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# Analysis on Random Fuzzy Queueing Systems with Finite Capacity

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## Abstract

This paper discusses random fuzzy queueing systems with finite capacity, where the interarrival times and service times are characterized as random fuzzy variables. Fuzzy simulation techniques are designed to estimate the membership degree, the expected value of system length, and the credibility measure that the system length does not exceed a predetermined level. Furthermore, the rough figures of the membership function and credibility distribution function of the system length can be obtained. Finally, an example is given to illustrate the effectiveness of the presented techniques.

**Keywords:** Queueing systems; Fuzzy variable; Random fuzzy variable; Fuzzy simulation; Credibility distribution

## 1 Introduction

The finite-capacity queueing systems have been widely studied by many researchers such as Gouweleeuw and Tijms [4], Bretthauer and Cote [1], Wagner [16], and Gross and Harris [5]. In the traditional queueing theory, the interarrival times and service times are characterized as random variables. It means that statistical data are needed to determine the distribution functions of interarrival times and service times. However, in many practical queueing systems, it is difficult or impossible to obtain the statistical data needed. In fact, in modelling queueing systems, one would describe the interarrival times and service times by linguistic terms such as very fast, fast, moderate or slow rather than by random variables. Thus, fuzzy queueing systems are much more realistic than the traditional queueing systems [2].

To handle fuzzy phenomena, Zadeh [17] initiated the concept of fuzzy set via membership function. In order to measure a fuzzy event, Zadeh [18] proposed the concepts of possibility measure and possibility space, which have been well developed by many researchers such as Nahmias [14], Dubois and Prade [3], Klir [7], Liu [8–11], and Liu and Liu [12]. Nowadays, the issue of applying the possibility theory to queueing systems with finite capacity has attracted a lot of researchers such as Li and Lee [13], Negi and Lee [15], Kao *et al* [6], and Chen [2].

More precisely, Li and Lee [13] proposed a general approach for analyzing fuzzy queueing systems. However, as commented by Negi and Lee [15], this approach is very complicated and generally unsuitable for computational purposes. Negi and Lee [15] proposed an approach by using the  $\alpha$ -level sets and two-variable simulation to analyze fuzzy queueing systems. Kao *et al* [6] adopted parametric programming to construct the membership functions of the performance measures for fuzzy queueing systems, and this approach was successfully applied to four simple fuzzy queueing systems with one or two fuzzy variables. Chen [2] developed a method that could construct the membership functions of the performance measures in finite-capacity queueing systems with fuzzified exponential arrival rate and service rate.

However, the possibility measure is not self-dual while a self-dual measure is very necessary in both theory and practical applications. This property of possibility measure will result in the phenomenon that an event may not occur even if its possibility is 1. This is the greatest shortcoming for possibility measure since it is very hard for a decision maker to make decision based on possibility measure. In order to overcome the drawback of possibility measure, Liu and Liu [12] proposed the concept of credibility measure of a fuzzy event, and introduced the concept of expected value of fuzzy variable based on the credibility measure. Liu [8] presented the concept of credibility distribution of a fuzzy variable. Furthermore, to handle the phenomena combined with randomness and fuzziness, Liu [8, 9] introduced the concept of random fuzzy variables. This paper discusses random fuzzy queueing systems with finite capacity, where the arrival rate and service rate are fuzzy variables with arbitrary membership functions, then presents fuzzy simulation techniques to analyze the queueing system performance.

The rest of this paper is organized as follows. Some concepts and results on fuzzy variables and random fuzzy variables are introduced in Section 2. The queueing systems with finite capacity containing random fuzzy variables are discussed in Section 3. The fuzzy simulation techniques are designed to analyze the queueing system performance in Section 4. Finally, an example is given to illustrate the effectiveness of the presented techniques.

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## 2 Fuzzy variables and random fuzzy variables

In this section, we review some basic concepts and results on fuzzy variables and random fuzzy variables, which are used throughout the paper.

**Definition 1** (Zadeh [18]) Let  $\Theta$  be a nonempty set,  $\mathcal{P}(\Theta)$  the power set of  $\Theta$ , and  $\text{Pos}$  a function from  $\mathcal{P}(\Theta)$  to the set of real numbers. Then  $\text{Pos}$  is called a possibility measure if it satisfies the following three axioms:

- 1)  $\text{Pos}\{\Theta\} = 1$ ;
- 2)  $\text{Pos}\{\emptyset\} = 0$ ;
- 3)  $\text{Pos}\{\bigcup_i A_i\} = \sup_i \text{Pos}\{A_i\}$ , for any set  $A_i$  in  $\mathcal{P}(\Theta)$ . Furthermore, the triplet  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  is called a possibility space.

