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Model and Algorithm for Sea Cargo Mix Problem

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Abstract

In this paper, we study a sea cargo mix problem that occurs in the context of container shipment planning and revenue analysis in ocean carriers. We describe structure and characteristics of the cargo mix problem, and formulate as a Multi-Dimension Multiple Knapsack Problem (MDMKP). In particular, the MDMKP is an optimization model that maximizes the total profit in several periods, subject to the limited shipping capacity and the limited number of empty containers in the origin port, etc. Algorithm is proposed to obtain the near optimal solution for the problem. Numerical experiments demonstrate the efficiency of the algorithm.


Keywords: sea cargo mix, container shipping industry, MDMKP.

1. Introduction and Problem Description

In the recent economic environment, it is often the case that profits can only be maintained or increased by improving efficiency and cutting costs. This is particularly proverbial in the shipping industry, where it has been seen that the competition is very intensive among container carriers, thus alliances and partnerships are resulting for cost effective services in recent years. In this scenario, effective planning methods are important not only for strategic but also operating tasks, covering their entire transportation business (see [2][4][13][18][19]). Sea cargo mix planning is an important part of the operations of container shipping lines. This paper addresses the problem of sea cargo mix planning for containerized cargoes, to achieve the most profitable, time definite and cost-effective services in a competitive ocean container shipping market.

Shipping industry has started utilizing internet B2B exchanges, a marketplace where many buyer and sellers post buy and sell bids to make trades at dynamically determined market prices. Both shipping lines and shippers are moving towards e-logistics in all dealings. All documents can be received electronically, captured and processed directly into the e-logistics system seamlessly. Online booking request is made available 24 hours a day. Within the e-booking, shipper has to select the cargo from the commodity list provided, specify the size and type of container, and enter port of receipt or delivery on the latest departure and arrival date. For examples, Maersk-Sealand, Hanjin, APL and Hyundai are among the carriers offering such service (see their websites for details). Maersk-Sealand uses digital certificate to prove one’s identity and enable access to protected area to make booking online faster and safer. The shipper suggests the most appropriate vessel sailing from global schedule system displayed on screen and finally save all information. The shipper’s request will then be transmitted to the local Maersk-Sealand office, where confirmation of requirement together with the booking reference number will be sent back to the shipper.

For many containerized cargo carriers, the majority of their bookings are under long-term contracts with shippers. Such long-term agreements help carriers support requests for the large amounts of capital required to run their businesses. These long-term contracts are typically for one to three years. For some carriers, annual contracts all date to the same time of year and may cause a flurry of activity to re-negotiate terms and solidify the next year’s business. Regardless of timing, these contracts form a significant part of the revenue stream for a container carrier and negotiating the right prices and terms can be a major contributor to overall profitability. In practice, booking requests are often made less than 14 days before the freight becomes available for shipment, in which case
the carrier may delay moving the freight if insufficient
capacity is available. For example, the shipper may
request a booking on a voyage that departs 9 days into
the future, but if that voyage is already too fully booked, then
the carrier may offer to transport the freight on a voyage
that departs a week later, thus 16 days into the future. The
shipper may then accept the booking offer, or decline the
booking offer and request a booking with another carrier.
Also, often shippers’ cancel previously made bookings. It
also happens that freight scheduled to depart on a voyage
does not arrive at the port in time for the voyage.
Approximately 30% of booked freight ends up not
transported as originally booked. The booking control
decision maker at the carrier has to decide, whenever a
shipper requests a booking, what booking to offer the
shipper. For example, the carrier may offer the shipper a
booking on the voyage 9 days into the future, or on a
voyage scheduled to depart a week later. The decision
maker should take into account the revenue specified in
the contract with the shipper, the probability of the shipper
decining the booking offer, as well as uncertain future
booking requests, cancellations and no-shows.

In practical situation, less-than-container-load (LCL)
cargo orders come through telephone, email or website
from established customers (these usually have their own
shipping department), freight forwarders who act for
customers or some new customers. Full-container-load
(FCL) cargo orders are usually from big firms such as
many electronics manufacturing companies. These big
customers usually let the carrier know approximately the
number of containers they will ship over the next three or
four weeks (need to check if this practice still is in place).
Agreement is loose and not legally binding.
The carrier will have the following priorities:
1) High paying FCL cargo (e.g., electronics)
2) Medium paying
3) Low paying (e.g., gypsum board)
If times are good and demand exceeds container space,
the shipping company wills rollover those low paying
cargo (i.e. not ship on schedule but postpone shipment to a
later date). Under the contract, customers cannot sue the
shipping company. If times are bad, the shipping company
will look for low-paying cargo to fill the vessel rather than
ship empties.

