Winter 12-10-2002

Quality Management for an E-Commerce Network under Budget Constraint

Yi-Kuei Lin

Quality Management for an E-Commerce Network Under Budget Constraint

Yi-Kuei Lin
Department of Information Management
Van Nung Institute of Technology
Chung-Li, Tao-Yuan, Taiwan
yklin@cc.vit.edu.tw

Abstract

In general, several types of information data are transmitted through an E-Commerce network simultaneously. Each type of information data is set to one type of commodity. Under the budget constraint, this paper studies the probability that a given amount of multicommodity can be transmitted through an E-Commerce network, where each node and each arc has several possible capacities. We may take this probability as a performance index for this network. Based on the properties of minimal paths, a simple algorithm is proposed to generate all lower boundary points for $(d^1,d^2,\ldots,d^p;C)$ where d^i is the demand of commodity i and C is the budget. The probability can then be calculated in terms of such points.

1. Introduction

The capacity of each arc (the maximum flow passing the arc per unit time) in a binary-state flow network has two levels, 0 and a positive integer. For perfect nodes case, Aggarwal *et al.* [1] computed the system reliability, the probability that the maximum flow of the network is not less than the demand, in terms of minimal paths (MPs). A MP is an ordered sequence of arcs from the source s to the sink t that has no cycle. Lee *et al.* [10] and Rueger [15] extended the system reliability problem to the case that nodes and arcs have a positive-integer capacity and may fail. A stochastic-flow network is a multi-state network in which each arc has several states or capacities. The system reliability is the probability that the maximum flow of single-commodity through the network is not less than the demand d. Without the budget constraint, several authors [11,12,14,19,21] had presented algorithms to generate lower boundary points for d in order to evaluate the system reliability for perfect node case. Lin [13] and Yeh [22] extended the problem to the more general case that nodes have several capacities as arcs do.

However, in real world, many stochastic-flow networks allow multicommodity to be transmitted from s to t simultaneously. Assuming the flow network is deterministic (i.e., the capacity of each arc is a constant), many authors [3,4,8,17,18] studied the multicommodity minimum cost flow problem, which is to minimize the total cost of multicommodity. The purpose of this paper is to extend the system reliability problem to a multicommodity case, named multicommodity reliability here, for a stochastic-flow network with node failure under budget constraint. Then a MP is an ordered sequence of arcs and nodes from s to t that has no cycle. The system reliability is the probability that the given demand $(d^1,d^2,\ldots,d^p)$ can be transmitted through the stochastic-flow network under budget C, where d^k, $k = 1, 2, \ldots, p$, is the required demand of commodity k. A simple algorithm is proposed to generate all lower boundary points for $(d^1,d^2,\ldots,d^p;C)$, then the multicommodity reliability can be computed in terms of all lower boundary points for $(d^1,d^2,\ldots,d^p;C)$.

2. Multicommodity Model Under Budget Constraint

$G = (A, N, M)$ is a stochastic-flow network with source s and sink t where $A = \{a_i|1 \le i \le n\}$ the set of arcs, $N = \{a_i|n + 1 \le i \le n + r\}$ the set of nodes and $M = (M_1, M_2, \ldots, M_{n+r})$ with M_i the maximal capacity of a_i. Let x_i denote the (current) capacity of a_i, and it takes values from $\{0, 1, 2, \ldots, M_i\}$ with a given probability distribution.

2.1 Assumptions and Nomenclature

1. All commodities are transmitted from s to t.
2. The capacities of different arcs are statistically independent.

$\lceil x \rceil$ the smallest integer such that $\lceil x \rceil \ge x$

$Y \ge X$ $(y_1, y_2, \ldots, y_{n+r}) \le (x_1, x_2, \ldots, x_{n+r})$ if and only if $y_i \ge x_i$ for $i = 1, 2, \ldots, n + r$

$Y > X$ $(y_1, y_2, \ldots, y_{n+r}) > (x_1, x_2, \ldots, x_{n+r})$ if and only if $Y \ge X$ and $y_i > x_i$ for at least one i