**Definition 2** (Nahmias [14]) A fuzzy variable is defined as a function from the possibility space  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  to the set of real numbers.

**Definition 3** Let  $\xi$  be a fuzzy variable on the possibility space  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ . Then its membership function is derived from the possibility measure by

$$\mu(x) = \text{Pos}\{\theta \in \Theta \mid \xi(\theta) = x\}, \quad x \in \mathfrak{R}.$$

**Example 1** Let  $\xi = (a, b, c)$  be a triangular fuzzy variable with  $a < b < c$ . Then its membership function is

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}, & \text{if } a \leq x \leq b \\ \frac{x-c}{b-c}, & \text{if } b < x \leq c \\ 0, & \text{otherwise.} \end{cases}$$

**Definition 4** (Zadeh [19]) Let  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  be a possibility space, and  $A$  a set in  $\mathcal{P}(\Theta)$ . Then the necessity measure of  $A$  is defined by

$$\text{Nec}\{A\} = 1 - \text{Pos}\{A^c\},$$

where  $A^c$  is the complementary set of  $A$ .

**Remark 1** (Liu and Liu [12]) It is obvious that neither the possibility measure nor the necessity measure is self-dual, i.e., the possibility (necessity) of a fuzzy event plus the possibility (necessity) of its opposite event is not equal to 1.

Based on possibility measure and necessity measure, Liu and Liu [12] proposed the concept of credibility measure.

**Definition 5** (Liu and Liu [12]) Let  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  be a possibility space, and  $A$  a set in  $\mathcal{P}(\Theta)$ . Then the credibility measure of  $A$  is defined by

$$\text{Cr}\{A\} = \frac{1}{2}(\text{Pos}\{A\} + \text{Nec}\{A\}).$$

**Remark 2** (Liu and Liu [12]) The credibility measure is self-dual, i.e.,

$$\text{Cr}\{A\} + \text{Cr}\{A^c\} = 1.$$

**Example 2** Let  $\xi = (1, 2, 3)$  be a triangular fuzzy variable. It is easy to obtain that  $\text{Pos}\{\xi \leq 2\} = 1$ ,  $\text{Nec}\{\xi \leq 2\} = 0$ , and  $\text{Cr}\{\xi \leq 2\} = 0.5$ , while  $\text{Pos}\{\xi > 2\} = 1$ ,  $\text{Nec}\{\xi > 2\} = 0$ , and  $\text{Cr}\{\xi > 2\} = 0.5$ .

**Proposition 1** (Liu [10]) Suppose that  $(\Theta_i, \mathcal{P}(\Theta_i), \text{Pos}_i)$  are possibility spaces,  $i = 1, 2, \dots, n$ . Let  $\Theta = \Theta_1 \times \Theta_2 \times \dots \times \Theta_n$  and  $\text{Pos} = \text{Pos}_1 \wedge \text{Pos}_2 \wedge \dots \wedge \text{Pos}_n$ . Then the set function  $\text{Pos}$  is a possibility measure, and  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  is a possibility space. Furthermore,  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  is called the product possibility space.

**Definition 6** (Liu and Liu [12]) Let  $\xi$  be a fuzzy variable on the possibility space  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ . Then the expected value  $E[\xi]$  is defined as

$$E[\xi] = \int_0^{+\infty} \text{Cr}\{\xi \geq r\}dr - \int_{-\infty}^0 \text{Cr}\{\xi \leq r\}dr \quad (1)$$

provided that at least one of the two integrals is finite. Especially, if  $\xi$  is a nonnegative fuzzy variable, then

$$E[\xi] = \int_0^{+\infty} \text{Cr}\{\xi \geq r\}dr. \quad (2)$$

**Example 3** (Liu and Liu [12]) The triangular fuzzy variable  $\xi = (a, b, c)$  has an expected value

$$E[\xi] = \frac{1}{4}(a + 2b + c).$$

**Definition 7** (Liu [8]) The credibility distribution  $\Phi: \mathfrak{R} \rightarrow [0, 1]$  of a fuzzy variable  $\xi$  is defined by

$$\Phi(x) = \text{Cr}\{\theta \in \Theta \mid \xi(\theta) \leq x\}.$$

That is,  $\Phi(x)$  is the credibility that the fuzzy variable  $\xi$  takes a value less than or equal to  $x$ .