FCL cargo usually has no weight/volume
specification because customers always pack to the fullest.
Because maximum weight of 20-foot and 40-foot
container are 18 MT and 40MT, respectively, so if cargo is
volume cargo, you can pick in something like 40 cu
meters and yet the cargo weighs less 20 tons. For weight
cargo (e.g., steel), 20 tons is easily reached when the
container is 1/3 full. You just cannot pack in anymore
/legal requirement). Most LCL cargo tends to be volume
cargo and the shipping company prefers that because they
charge by weight or volume. The shipping company takes
all orders but those they can’t ship, they will inform the
customers after the ship sails.

Sea cargo mix problem arises when a liner operator
wants to select cargoes in order to maximize his revenue
for some particular trips. Optimal cargo mix technique
needs data from the online booking system and container
tracking system to provide current status and location of
container inventory as well as details of all costs
associated with shipments. Historical data is gathered or
built up over time to permit forecasting of container flows
and inventory levels. From demands, bookings, available
capacity and average contributions, it mathematically
determines an optimal target contribution to determine
whether to accept or reject a proposed booking, or to
suggest a surcharge or alternate product or service at a
higher/lower price. The result would be used to adjust
contribution calculations to favor certain types of freight
on certain routes and discourage others.

Each type of cargo has different volume/ weight
characteristic, different freight rate and generates different
revenue. For example, the tradeoff in FCL and LCL cargo
mix is important operational issue for liner operators.
Shipping companies offer customers priority, time-bound
forwarding for LCL and FCL shipments with fast transits,
reliable schedules and competitive price. A FCL of
electronics pays higher than a FCL of gypsum board.
Furthermore, if the inbound vessel is full, and if this is not
balanced by a full outbound, then there is the need to ship
empties necessitated by the need to re-position the empties
for use by the destination port. This too has to be taken
into account in the optimal revenue analysis. The
repositioning of empty containers is one of the single
largest expenses for most container carriers. Regarding
container demand and supply it minimizes empty
repositioning costs and maximize vessels utilization and it
lead to a pro-active pricing mechanism for improved
revenues and reduced costs. In order to optimally use
vessels optimally matched where possible according to
customer requirements to minimize empty repositioning
costs. This takes into account excess empties and will
result in optimal utilization of vessels while total
transportation costs (for both full and empty containers)
are reduced and trade imbalances will be compensated.
Many different models have been proposed in the
literature to obtain an optimal container repositioning
planning. However, most of them did not consider it with
transportation of laden containers and revenue
management (see [1][3][9]).

In our work, we abstract the FCL and LCL cargo and
the empty container as different type of cargo. Further
more, in the model proposed in our paper, we consider
more detailed information concerned with each type of
cargo. Such information include the weight, volume,
shipment price and shipment due date requirement, etc.
To our knowledge, up until now, there is no any
description and formulation for the multi-period sea cargo
mix problem in the literature. Single period sea cargo mix
problem is proposed by [5] and it is formulated as linear
programming model by [10], when all cargoes are
separable for shipment. Literature on related mathematical
programming models and algorithms see related Sections.
The rest of the paper is organized as follows. In
Section 2, the multi-period sea cargo mix problem is
formulated as the Multi-Dimension Multiple Knapsack
Problem (MDMKP). Section 3 presents algorithm for the
model. The heuristic algorithm with effective gradient and its modified algorithm are given in this section. We demonstrate the algorithm with effective gradient by a simple example in this section, followed in Section 4 by numerical experiments of the algorithm with a wide range of problem instances. Section 5 concludes the paper.

2. Mathematical Formulation

There are several technical constraints relative to how a ship should be loaded. The obvious constraints are that cargo weight and volume should not exceed the ship’s capacity. The operational objective is to select the loading parameters in order to maximize some profit criterion subject to the mathematical expression of the weight/volume constraints. If the freight rates are known and the operator can refuse or delay to carry cargoes, then the multi-period sea cargo mix problem can be formulated as MDMKP. The objective of the problem is to maximize a weighted sum of the quantities transported. The relative coefficients for the cargoes might reflect their unit profits.

In this section, we develop a mathematical model for the problem under consideration. For the ease of our discussion, we consider that one time period represents one day or one week and we use the term $t$ to represent the $t$th time period.

In order to present the mathematical formulation of the model for a given planning time horizon, we introduce some notation:

**Index Sets**
- $\bar{T}$ set of time periods $\{1, 2, \cdots, t, \cdots, T\}$
- $J$ set of ports $\{1, 2, \cdots, j, \cdots, J\}$
- $\bar{K}_t$ set of all cargoes received in period $t$, i.e., $\bar{K}_t = \{1, 2, \cdots, k, \cdots, K_t\}$