2.2 Multicommodity Flow

Suppose $P_1, P_2, \ldots, P_m$ are MPs form s to t. The multicommodity flow model for G is described in terms of the capacity vector $X = (x_1, x_2, \ldots, x_{n+r})$ and the flow assignment $(F^1, F^2,\ldots, F^p)$, where $F^k = (f_1^k, f_2^k,\ldots, f_m^k)$ with f_j^k denoting the flow (integer-value) of commodity k through P_j, $j = 1, 2, \ldots, m$, $k = 1, 2, \ldots, p$. Such an $(F^1, F^2,\ldots, F^p)$ which is feasible under X satisfies the following condition:

The Second International Conference on Electronic Business
Taipei, Taiwan, December 10-13, 2002

$$\left\lceil \sum_{k=1}^{p}(\varpi_i^k \cdot \sum_{a_i \in P_j} f_j^k) \right\rceil \le x_i \text{ for } i = 1, 2, ..., n + r, \quad (1)$$

where ϖ_i^k (real number) is the weight of commodity k on a_i i.e., the consumed amount of capacity on a_i per commodity k. For convenience, let ϕ_X denote the set of $(F^1, F^2,\ldots, F^p)$ feasible under X. Similarly, $(F^1, F^2,\ldots, F^p) \in \phi_M$ if it satisfies

$$\left\lceil \sum_{k=1}^{p}(\varpi_i^k \cdot \sum_{a_i \in P_j} f_j^k) \right\rceil \le M_i \text{ for } i = 1, 2, ..., n + r. \quad (2)$$

Let c_i^k denote the transportation cost of each commodity k through a_i. Under X, the network G satisfies the given demand $(d^1,d^2,\ldots,d^p)$ under the budget C if there exists an $(F^1, F^2,\ldots, F^p) \in \phi_X$ satisfying constraints (3) and (4);

$$\sum_{j=1}^{m} f_j^k = d^k,\ k = 1, 2, \ldots, p \quad (3)$$

$$\sum_{i=1}^{n+r}\sum_{k=1}^{p}\left(c_i^k \cdot \sum_{a_i \in mp_j} f_j^k \right) \le C. \quad (4)$$

Let $\Omega = \{X|$ there exists an $(F^1, F^2,\ldots, F^p) \in \phi_X$ satisfying constraints (3) and (4)$\}$. The multicommodity reliability $R_{d^1,d^2,\ldots,d^p;C}$ is thus

$$R_{d^1,d^2,\ldots,d^p;C} = \Pr\{\Omega\} = \sum_{X\in\Omega}\Pr\{X\}$$

Each minimal one in Ω is named a lower boundary point for $(d^1,d^2,\ldots,d^p;C)$ throughout this paper i.e., X is a lower boundary point for $(d^1,d^2,\ldots,d^p;C)$ if and only if i) $X \in \Omega$ and ii) $Y \notin \Omega$ for any capacity vector Y such that $Y < X$. Hence,

$R_{d^1,d^2,\ldots,d^p;C} = \Pr\{Y|Y \ge X$ for a lower boundary point for $(d^1,d^2,\ldots,d^p;C)\ X\}$.

2.3 Generate All Lower Boundary Points for $(d^1,d^2,\ldots,d^p;C)$

Let $\Phi = \{(F^1, F^2,\ldots, F^p)|\ (F^1, F^2,\ldots, F^p)$ satisfies constraints (2) - (4)$\}$. For each $(F^1, F^2,\ldots, F^p) \in \Phi$, generate the capacity vector $Z_{F^1,F^2,\ldots,F^p} = (z_1, z_2, \ldots, z_{n+r})$ via

$$z_i = \left\lceil \sum_{k=1}^{p}(\varpi_i^k \cdot \sum_{a_i \in P_j} f_j^k) \right\rceil \text{ for } i = 1, 2, ..., n + r. \quad (5)$$

For convenience, let $\Psi = \{ Z_{F^1,F^2,\ldots,F^p} |(F^1, F^2,\ldots, F^p) \in \Phi\}$. We will first see that Ψ contains all lower boundary points for $(d^1,d^2,\ldots,d^p;C)$ in the following lemma.

Lemma 1. Let X be a lower boundary point for $(d^1,d^2,\ldots,d^p;C)$. Then $X = Z_{F^1,F^2,\ldots,F^p}$ for each $(F^1, F^2,\ldots, F^p) \in \phi_X \cap \Phi$.