**Definition 8** (Liu [8]) A random fuzzy variable is defined as a function from the possibility space  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  to the set  $\mathcal{F}$  of random variables.

**Example 4** In a queueing system, let  $\xi$  be the interarrival time of customer arrivals. Usually, the probability distribution of  $\xi$  is assumed to be known completely except for the values of one or more parameters. For example, the interarrival time  $\xi$  might be known as an exponentially distributed random variable with an unknown mean  $\lambda$ , whose probability density function is

$$\phi(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } 0 \leq x < +\infty \\ 0, & \text{otherwise.} \end{cases}$$

If  $\lambda$  is characterized as a fuzzy variable defined on the possibility space  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ , then  $\xi$  is just a random fuzzy variable denoted by

$$\xi(\lambda(\theta)) \sim \mathcal{E}\mathcal{X}\mathcal{P}(\lambda(\theta)).$$

**Remark 3** (Liu [8]) *Roughly speaking, if  $\Theta$  consists of one single element only, then the random fuzzy variable degenerates to a random variable. If  $\mathcal{F}$  is a collection of real numbers (rather than random variables), then the random fuzzy variable degenerates to a fuzzy variable.*

### 3 Random fuzzy queueing systems with finite capacity

Consider a stochastic queueing system with finite capacity  $M/M/1/FCFS/K/\infty$ , in which there is one server. Customers arrive at the single server as a Poisson process with an arrival rate  $\lambda$ , and all service times are independent and identically distributed exponential random variables with service rate  $v$ . The queue discipline is first come, first served (*FCFS*). There is a limit  $K$  placed on the number allowed in the system at any time, and the size of source population is infinite.

In statistics, the values of  $\lambda$  and  $v$  are estimated by the interval estimate or point estimate method. However, in many practical queueing systems, the data needed in these methods can not be obtained. Therefore, it is very difficult or impossible to obtain the exact values of  $\lambda$  and  $v$ . In such a case, it is reasonable that  $\lambda$  and  $v$  are characterized as fuzzy variables according to experts' experiences. By Definition 8, the interarrival times and service times are all random fuzzy variables. Such queueing systems are called *random fuzzy queueing systems with finite capacity*, which is denoted by  $RF/RF/1/FCFS/K/\infty$ , where *RF* denotes that interarrival times and service times are random fuzzy variables. The following example explains the concept of random fuzzy queueing systems with finite capacity.

**Example 5** *There is a new petrol station with a single pump and finite space for no more than 10 cars (9 waiting, 1 being served), i.e.,  $K = 10$ . Cars arrive at the petrol station according to a Poisson process, and service time has an exponential distribution. Due to lack of historical data on arrivals and service time, the arrival rate  $\lambda$  and service rate  $v$  can not be determined exactly. In such a case, by some relevant experts' experiences,  $\lambda$  and  $v$  are characterized as the fuzzy variables with the membership functions  $\mu_1(x) = [1 - (x - 1)^2] \vee 0$  and  $\mu_2(x) = e^{-(x-2)^2}$ , respectively. Then the queueing system is just a random fuzzy queueing system with finite capacity.*

**Remark 4** *If  $\lambda$  and  $v$  degenerate to crisp numbers, then the random fuzzy queueing system  $RF/RF/1/FCFS/K/\infty$  is just the traditional queueing system  $M/M/1/FCFS/K/\infty$ .*

**Remark 5** *If we ignore the fuzziness of the arrival rate and service rate when they can not be obtained exactly, it is reasonable for us to employ their expected values to*

*denote them. Such a case is very similar to the case of using a deterministic value instead of a random variable in stochastic environments. Therefore, it is much more realistic that the arrival rate and service rate are characterized as fuzzy variables when they can not be obtained exactly.*

### 4 Analysis on random fuzzy queueing systems with finite capacity

There are many indices to measure the queueing system performance, such as system length, queue length, waiting time, etc. Without loss of generality, we take system length as the performance measure of queueing systems. In stochastic environments, the expected value of system length of the queueing system  $M/M/1/FCFS/K/\infty$  is

$$L(\lambda, v) = \frac{\lambda [K\lambda^{K+1} - v(K+1)\lambda^K + v^{K+1}]}{(\lambda - v)(\lambda^{K+1} - v^{K+1})}, \quad (3)$$

where  $\lambda < v$  (see [5]).

