Quality Management for an E-Commerce Network under Budget Constraint

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Abstract

In general, several types of information data are transmitted through an E-Commerce network simultaneously. Each type of information data is set to one type of commodity. Under the budget constraint, this paper studies the probability that a given amount of multicommodity can be transmitted through an E-Commerce network, where each node and each arc has several possible capacities. We may take this probability as a performance index for this network. Based on the properties of minimal paths, a simple algorithm is proposed to generate all lower boundary points for \((d^1,d^2,...,d^p;C)\) where \(d^i\) is the demand of commodity \(i\) and \(C\) is the budget. The probability can then be calculated in terms of such points.

1. Introduction

The capacity of each arc (the maximum flow passing the arc per unit time) in a binary-state flow network has two levels, 0 and a positive integer. For perfect nodes case, Aggarwal et al. [1] computed the system reliability, the probability that the maximum flow of single-commodity through the network is not less than the demand \(d\). Without the budget constraint, several authors [3,4,8,17,18] studied the multicommodity minimum cost flow problem, which is to minimize the total cost of multicommodity. The purpose of this paper is to extend the system reliability problem to a multicommodity case, named multicommodity reliability here, for a stochastic-flow network with node failure under budget constraint. Then a MP is an ordered sequence of arcs and nodes from \(s\) to \(t\) that has no cycle. The system reliability is the probability that the given demand \((d^1,d^2,...,d^p)\) can be transmitted through the stochastic-flow network under budget \(C\), where \(d^k, k = 1, 2, ..., p\), is the required demand of commodity \(k\). A simple algorithm is proposed to generate all lower boundary points for \((d^1,d^2,...,d^p;C)\), then the multicommodity reliability can be computed in terms of all lower boundary points for \((d^1,d^2,...,d^p;C)\).

2. Multicommodity Model Under Budget Constraint

\(G = (A, N, M)\) is a stochastic-flow network with source \(s\) and sink \(t\) where \(A = \{a_i|1 \leq i \leq n\}\) the set of arcs, \(N = \{a_i|n + 1 \leq i \leq n + r\}\) the set of nodes and \(M = (M_1, M_2, ..., M_{n+r})\) with \(M_i\) the maximal capacity of \(a_i\). Let \(x\) denote the (current) capacity of \(a_i\), and it takes values from \(\{0, 1, 2, ..., M_i\}\) with a given probability distribution.

2.1 Assumptions and Nomenclature

1. All commodities are transmitted from \(s\) to \(t\).
2. The capacities of different arcs are statistically independent.
   \(\lceil x \rceil\) the smallest integer such that \(\lceil x \rceil \geq x\)
   \(Y \geq X (y_1, y_2, ..., y_n) \leq (x_1, x_2, ..., x_n)\) if and only if \(y_i \geq x_i\) for \(i = 1, 2, ..., n + r\)
   \(Y > X (y_1, y_2, ..., y_n) > (x_1, x_2, ..., x_n)\) if and only if \(Y \geq X\) and \(y_i > x_i\) for at least one \(i\)

2.2 Multicommodity Flow

Suppose \(P_1, P_2, ..., P_m\) are M Ps form \(s\) to \(t\). The multicommodity flow model for \(G\) is described in terms of the capacity vector \(X = (x_1, x_2, ..., x_{n+r})\) and the flow assignment \((F^1, F^2, ..., F^p)\), where \(F^k = (f^k_1, f^k_2, ..., f^k_m)\) with \(f^k_j\) denoting the flow (integer-value) of commodity \(k\) through \(P_j\) for \(j = 1, 2, ..., m, k = 1, 2, ..., p\). Such an \((F^1, F^2, ..., F^p)\) which is feasible under \(X\) satisfies the following condition:
where $\sigma_k^i$ (real number) is the weight of commodity $k$ on $a_i$ i.e., the consumed amount of capacity on $a_i$ per commodity $k$. For convenience, let $\phi_i$ denote the set of $(F^1, F^2, \ldots, F^p)$ feasible under $X$. Similarly, $(F^1, F^2, \ldots, F^p) \in \phi_i$ if it satisfies

$$\sum_{k=1}^{p} (\sigma_k^i \cdot f_{ij}^k) \leq M_i \text{ for } i = 1, 2, \ldots, n + r.$$ \hspace{1cm} (1)