Proof: For each $(F^1, F^2,\ldots, F^p) \in \phi_X \cap \Phi$, constraint (1) says that $Z_{F^1,F^2,\ldots,F^p} \le X$. Suppose that $Z_{F^1,F^2,\ldots,F^p} < X$, then $Z_{F^1,F^2,\ldots,F^p} \notin \Omega$ as X is minimal in Ω. This is a contradiction. Hence, $X = Z_{F^1,F^2,\ldots,F^p}$.

□

The following lemma further shows that $\Psi_{\min} \equiv \{X|X$ is minimal in $\Psi\}$ is the set of lower boundary point for $(d^1,d^2,\ldots,d^p;C)$.

Lemma 2. $\{X|X$ is a lower boundary point for $(d^1,d^2,\ldots,d^p;C)\} = \Psi_{\min}$.

Proof: Firstly, suppose that X is a lower boundary point for $(d^1,d^2,\ldots,d^p;C)$ (note that $X \in \Psi$ by lemma 1) but $X \notin \Psi_{\min}$ i.e., there exist a $Y \in \Psi$ such that $Y < X$. Then $Y \in \Omega$, which contradicts to that X is a lower boundary point for $(d^1,d^2,\ldots,d^p;C)$. Hence, $X \in \Psi_{\min}$.

Conversely, suppose that $X \in \Psi_{\min}$ (note that $X \in \Omega$) but it is not a lower boundary point for $(d^1,d^2,\ldots,d^p;C)$. Then there exists a lower boundary point for $(d^1,d^2,\ldots,d^p;C)$ Y s.t. $Y < X$. By lemma 1, $Y \in \Psi$ that contradicts to that $X\in\Psi_{\min}$. Hence, X is a lower boundary point for $(d^1,d^2,\ldots,d^p;C)$.

□

3. Algorithm

As those approaches of [11,13,14,19,21,22] we suppose all MPs have been pre-computed. Minimal paths can be efficiently derived from those algorithms discussed in [2,9,16]. The algorithm of Al-Ghanim [2] showed an approximate linear time response versus the number of network nodes. Kobayashi and Yamamoto [9] showed that to generate all minimal paths for a random network with 30 nodes and 100 arcs takes no more than 1300 seconds.

Step 1. Obtain all $(F^1, F^2,\ldots, F^p)$ with $F^k = (f_1^k, f_2^k,..., f_m^k)$, $k = 1, 2, \ldots, p$, of the following constraints:

$$\left\lceil \sum_{k=1}^{p}(\varpi_i^k \cdot \sum_{a_i \in P_j} f_j^k) \right\rceil \le M_i \quad \text{for } i = 1, 2, ..., n + r (5)$$

$$\sum_{j=1}^{m} f_j^k = d^k,\ k = 1, 2, \ldots, p \quad (6)$$

$$\sum_{i=1}^{n+r}\sum_{k=1}^{p}\left(c_i^k \cdot \sum_{a_i \in mp_j} f_j^k \right) \le C \quad (7)$$

Step 2. Transform each $(F^1, F^2,\ldots, F^p)$ into $X = (x_1, x_2, \ldots, x_{n+r})$ according to

$$x_i = \left\lceil \sum_{k=1}^{p} (\varpi_i^k \cdot \sum_{a_i \in P_j} f_j^k) \right\rceil \quad \text{for } i = 1, 2, ..., n + r \qquad (8)$$

Step 3. Suppose $\Psi = \{X_1, X_2, \ldots, X_v\}$.

3.1) $I = \phi$ (I is the stack which stores the index of each non-minimal X after checking. Initially, $I = \phi$.)
3.2) For i = 1 To v and $i \notin I$
3.3) For $j = i + 1$ To v with $j \notin I$
3.4) If $X_i \geq X_j$, $I = I \cup \{i\}$ and go to step 3.7) Elseif $X_j > X_i$, $I = I \cup \{j\}$
3.5) $j = j + 1$
3.6) X_i is a lower boundary point for $(d^1, d^2, \ldots, d^p; C)$
3.7) $i = i + 1$
3.8) End.