In the random fuzzy queueing system  $RF/RF/1/FCFS/K/\infty$ ,  $\lambda$  and  $v$  are characterized as fuzzy variables on the possibility spaces  $(\Theta_1, \mathcal{P}(\Theta_1), \text{Pos}_1)$  and  $(\Theta_2, \mathcal{P}(\Theta_2), \text{Pos}_2)$ , respectively. Therefore,  $L(\lambda, v)$  is a fuzzy variable defined on the product possibility space  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ , where  $\Theta = \Theta_1 \times \Theta_2$  and  $\text{Pos} = \text{Pos}_1 \wedge \text{Pos}_2$ . Apparently,  $L(\lambda, v)$  is a nonnegative fuzzy variable. By Definition 6, the expected value of  $L(\lambda, v)$  can be written as

$$E[L(\lambda, v)] = \int_0^{+\infty} \text{Cr}\{L(\lambda, v) \geq r\} dr. \quad (4)$$

The credibility distribution  $\Phi: \mathcal{R} \rightarrow [0, 1]$  of the system length  $L(\lambda, v)$ , by Definition 7, is

$$\Phi(x) = \text{Cr}\{L(\lambda, v) \leq x\}.$$

That is,  $\Phi(x)$  is the credibility measure that the system length  $L(\lambda, v)$  does not exceed  $x$ .

In many real queueing systems, it is very hard to compute the membership degree  $\mu_L(x)$ , the expected value  $E[L(\lambda, v)]$  and the credibility measure  $\text{Cr}\{L(\lambda, v) \leq x\}$ . Therefore, it is necessary to design fuzzy simulation techniques to estimate these values. For more details on fuzzy simulation techniques, see [12].

#### 4.1 Fuzzy simulation for the membership degree $\mu_L(x)$

The fuzzy simulation technique for estimating the membership degree  $\mu_L(x)$  is described as follows.

- 1) Set  $k = 1$ .
- 2) Let  $x_k = \frac{kK}{M}$ , and  $\mu_k = 0$ , where  $M$  is a sufficiently large integer.

- 3) Uniformly generate a number  $\lambda_1$  from the support of  $\lambda$ , then calculate the membership degree of  $\lambda_1$  according to the membership function of  $\lambda$ , written as  $v_1$ .
- 4) Replace  $L(\lambda, v)$ ,  $\lambda$  and  $v$  in Eq. (3) with  $x_k, \lambda_1$  and  $y$ , respectively, then Eq. (3) becomes the following polynomial equation

$$\begin{aligned}
 &x_k y^{K+2} - (\lambda_1 + x_k \lambda_1) y^{K+1} \\
 &+ (K + 1 - x_k) \lambda_1^{K+1} y \\
 &+ (x_k - K) \lambda_1^{K+2} = 0.
 \end{aligned}
 \tag{5}$$

Solve Eq. (5) by the dichotomy method, and calculate the membership degree of the solution according to the membership function of  $v$ , written as  $v_2$ .

- 5) Set  $temp = v_1 \wedge v_2$ .
- 6) If  $\mu_k < temp$ , then set  $\mu_k = temp$ .
- 7) Repeat Steps 3 to 6  $N$  times, where  $N$  is a sufficiently large integer.
- 8) Set  $k = k + 1$ . If  $k \leq M$ , return to Step 2. Otherwise, output  $(x_i, \mu_i)$ , where  $i = 1, 2, \dots, M$ .

**Example 6** Let us continue to discuss the random fuzzy queueing system in Example 5. It is easy to obtain that

$$L(\lambda, v) = \frac{10\lambda^{12} - 11v\lambda^{11} + \lambda v^{11}}{(\lambda - v)(\lambda^{11} - v^{11})},$$

where  $\lambda$  and  $v$  are the fuzzy variables with the membership functions  $\mu_1(x) = [1 - (x - 1)^2] \vee 0$  and  $\mu_2(x) = e^{-(x-2)^2}$ , respectively. The rough figure of the membership function of  $L(\lambda, v)$  is shown in Figure 1.