Let $c_i^k$ denote the transportation cost of each commodity $k$ through $a_i$. Under $X$, the network $G$ satisfies the given demand $(d_1, d_2, \ldots, d_p)$ under the budget $C$ if there exists an $(F^1, F^2, \ldots, F^p) \in \phi_i$ satisfying constraints (3) and (4);

$$\sum_{j=1}^{k} f_{ij}^k = d_i, k = 1, 2, \ldots, p \hspace{1cm} (3)$$

$$\sum_{j=1}^{k} \sum_{i \in \phi_j^k} (c_i^k \cdot f_{ij}^k) \leq C. \hspace{1cm} (4)$$

Let $\Omega = \{X\}$ there exists an $(F^1, F^2, \ldots, F^p) \in \phi_i$ satisfying constraints (3) and (4). The multicommodity reliability $R_{d_1, d_2, \ldots, d_p}^{\Omega}$ is thus

$$R_{d_1, d_2, \ldots, d_p}^{\Omega} = \Pr\{\Omega\} = \sum_{x \in \Omega} \Pr\{X\}$$

Each minimal one in $\Omega$ is named a lower boundary point for $(d_1, d_2, \ldots, d_p; C)$ throughout this paper. Hence, $X$ is a lower boundary point for $(d_1, d_2, \ldots, d_p; C)$. This is a contradiction. Hence, $X = \sum_{i=1}^{p} M_i$.

**Lemma 1.** Let $X$ be a lower boundary point for $(d_1, d_2, \ldots, d_p; C)$. Then $X = \sum_{i=1}^{p} M_i$ for each $(F^1, F^2, \ldots, F^p) \in \phi_i \cap \Phi$.

**Proof:** For each $(F^1, F^2, \ldots, F^p) \in \phi_i \cap \Phi$, constraint (1) says that $Z_{p, p', \ldots, p''} \leq X$. Suppose that $Z_{p, p', \ldots, p''} < X$, then $Z_{p, p', \ldots, p''} \notin \Omega$ as $X$ is minimal in $\Omega$. This is a contradiction. Hence, $X = \sum_{i=1}^{p} M_i$.

The following lemma further shows that $\Psi_{\min} = \{X \mid X \text{ is minimal in } \Psi\}$ is the lower set of lower boundary points for $(d_1, d_2, \ldots, d_p; C)$.

**Lemma 2.** $\{X \mid X \text{ is a lower boundary point for } (d_1, d_2, \ldots, d_p; C)\} = \Psi_{\min}.$

**Proof:** Firstly, suppose that $X$ is a lower boundary point for $(d_1, d_2, \ldots, d_p; C)$ (note that $X \in \Psi$ by lemma 1) but $X \notin \Psi_{\min}$ i.e., there exist a $Y \in \Psi$ such that $Y < X$. Then $Y \in \Omega$, which contradicts to that $X$ is a lower boundary point for $(d_1, d_2, \ldots, d_p; C)$. Hence, $X \in \Psi_{\min}$.

Conversely, suppose that $X \in \Psi_{\min}$ (note that $X \in \Omega$) but it is not a lower boundary point for $(d_1, d_2, \ldots, d_p; C)$. Then there exists a lower boundary point for $(d_1, d_2, \ldots, d_p; C) Y$ s.t. $Y < X$. By lemma 1, $Y \in \Psi$ that contradicts to that $X \in \Psi_{\min}$. Hence, $X$ is a lower boundary point for $(d_1, d_2, \ldots, d_p; C)$.

3. Algorithm

As those approaches of [11,13,14,19,21,22] we suppose all MPs have been pre-computed. Minimal paths can be efficiently derived from those algorithms discussed in [2,9,16]. The algorithm of Al-Ghanim [2] showed an approximate linear time response versus the number of network nodes. Kobayashi and Yamamoto [9] showed that to generate all minimal paths for a random network with 30 nodes and 100 arcs takes no more than 1300 seconds.