4. A numerical example

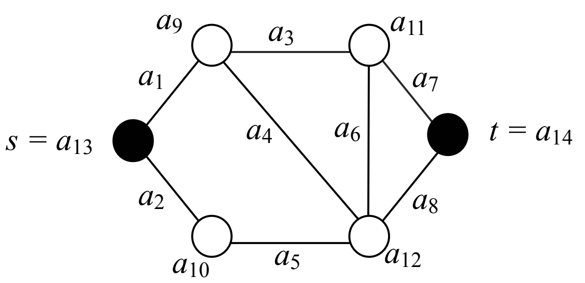

Figure 1. A benchmark [14,20]

We use the benchmark [14,20] in Figure 1 to illustrate the proposed approach. There are 7 MPs: $P_1 = \{a_{13}, a_1, a_9, a_3, a_{11}, a_7, a_{14}\}$, $P_2 = \{a_{13}, a_1, a_9, a_3, a_{11}, a_6, a_{12}, a_8, a_{14}\}$, $P_3 = \{a_{13}, a_1, a_9, a_4, a_{12}, a_8, a_{14}\}$, $P_4 = \{a_{13}, a_1, a_9, a_4, a_{12}, a_6, a_{11}, a_7, a_{14}\}$, $P_5 = \{a_{13}, a_2, a_{10}, a_5, a_{12}, a_8, a_{14}\}$, $P_6 = \{a_{13}, a_2, a_{10}, a_5, a_{12}, a_6, a_{11}, a_7, a_{14}\}$ and $P_7 = \{a_{13}, a_2, a_{10}, a_5, a_{12}, a_4, a_9, a_3, a_{11}, a_7, a_{14}\}$. The data of arcs and nodes for 2-commodity case are shown in Table 1. We assume the source and the sink both have infinite capacity and are perfect. If the demand (d_1, d_2) is set to be (3,3) and C = 810 US dollars, then the multicommodity reliability $R_{3,3;810}$ can be calculated by the following steps.

Step 1. Obtain all $F^1 = (f_1^1, f_2^1, f_3^1, f_4^1, f_5^1, f_6^1, f_7^1)$, and $F^2 = (f_1^2, f_2^2, f_3^2, f_4^2, f_5^2, f_6^2, f_7^2)$ of the following integer-programming:

(9) a_1: $\lceil f_1^1 + f_2^1 + f_3^1 + f_4^1 + 2f_1^2 + 2f_2^2 + 2f_3^2 + 2f_4^2 \rceil \leq 5$

a_2: $\lceil f_5^1 + f_6^1 + f_7^1 + 2f_5^2 + 2f_6^2 + 2f_7^2 \rceil \leq 5$

a_3: $\lceil f_1^1 + f_2^1 + f_7^1 + 2f_1^2 + 2f_2^2 + 2f_7^2 \rceil \leq 5$

a_4: $\lceil f_3^1 + f_4^1 + f_7^1 + 2f_3^2 + 2f_4^2 + 2f_7^2 \rceil \leq 5$

a_5: $\lceil f_5^1 + f_6^1 + f_7^1 + 2f_5^2 + 2f_6^2 + 2f_7^2 \rceil \leq 5$

a_6: $\lceil f_2^1 + f_4^1 + f_6^1 + 2f_2^2 + 2f_4^2 + 2f_6^2 \rceil \leq 5$

a_7: $\lceil f_1^1 + f_4^1 + f_6^1 + f_7^1 + 2f_1^2 + 2f_4^2 + 2f_6^2 + 2f_7^2 \rceil \leq 5$

a_8: $\lceil f_2^1 + f_3^1 + f_5^1 + 2f_2^2 + 2f_3^2 + 2f_5^2 \rceil \leq 5$

a_9: $\lceil f_1^1 + f_2^1 + f_3^1 + f_4^1 + f_7^1 + 2f_1^2 + 2f_2^2 + 2f_3^2 + 2f_4^2 + 2f_7^2 \rceil \leq 9$

a_{10}: $\lceil f_5^1 + f_6^1 + f_7^1 + 2f_5^2 + 2f_6^2 + 2f_7^2 \rceil \leq 9$

a_{11}: $\lceil f_1^1 + f_2^1 + f_4^1 + f_6^1 + f_7^1 + 2f_1^2 + 2f_2^2 + 2f_4^2 + 2f_6^2 + 2f_7^2 \rceil \leq 9$

a_{12}: $\lceil f_2^1 + f_3^1 + \ldots + f_7^1 + 2f_2^2 + 2f_3^2 + \ldots + 2f_7^2 \rceil \leq 9$

(10) $f_1^1 + f_2^1 + f_3^1 + f_4^1 + f_5^1 + f_6^1 + f_7^1 = 3$
$f_1^2 + f_2^2 + f_3^2 + f_4^2 + f_5^2 + f_6^2 + f_7^2 = 3$