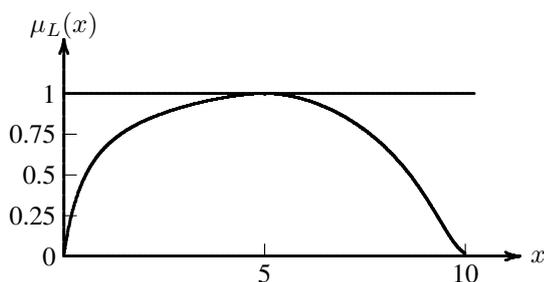


Figure 1: Rough Shape of  $\mu_L(x)$  in Example 6

**4.2 Fuzzy simulation for the expected value**

$$E[L(\lambda, v)]$$

The fuzzy simulation technique for estimating  $E[L(\lambda, v)]$  is summarized as follows.

- 1) Set  $e = 0$ .
- 2) Uniformly generate  $(\theta_{k1}, \theta_{k2})$  from  $\Theta_1 \times \Theta_2$  such that  $p_k \geq \varepsilon$  for  $k = 1, 2, \dots, N$ , where  $p_k = \text{Pos}\{\theta_{k1}\} \wedge \text{Pos}\{\theta_{k2}\}$ ,  $\varepsilon$  is a sufficiently small positive number, and  $N$  is a sufficiently large integer.
- 3) Set
 
$$\begin{aligned}
 a &= L(\lambda(\theta_{11}), v(\theta_{12})) \wedge \dots \wedge L(\lambda(\theta_{N1}), v(\theta_{N2})), \\
 b &= L(\lambda(\theta_{11}), v(\theta_{12})) \vee \dots \vee L(\lambda(\theta_{N1}), v(\theta_{N2})).
 \end{aligned}$$
- 4) Uniformly generate  $r$  from  $[a, b]$ . Set  $e = e + \text{Cr}\{L(\lambda, v) \geq r\}$ , where
 
$$\begin{aligned}
 &\text{Cr}\{L(\lambda, v) \geq r\} \\
 &= \frac{1}{2} \left( \max_{1 \leq k \leq N} \{p_k \mid L(\lambda(\theta_{k1}), v(\theta_{k2})) \geq r\} \right. \\
 &\quad \left. + \min_{1 \leq k \leq N} \{1 - p_k \mid L(\lambda(\theta_{k1}), v(\theta_{k2})) < r\} \right).
 \end{aligned}$$
- 5) Repeat the fourth step for  $N$  times.
- 6) Compute  $E = a \vee 0 + b \wedge 0 + e \cdot (b - a)/N$ , then output  $E$ .

**Example 7** Let us continue to discuss the random fuzzy queueing system in Example 5. After 5000 cycles, the estimate of  $E[L(\lambda, v)]$  is reported by fuzzy simulation technique as 3.5201. The variations of the estimate with different numbers of cycles are shown in Figure 2, where the straight line represents the real value of  $E[L(\lambda, v)]$ , and the curve represents the variations of the estimates with different numbers of cycles. It is easy to see that the estimates approach the real value when the number of cycles is larger than 2000.

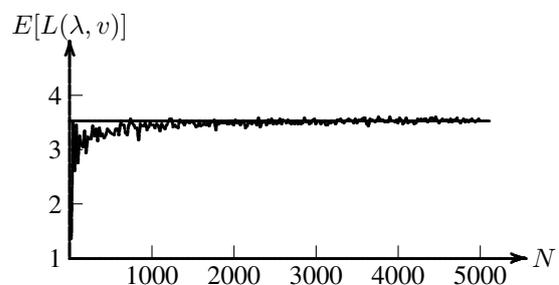


Figure 2: Variations of the Estimates of  $E[L(\lambda, v)]$  in Example 7

### 4.3 Fuzzy simulation for the credibility measure

$$\text{Cr}\{L(\lambda, v) \leq x\}$$

The fuzzy simulation technique for estimating  $\text{Cr}\{L(\lambda, v) \leq x\}$  is summarized as follows.

1) Uniformly generate  $(\theta_{k1}, \theta_{k2})$  from  $\Theta_1 \times \Theta_2$  such that  $p_k \geq \varepsilon$  for  $k = 1, 2, \dots, N$ , where  $p_k = \text{Pos}\{\theta_{k1}\} \wedge \text{Pos}\{\theta_{k2}\}$ ,  $\varepsilon$  is a sufficiently small positive number, and  $N$  is a sufficiently large integer.