**Step 1.** Obtain all $(F^1, F^2, \ldots, F^p)$ with $F^k = (f_{ij}^k, f_{ij}^{k'}, \ldots, f_{ij}^{k''})$, $k = 1, 2, \ldots, p$, of the following constraints:

$$\sum_{i=1}^{p} (\sigma_i^k \cdot f_{ij}^k) \leq M_i \text{ for } i = 1, 2, \ldots, n + r.$$ \hspace{1cm} (5)

$$\sum_{j=1}^{k} f_{ij}^k = d_i, k = 1, 2, \ldots, p \hspace{1cm} (6)$$

$$\sum_{j=1}^{k} \sum_{i \in \phi_j^k} (c_i^k \cdot f_{ij}^k) \leq C. \hspace{1cm} (7)$$

**Step 2.** Transform each $(F^1, F^2, \ldots, F^p)$ into $x = (x_1, x_2, \ldots, x_{n+1})$ according to
Step 3. Suppose $\Psi = \{X_1, X_2, \ldots, X_\ell\}$.

3.1) $I = \phi (I$ is the stack which stores the index of each non-minimal $X$ after checking. Initially, $I = \phi$)

3.2) For $i = 1$ to $v$ and $i \notin I$

3.3) For $j = i + 1$ to $v$ with $j \notin I$

3.4) If $X_i \geq X_j$, $I = I \cup \{i\}$ and go to step 3.7)

Else $X_i > X_j$, $I = I \cup \{j\}$

3.5) $j = j + 1$

3.6) $X_i$ is a lower boundary point for $(d^0, d^1, \ldots, d^p; C)$

3.7) $i = i + 1$

3.8) End.

4. A numerical example

![Diagram](image1.png)

We use the benchmark [14,20] in Figure 1 to illustrate the proposed approach. There are 7 MPs: $P_1 = \{a_{15}, a_{11}, a_9, a_{13}, a_5, a_3, a_{10}, a_{12}, a_8, a_{14}\}$, $P_2 = \{a_{15}, a_1, a_9, a_3, a_{11}, a_0, a_{12}, a_8, a_{12}, a_8, a_{14}\}$, $P_3 = \{a_{15}, a_1, a_9, a_3, a_{11}, a_0, a_{12}, a_8, a_{12}, a_8, a_{14}\}$, $P_4 = \{a_{15}, a_1, a_9, a_3, a_{11}, a_0, a_{12}, a_8, a_{12}, a_8, a_{14}\}$, $P_5 = \{a_{15}, a_1, a_9, a_3, a_{11}, a_0, a_{12}, a_8, a_{12}, a_8, a_{14}\}$, $P_6 = \{a_{15}, a_1, a_9, a_3, a_{11}, a_0, a_{12}, a_8, a_{12}, a_8, a_{14}\}$, and $P_7 = \{a_{15}, a_1, a_9, a_3, a_{11}, a_0, a_{12}, a_8, a_{12}, a_8, a_{14}\}$. The data of arcs and nodes for 2-commodity case are shown in Table 1.

We assume the source and the sink both have infinite capacity and are perfect. If the demand $(d_1, d_3)$ is set to be (3,3) and $C = 810$ US dollars, then the multimodularity reliability $R_{1,3,810}$ can be calculated by the following steps.

**Step 1.** Obtain all $F^1 = (f^1_1, f^1_2, f^1_3, f^1_4, f^1_5, f^1_6)$, and

$$F^2 = (f^2_1, f^2_2, f^2_3, f^2_4, f^2_5, f^2_6, f^2_7)$$

of the following integer-programming:

\[
\begin{align*}
\text{Step 1:} & \quad \begin{array}{l}
\text{maximize} \quad c^1_1 x_1^1 + c^1_2 x_2^1 + c^1_3 x_3^1 + c^1_4 x_4^1 + c^1_5 x_5^1 + c^1_6 x_6^1 \\
\text{subject to} \quad \begin{array}{l}
\sum_{i=1}^{n} (a_i^1 + \sum_{r=1}^{r} f_i^1) \leq 5 \\
\sum_{i=1}^{n} (a_i^2 + f_{i}^2) \leq 5 \\
\sum_{i=1}^{n} (a_i^3 + f_{i}^3) \leq 5 \\
\sum_{i=1}^{n} (a_i^4 + f_{i}^4) \leq 5 \\
\sum_{i=1}^{n} (a_i^5 + f_{i}^5) \leq 5 \\
\sum_{i=1}^{n} (a_i^6 + f_{i}^6) \leq 5 \\
\sum_{i=1}^{n} (a_i^7 + f_{i}^7) \leq 5 \\
\end{array}
\end{array}
\end{align*}
\]

\[
\begin{align*}
\text{Step 2:} & \quad \begin{array}{l}
\text{maximize} \quad d^1_1 x_1^1 + d^1_2 x_2^1 + d^1_3 x_3^1 + d^1_4 x_4^1 + d^1_5 x_5^1 + d^1_6 x_6^1 \\
\text{subject to} \quad \begin{array}{l}
\sum_{i=1}^{n} (a_i^1 + \sum_{r=1}^{r} f_i^1) \leq 5 \\
\sum_{i=1}^{n} (a_i^2 + f_{i}^2) \leq 5 \\
\sum_{i=1}^{n} (a_i^3 + f_{i}^3) \leq 5 \\
\sum_{i=1}^{n} (a_i^4 + f_{i}^4) \leq 5 \\
\sum_{i=1}^{n} (a_i^5 + f_{i}^5) \leq 5 \\
\sum_{i=1}^{n} (a_i^6 + f_{i}^6) \leq 5 \\
\sum_{i=1}^{n} (a_i^7 + f_{i}^7) \leq 5 \\
\end{array}
\end{array}
\end{align*}
\]

Seven $(F^1, F^2)$ are obtained: $(3, 0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 0), (0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 2, 0, 0), (2, 0, 0, 1, 0, 0, 0, 0, 2, 0, 0), (0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 0), (0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0), (0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 0), (0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 0), (0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 0), (0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 0)

And the corresponding costs are 790, 790, 780, 780, 770, and 780, respectively.

**Step 2.** Transform all $(F^1, F^2)$ into $X = (x_1, x_2, \ldots, x_{12})$ according to

\[
\begin{align*}
\text{Step 2:} & \quad \begin{array}{l}
\text{maximize} \quad \begin{array}{l}
\sum_{i=1}^{n} (a_i^1 + \sum_{r=1}^{r} f_i^1) \leq 5 \\
\sum_{i=1}^{n} (a_i^2 + f_{i}^2) \leq 5 \\
\sum_{i=1}^{n} (a_i^3 + f_{i}^3) \leq 5 \\
\sum_{i=1}^{n} (a_i^4 + f_{i}^4) \leq 5 \\
\sum_{i=1}^{n} (a_i^5 + f_{i}^5) \leq 5 \\
\sum_{i=1}^{n} (a_i^6 + f_{i}^6) \leq 5 \\
\sum_{i=1}^{n} (a_i^7 + f_{i}^7) \leq 5 \\
\end{array}
\end{array}
\end{align*}
\]
\[ x_2 = \left[ f_2^1 + f_6^1 + f_1^1 + 2f_3^2 + 2f_6^2 + 2f_7^2 \right] \]
\[ x_3 = \left[ f_1^1 + f_7^1 + f_2^1 + 2f_3^2 + 2f_6^2 + 2f_7^2 \right] \]
\[ x_4 = \left[ f_1^1 + f_7^1 + f_2^1 + 2f_3^2 + 2f_6^2 + 2f_7^2 \right] \]
\[ x_5 = \left[ f_1^1 + f_7^1 + f_2^1 + 2f_3^2 + 2f_6^2 + 2f_7^2 \right] \]
\[ x_6 = \left[ f_1^1 + f_7^1 + f_2^1 + 2f_3^2 + 2f_6^2 + 2f_7^2 \right] \]
\[ x_7 = 2 \left[ f_1^1 + f_7^1 + f_2^1 + 2f_3^2 + 2f_6^2 + 2f_7^2 \right] \]  \hspace{0.5cm} (12)
\[ x_8 = 2 \left[ f_1^1 + f_7^1 + f_2^1 + 2f_3^2 + 2f_6^2 + 2f_7^2 \right] \]
\[ x_9 = 2 \left[ f_1^1 + f_7^1 + f_2^1 + 2f_3^2 + 2f_6^2 + 2f_7^2 \right] \]
\[ x_{10} = 2 \left[ f_1^1 + f_7^1 + f_2^1 + 2f_3^2 + 2f_6^2 + 2f_7^2 \right] \]
\[ x_{11} = 2 \left[ f_1^1 + f_7^1 + f_2^1 + 2f_3^2 + 2f_6^2 + 2f_7^2 \right] \]
\[ x_{12} = 2 \left[ f_1^1 + f_7^1 + f_2^1 + 2f_3^2 + 2f_6^2 + 2f_7^2 \right] \]