(11) $\{10(f_1^1 + f_2^1 + f_3^1 + f_4^1) + 20(f_1^2 + f_2^2 + f_3^2 + f_4^2)\} +$
$\{20(f_5^1 + f_6^1 + f_7^1) + 30(f_5^2 + f_6^2 + f_7^2)\} +$
$\{30(f_1^1 + f_2^1 + f_7^1) + 40(f_1^2 + f_2^2 + f_7^2)\} +$
$\{20(f_3^1 + f_4^1 + f_7^1) + 40(f_3^2 + f_4^2 + f_7^2)\} +$
$\{10(f_5^1 + f_6^1 + f_7^1) + 20(f_5^2 + f_6^2 + f_7^2)\} +$
$\{20(f_2^1 + f_4^1 + f_6^1) + 30(f_2^2 + f_4^2 + f_6^2)\} +$
$\{30(f_1^1 + f_4^1 + f_6^1 + f_7^1) + 40(f_1^2 + f_4^2 + f_6^2 + f_7^2)\} +$
$\{20(f_2^1 + f_3^1 + f_5^1) + 40(f_2^2 + f_3^2 + f_5^2)\} +$
$\{20(f_1^1 + f_2^1 + f_3^1 + f_4^1 + f_7^1) + 30(f_1^2 + f_2^2 + f_3^2 + f_4^2 + f_7^2)\} +$
$\{20(f_5^1 + f_6^1 + f_7^1) + 30(f_5^2 + f_6^2 + f_7^2)\} +$
$\{20(f_1^1 + f_2^1 + f_4^1 + f_6^1 + f_7^1) + 30(f_1^2 + f_2^2 + f_4^2 + f_6^2 + f_7^2)\} +$
$\{20(f_2^1 + f_3^1 + \ldots + f_7^1) + 30(f_2^2 + f_3^2 + \ldots + f_7^2)\}$
≤ 810

Seven (F^1, F^2) are obtained: (3, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 0), (2, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 0), (2, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 2, 0, 0), (1, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 1, 0, 0), (0, 0, 1, 0, 2, 0, 0, 2, 0, 0, 0, 1, 0, 0), (0, 0, 0, 0, 3, 0, 0, 2, 0, 0, 0, 1, 0, 0) and (0, 0, 0, 0, 2, 1, 0, 2, 0, 0, 0, 1, 0, 0). And the corresponding costs are 790, 790, 780, 780, 780, 770 and 810, respectively.

Step 2. Transform all (F^1, F^2) into $X = (x_1, x_2, \ldots, x_{12})$ according to

$$x_1 = \lceil f_1^1 + f_2^1 + f_3^1 + f_4^1 + 2f_1^2 + 2f_2^2 + 2f_3^2 + 2f_4^2 \rceil$$

$x_2 = \lceil f_5^1 + f_6^1 + f_7^1 + 2f_5^2 + 2f_6^2 + 2f_7^2 \rceil$
$x_3 = \lceil f_1^1 + f_2^1 + f_7^1 + 2f_1^2 + 2f_2^2 + 2f_7^2 \rceil$
$x_4 = \lceil f_3^1 + f_4^1 + f_7^1 + 2f_3^2 + 2f_4^2 + 2f_7^2 \rceil$
$x_5 = \lceil f_5^1 + f_6^1 + f_7^1 + 2f_5^2 + 2f_6^2 + 2f_7^2 \rceil$
$x_6 = \lceil f_2^1 + f_4^1 + f_6^1 + 2f_2^2 + 2f_4^2 + 2f_6^2 \rceil$ (12)
$x_7 = \lceil f_1^1 + f_4^1 + f_6^1 + f_7^1 + 2f_1^2 + 2f_4^2 + 2f_6^2 + 2f_7^2 \rceil$
$x_8 = \lceil f_2^1 + f_3^1 + f_5^1 + 2f_2^2 + 2f_3^2 + 2f_5^2 \rceil$
$x_9 = \lceil f_1^1 + f_2^1 + f_3^1 + f_4^1 + f_7^1 + 2f_1^2 + 2f_2^2 + 2f_3^2 + 2f_4^2 + 2f_7^2 \rceil$
$x_{10} = \lceil f_5^1 + f_6^1 + f_7^1 + 2f_5^2 + 2f_6^2 + 2f_7^2 \rceil$
$x_{11} = \lceil f_1^1 + f_2^1 + f_4^1 + f_6^1 + f_7^1 + 2f_1^2 + 2f_2^2 + 2f_4^2 + 2f_6^2 + 2f_7^2 \rceil$
$x_{12} = \lceil f_2^1 + f_3^1 + ... + f_7^1 + 2f_2^2 + 2f_3^2 + ... + 2f_7^2 \rceil$