2) Compute

$$M = \frac{1}{2} \left( \max_{1 \leq k \leq N} \{p_k \mid L(\lambda(\theta_{k1}), v(\theta_{k2})) \leq x\} + \min_{1 \leq k \leq N} \{1 - p_k \mid L(\lambda(\theta_{k1}), v(\theta_{k2})) > x\} \right),$$

then output  $M$ .

**Example 8** Let us continue to discuss the random fuzzy queueing system in Example 5. We use the fuzzy simulation technique proposed to estimate the credibility measure  $\text{Cr}\{L(\lambda, v) \leq 5\}$ . After 5000 cycles, the estimate of  $\text{Cr}\{L(\lambda, v) \leq 5\}$  is 0.6305. The variations of the estimates with different numbers of cycles are shown in Figure 3, where the straight line represents the real value of  $\text{Cr}\{L(\lambda, v) \leq 5\}$ , and the curve represents the variations of the estimates. It is easy to see that the estimates approach the real value when the number of cycles is larger than 2000.

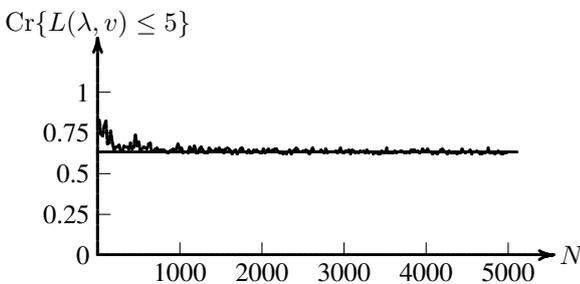


Figure 3: Variations of the Estimates of  $\text{Cr}\{L(\lambda, v) \leq 5\}$  in Example 8

By using the fuzzy simulation technique proposed to estimate  $\text{Cr}\{L(\lambda, v) \leq x\}$ , where  $x \in (0, 10)$ , the rough figure of the credibility distribution function  $\Phi(x)$  of the system length  $L(\lambda, v)$  can be seen, which is depicted in Figure 4 (5000 cycles in fuzzy simulation for every credibility measure  $\text{Cr}\{L(\lambda, v) \leq x\}$ ).

### 5 Numerical example

**Example 9** Consider a random fuzzy queueing system  $RF/RF/1/FCFS/K/\infty$ . Let the arrival rate be the fuzzy

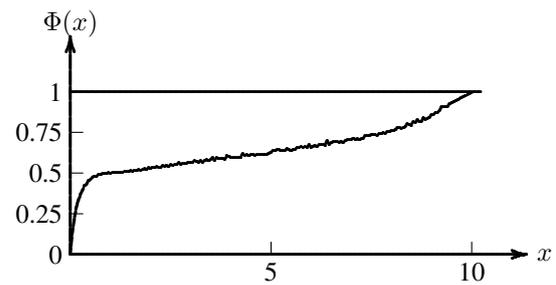


Figure 4: The Credibility Distribution Function  $\Phi(x)$  in Example 8

variable  $\lambda$  with the membership function

$$\mu_1(x) = \begin{cases} 1 - (x - 1)^2, & \text{if } 0 \leq x \leq 2 \\ 1 - (x - 3)^2, & \text{if } 2 < x \leq 4 \\ 0, & \text{otherwise,} \end{cases}$$

the service rate be the triangular fuzzy variable  $v = (3, 5, 7)$ , and the system capacity  $K = 100$ .

It is easy to obtain that

$$L(\lambda, v) = \frac{100\lambda^{102} - 101v\lambda^{101} + \lambda v^{101}}{(\lambda - v)(\lambda^{101} - v^{101})}.$$

Apparently, since the membership function of arrival rate  $\lambda$  is multimodal, the methods presented in [2] and [6] can not apply to this queueing system. The fuzzy simulation techniques proposed in Section 4 can be used to estimate the membership degree  $\mu_L(x)$ , the expected value  $E[L(\lambda, v)]$ , and the credibility measure  $\text{Cr}\{L(\lambda, v) \leq x\}$ .

The rough figure of the membership function of  $L(\lambda, v)$  is shown in Figure 5.