We obtain \( X_1 = (5, 4, 5, 0, 4, 0, 5, 4, 5, 0, 4, 5, 4, 5) \), \( X_2 = (5, 4, 4, 1, 0, 4, 5, 4, 4, 5) \), \( X_3 = (4, 5, 4, 0, 5, 4, 5, 4, 5) \), \( X_4 = (5, 4, 5, 0, 4, 5, 4, 5, 4, 5) \), \( X_5 = (5, 4, 4, 1, 0, 4, 5, 4, 4, 5) \), \( X_6 = (4, 5, 4, 0, 5, 0, 4, 5, 4, 5) \) and \( X_7 = (4, 5, 4, 0, 5, 1, 5, 4, 4, 5) \).

**Step 3.** Check each \( X_i \) whether it is a lower boundary point for (3,3;810) or not.

3.1) \( I = \phi \)

3.2) \( i = 1 \)

3.3) \( j = 2 \)

3.4) \( X_1 \) \( \not\geq \) \( X_2 \) and \( X_2 \not\geq X_1 \), \( I = \{1\} \).

3.3) \( j = 4 \)

3.4) \( X_1 \not\geq X_4 \), \( I = \{1\} \).

3.2) \( i = 2 \)

After further checking, \( X_4, X_5, X_6 \) and \( X_7 \) are all lower boundary points for (3,3;810). Let \( B_1 = \{X \geq X_1\} \), \( B_2 = \{X \geq X_2\} \), \( B_3 = \{X \geq X_3\} \) and \( B_4 = \{X \geq X_4\} \). Hence, the multicommodity reliability \( R_{3,3;810} \) is computed by the inclusion-exclusion method.

5. Conclusions

This article extends the system reliability problem to the multicommodity reliability for a stochastic-flow network with node failure under budget constraint. The multicommodity reliability is the probability that the demand \((d^1,d^2,\ldots,d^n)\) can be transmitted through the stochastic-flow network under budget \(C\). Based on the properties of minimal paths, we propose a simple algorithm to generate all lower boundary point for \((d^1,d^2,\ldots,d^n,C)\). Then the multicommodity reliability can be calculated in terms of lower boundary points for \((d^1,d^2,\ldots,d^n,C)\) by applying the inclusion-exclusion method. In our model the transportation cost \(c_i\) is not assumed to be linear in \( \sigma_i^t \). The main reason is that the transportation cost is not only dependent on the dimension of commodity but also on other attributes of commodity. For example, poison, vulnerable, fragile, etc. For the case that the transportation cost is only charged in terms of consumed capacity, \(c_i\) is linear in \( \sigma_i^t \).

However, this condition is a special case of the proposed model.

References


Table 1. The data of arcs and nodes for 2-commodity example

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<tr>
<th>Arc</th>
<th>Capacity</th>
<th>Probability</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$c_1$ (US dollar)</th>
<th>$c_2$ (US dollar)</th>
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*Pr{the capacity of $a_1$ is 0} = 0.01.