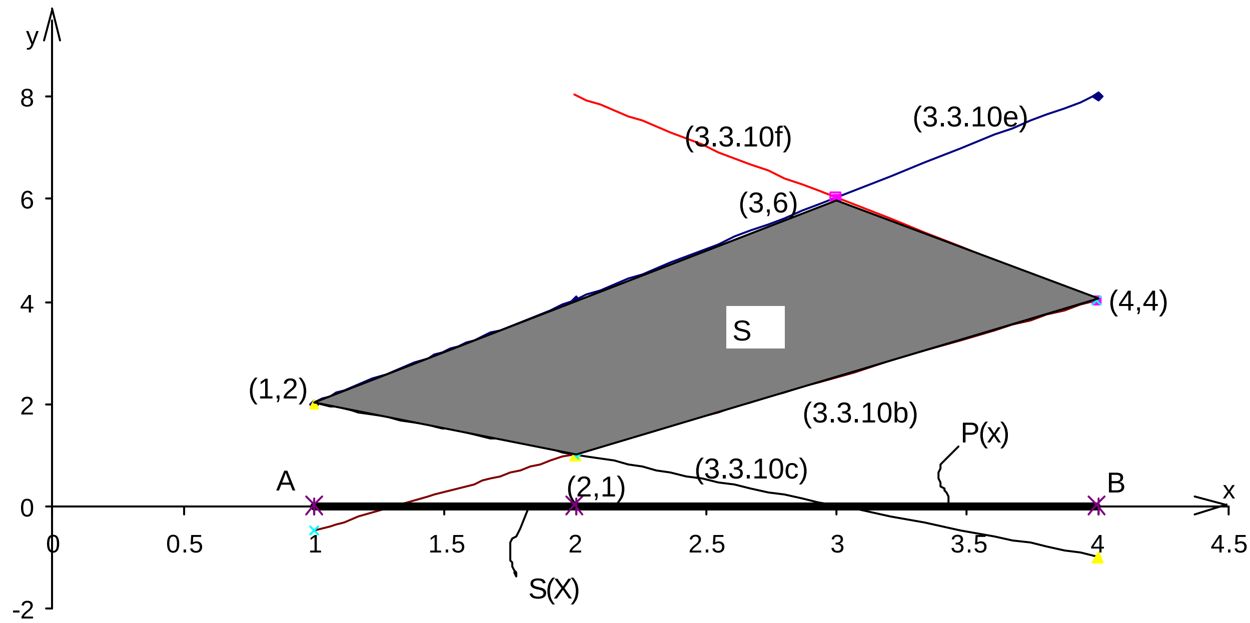

Fig.5 Illustration of S, S(x), S(X) and P(x) for sample BLPP 3 (case 2)

region IR should be S.

$$IR = \{(x, y) : (x, y) \in S\} \quad (3.3.14)$$

The problem can then be written as

$$\min\{F(x, y) = x - 4y : (x, y) \in IR\} \quad (3.3.15)$$

The solution to the problem (3.3.10) can be found by comparing the F(x,y) values over the vertexes of IR as shown in Table 6.

Table 6 Finding the solution to (3.3.10)

x	y	F(x,y)=x-4y
1	2	-7
2	1	-2
3	6	-21
4	4	-12

It can be seen that the optimal solution to (3.3.10) occurs at x=3. The leader selects x=3 and the follower responses y=6 to make F(x,y) = -21.

Table 7 tabulates the solutions to the sample BLPP 3 using the new model for the cases 1 and 2 and the existing model. It can be seen that the solution to the sample BLPP 2 using the new model with perfect information for the leader gives the most reasonable result.

Table 7 Comparison of solutions to sample BLPP3 using different models

	x	y	F(x,y)=x-2y
New model (case 1)	4	4	-12
New model (case 2)	3	6	-21
Existing model	No solution		

4. Conclusion

The follower's objective function is modelled by a new kind of constraint, *reference to*, in the BLPPs to reflects the predetermined hierarchy of the bilevel decision making process. The newly introduced constraint is the same as the one in the existing model if the set $S_f = \{(x, y) : x \in S(X), y \in P(x)\}$ belongs to S. When S_f does not belong to S, the follower's objective function will be ignored. The new model of BLPPs can lead to a reasonable solution when S_f is out of S. Three typical sample BLPPs have been solved by using the new model and the results have been compared against the results gained from the existing model. It have been shown that the new model can be expected to be suitable for a wider range of BLPPs and be able to produce more reasonable solutions.

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