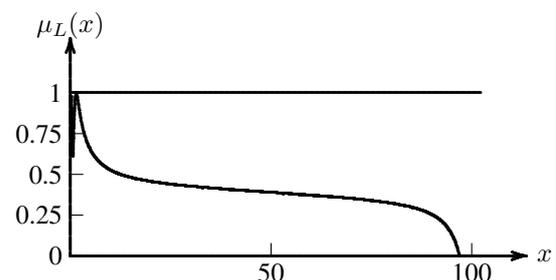


Figure 5: Rough Figure of  $\mu_L(x)$  in Example 9

After 5000 cycles, the estimate of  $E[L(\lambda, v)]$  is reported by fuzzy simulation technique as 49.8414. The variations of the estimate of  $E[L(\lambda, v)]$  with different numbers of cycles are shown in Figure 6, where the straight line represents the real value of  $E[L(\lambda, v)]$ , and the curve represents the variations of the estimate of

$E[L(\lambda, v)]$ . It is easy to see that the estimates approach the real value when the number of cycles is larger than 2000.

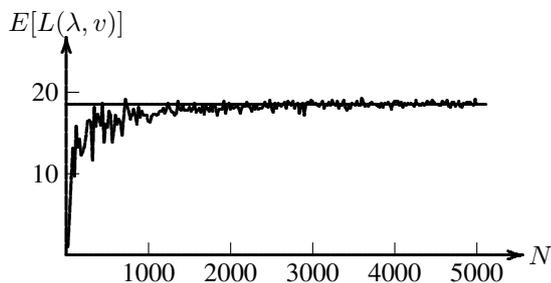


Figure 6: Variations of the Estimates of  $E[L(\lambda, v)]$  in Example 9

After 5000 cycles, the estimate of  $\text{Cr}\{L(\lambda, v) \leq 50\}$  is 0.5001. The variations of the estimate with different numbers of cycles are shown in Figure 7, where the straight line represents the real value of the credibility measure  $\text{Cr}\{L(\lambda, v) \leq 50\}$ , and the curve represents the variations of the estimates obtained by the fuzzy simulation technique. It is easy to see that the estimates approach the real value when the number of cycles is larger than 2000.

By using fuzzy simulation technique to estimate  $\text{Cr}\{L(\lambda, v) \leq x\}$ , where  $x \in (0, 100)$ , the rough figure of the credibility distribution function  $\Phi(x)$  of the system length  $L(\lambda, v)$  can be seen, which is depicted in Figure 8 (5000 cycles in fuzzy simulation for every credibility measure  $\text{Cr}\{L(\lambda, v) \leq x\}$ ).

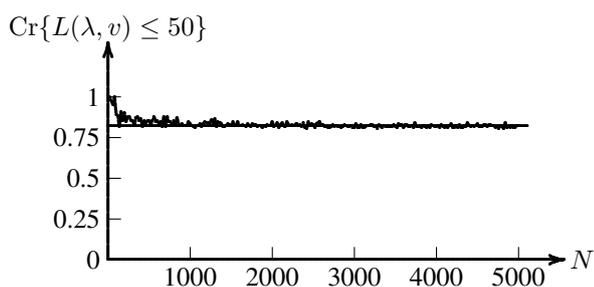


Figure 7: Variations of the Estimates of  $\text{Cr}\{L(\lambda, v) \leq 50\}$  in Example 9

## 6 Conclusion

This paper discussed random fuzzy queueing systems with finite capacity. Fuzzy simulation techniques were designed to estimate the membership degree, the expected value of the system length, and the credibility measure

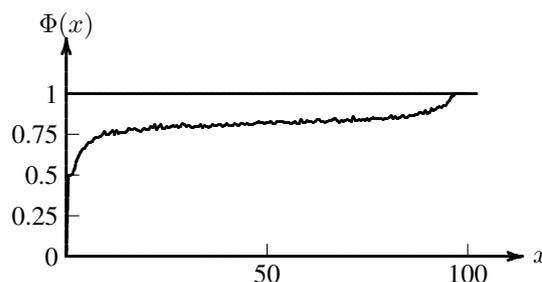


Figure 8: The Credibility Distribution Function  $\Phi(x)$  in Example 9

that the system length was less than or equal to a predetermined level. Finally, an example was given to illustrate the effectiveness of the proposed techniques. The methods can be easily employed to analyze the other performance measures such as waiting time, queue length, and so on. Similarly, the proposed methods can be employed to analyze the other random fuzzy queueing systems.

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