We obtain X_1 = (5, 4, 5, 0, 4, 0, 5, 4, 5, 4, 5, 4), X_2 = (5, 4, 4, 1, 4, 0, 4, 5, 5, 4, 4, 5), X_3 = (4, 5, 4, 0, 5, 0, 4, 5, 4, 5, 4, 5), X_4 = (5, 4, 5, 0, 4, 0, 5, 4, 5, 4, 5, 4), X_5 = (5, 4, 4, 1, 4, 0, 4, 5, 5, 4, 4, 5), X_6 = (4, 5, 4, 0, 5, 0, 4, 5, 4, 5, 4, 5) and X_7 = (4, 5, 4, 0, 5, 1, 5, 4, 4, 5, 5, 5).

Step 3. Check each X_i whether it is a lower boundary point for (3,3;810) or not.
3.1) $I = \phi$
3.2) $i = 1$
3.3) $j = 2$
3.4) $X_1 \ngeq X_2$ and $X_2 \ngtr X_1$. $I = \{\phi\}$.
3.3) $j = 3$
3.4) $X_1 \ngeq X_3$ and $X_3 \ngtr X_1$. $I = \{\phi\}$.
3.5) $j = 4$
3.4) $X_1 \geq X_4$. $I = \{1\}$.
3.2) $i = 2$
⋮

After further checking, X_4, X_5, X_6 and X_7 are all lower boundary points for (3,3;810). Let $B_1 = \{X|X \geq X_4\}$, $B_2 = \{X|X \geq X_5\}$, $B_3 = \{X|X \geq X_6\}$ and $B_4 = \{X|X \geq X_7\}$. Hence, the multicommodity reliability $R_{3,3;810} = \Pr\{B_1 \cup B_2 \cup B_3 \cup B_4\} = 0.61623376$ can be computed by the inclusion-exclusion method.

5. Conclusions

This article extends the system reliability problem to the multicommodity reliability for a stochastic-flow network with node failure under budget constraint. The multicommodity reliability is the probability that the demand $(d^1,d^2,\ldots,d^p)$ can be transmitted through the stochastic-flow network under budget C. Based on the properties of minimal paths, we propose a simple algorithm to generate all lower boundary point for $(d^1,d^2,\ldots,d^p;C)$. Then the multicommodity reliability can be calculated in terms of lower boundary points for $(d^1,d^2,\ldots,d^p;C)$ by applying the inclusion-exclusion method. In our model the transportation cost c_i^k is not assumed to be linear in ϖ_i^k. The main reason is that the transportation cost is not only dependent on the dimension of commodity but also on other attributes of commodity. For example, poison, vulnerable, fragile, etc. For the case that the transportation cost is only charged in terms of consumed capacity, c_i^k is linear in ϖ_i^k. However, this condition is a special case of the proposed model.

References

[1] Aggarwal, K. K., Chopra, Y. C. and Bajwa, J. S., "Capacity consideration in reliability analysis of communication systems," *IEEE Trans. Reliability*, 1982, 31, 177-180.

[2] Al-Ghanim, A. M., "A heuristic technique for generating minimal paths and cutsets of a general network," *Computers and Industrial Engineering*, 1999, 36, 45-55.

[3] Cremeans, J. E., Smith, R, A. and Tyndall, G. R., "Optimal multicommodity network flows with resource allocation," *Naval Research Logistics Quarterly*, 1970, 17, 269-279.

[4] Evans, J. R., "A combinatorial equivalence between a class of multicommodity flow problems and the capacitated transportation problem," *Mathematical Programming*, 1976, 10, 401-404.

[5] Ford, L. R. and Fulkerson, D. R., Flows in networks, Princeton University Press, Princeton, New Jersey, 1962.

[6] Griffith, W. S., "Multistate reliability models," *Journal of Applied Probability*, 1980, 17, 735-744.

[7] Hudson, J. C. and Kapur, K. C., "Reliability bounds for multistate systems with multistate components," *Operations Research*, 1985, 33, 153-160.

[8] Kennington, J. L., "Solving multicommodity transportation problems using a primal partitioning simplex technique," *Naval Research Logistics Quarterly*, 1977, 24, 309-325.

[9] Kobayashi, K. and Yamamoto, H., "A new algorithm in enumerating all minimal paths in a sparse network," *Reliability Engineering and System Safety*, 1999, 65, 11-15.

[10] Lee, D. W. and Yum, B. J., "Determination of minimal upper paths for reliability analysis of planar flow networks," *Reliability Engineering & System Safety*, 1993, 39, 1-10.

[11] Lin, J. S., Jane, C. C. and Yuan, J., "On reliability evaluation of a capacitated-flow network in terms of minimal pathsets," *Networks*, 1995, 25, 131-138.

[12] Lin, Y. K., "On Reliability Evaluation of a Stochastic-Flow Network in Terms of Minimal Cuts," *Journal of Chinese Institute of Industrial Engineers*, 2001, 18, 49-54.

[13] Lin, Y. K., "A Simple Algorithm for Reliability Evaluation of a Stochastic-Flow Network with Node Failure," *Computers and Operations Research*, 2001, 28(13), 1277-1285.

[14] Lin, Y. K. and Yuan, J., "A new algorithm to generate d-minimal paths in a multistate flow network with noninteger arc capacities," *Int. J. Reliability, Quality, and Safety Engineering*, 1998, 5, 269-285.

[15] Rueger, W. J., "Reliability analysis of networks with capacity-constraints and failures at branches and nodes," *IEEE Trans. Reliability*, 1986, 35, 523-528.

[16] Shen, Y., "A new simple algorithm for enumerating all minimal paths and cuts of a graph," *Microelectronics and Reliability*, 1995, 35, 973-976.

[17] Tomlin, J. A., "Minimum-cost multicommodity network flows," *Operations Research*, 1966, 14, 45-51.

[18] Weigel, H. S. and Cremeans, J. E., "The multicommodity network flow model revised to include vehicle per time period and node constraints," *Naval Research Logistics Quarterly*, 1972, 19, 77-89.

[19] Xue, J., "On multistate system analysis," *IEEE Trans. Reliability*, 1985, 34, 329-337.

[20] Soh, S. and Rai, S., "CAREL: Computer aided reliability evaluation for distributed computing networks," *IEEE Trans. Parallel & Distributed Systems* 1991, 2 199-213.

[21] Yeh, W. C., "A revised layered-network algorithm to search for all d-minpaths of a limited-flow acyclic network," *IEEE Transactions on Reliability*, 1998, 47, 436-442.

[22] Yeh, W. C., "A simple algorithm to search for all d-MPs with unreliable nodes," *Reliability Engineering & System Safety*, 2001, 73, 49-54.

Table 1. The data of arcs and nodes for 2-commodity example

Arc	Capacity	Probability	ϖ_i^1	ϖ_i^2	c_i^1 (US dollar)	c_i^2 (US dollar)
a_1	0*	.01	1	2	10	20
	1	.01				
	2	.01				
	3	.02				
	4	.02				
	5	.93				
a_2	0	.01	1	2	20	30
	1	.01				
	2	.02				
	3	.03				
	4	.03				
	5	.90				
a_3	0	.01	1	2	30	40
	1	.01				
	2	.01				
	3	.02				
	4	.02				
	5	.93				
a_4	0	.01	1	2	20	40
	1	.01				
	2	.02				
	3	.03				
	4	.03				
	5	.90				
a_5	0	.01	1	2	10	20
	1	.01				
	2	.01				
	3	.02				
	4	.02				
	5	.93				
a_6	0	.01	1	2	20	30
	1	.01				
	2	.02				
	3	.03				
	4	.03				
	5	.90				
a_7	0	.01	1	2	30	40
	1	.01				
	2	.01				
	3	.02				
	4	.02				
	5	.93				
a_8	0	.01	1	2	20	40
	1	.01				
	2	.02				
	3	.03				
	4	.03				
	5	.90				
$a_9\sim$	0	.01	1	2	20	30
a_{12}	1	.01				
	3	.01				
	5	.01				
	7	.02				
	9	.94				

*Pr{the capacity of a_1 is 0} = 0.01.