**Parameters**
- $r_{tkd_{j,t}}$ per volume revenue of cargo $k$ which is received in period $t$ and shipped to port $j$ in period $d$
- $c_{tkd_{j,t}}$ per volume cost of cargo $k$ which is received in period $t$ and shipped to port $j$ in period $d$
- $h_{tk}$ per volume inventory cost of cargo $k$ which is received in period $t$ and delayed to carry in next period
- $\tau_k$ due date of cargo $k$
- $E_t$ total volume of available empty containers at origin port in period $t$
- $V_{j,t}$ total available volume capacity of shipment to port $j$ in period $t$
- $W_{j,t}$ maximum allowable weight capacity of shipment to port $j$ in period $t$
- $v_{tkd_{j,t}}$ volume of cargo $k$ received in period $t$ ready for shipment to port $j$ before its due date $\tau_k$
- $w_{tkd_{j,t}}$ weight of cargo $k$ received in period $t$ ready for shipment to port $j$ before its due date $\tau_k$

**Decision Variables**
- $x_{tkd_{j,t}}$ binary variable, i.e., $x_{tkd_{j,t}} = 1$ if cargo $k$ is received in period $t$ and is ready for shipment to port $j$ in period $d$ before its due date $\tau_k$, 0, otherwise.

In this paper, we assume that there are one origin port and $J$ destination ports. All cargoes are received at origin port in a certain period of time horizon $T$. The shipping company decide which cargoes carry in the current period and which cargoes are delay to carry in the other period within their due date and which cargoes are refused to carry in the time horizon $\bar{T}$.

Let $r_{tkd_{j,t}} = r_{tkd_{j,t}} - c_{tkd_{j,t}} - (d - t)h_{tk}$, $\forall t \in \bar{T}$,

$$k \in \bar{K}_t, \quad j \in J, \quad t \leq d \leq \tau_k,$$

our multi-period sea cargo mix problem can then be formulated as follows:

(M1): Maximize $z = \sum_{t=1}^{\bar{T}} \sum_{k \in K_t} \sum_{d=1}^{T} v_{tkd_{j,t}} x_{tkd_{j,t}}$ \hspace{1cm} (1)

subject to

$$\sum_{t=1}^{\bar{T}} \sum_{k \in K_t} \sum_{d=1}^{T} v_{tkd_{j,t}} x_{tkd_{j,t}} \leq E_t \quad \forall d \in \bar{T}$$ \hspace{1cm} (2)

$$\sum_{t=1}^{\bar{T}} \sum_{k \in K_t} \sum_{j \in J} \sum_{d=1}^{T} v_{tkd_{j,t}} x_{tkd_{j,t}} \leq V_{j,t} \quad \forall d \in \bar{T}, \quad j \in J$$ \hspace{1cm} (3)

$$\sum_{t=1}^{\bar{T}} \sum_{k \in K_t} \sum_{d=1}^{T} w_{tkd_{j,t}} x_{tkd_{j,t}} \leq W_{j,t} \quad \forall d \in \bar{T}, \quad j \in J$$ \hspace{1cm} (4)

$$\sum_{d=1}^{T} x_{tkd_{j,t}} \leq 1 \quad \forall t \in \bar{T}, \quad k \in \bar{K}_t$$ \hspace{1cm} (5)

$$x_{tkd_{j,t}} \in \{0, 1\} \quad \forall t \in \bar{T}, k \in \bar{K}_t, \quad t \leq d \leq \tau_k$$ \hspace{1cm} (6)

The objective function (1) maximizes the total profit in the time horizon $\bar{T}$. Constraints (2) ensure that the demand for empty containers at origin port is less than the number of available empty containers at origin port in each period. Constraints (3) ensure that the total volume of cargoes which will be carried to port $j$ in period $d$ is less than the total available volume capacity of shipment to port $j$ in period $d$. Constraints (4) ensure that the total weight of cargoes which will be carried to port $j$ in period $d$ is less than the total available weight capacity of shipment to port $j$ in period $d$. Constraints (5)
impose that each cargo may be carried in a certain period before their due date or be refused to carry in the time horizon $T$. Constraints (6) assure that each cargo can be either accepted at its total quantity or be turned down.

We may notice that the inequality sign in (5) is replaced by equality, if all received cargoes have to be carried in the time horizon $T$. In this case, the MDMKP (M1) may have not feasible solution, because the total available capacity of shipment may be less than the amount of demands in the time horizon $T$. On the other hand, MDMKP (M1) always has the optimal solution, because it has a trivial feasible solution $x_{tkdj} = 0$, for all $t \in \tilde{T}$, $k \in \tilde{K}$, $d \in \{1, t + 1, \cdots, \tau_k\}$, and its LP relaxation optimal value is an upper bound of its optimal value.

3. Heuristic Algorithm for MDMKP

The 0-1 or binary integer programming is an integer programming whose all decision variables are 0-1 values. An important class of binary integer programming problems is the Knapsack Problem (KP). The Single Knapsack Problem (SKP) is one kind of KP where only one knapsack needs to be filled and each item may be chosen at most once; if the items should be chosen from disjoint classes and exactly one item from each class, we get the Multiple-Choice Knapsack Problem (MCKP); The Multiple Knapsack Problem (MKP) is also a kind of KP where several knapsacks are to be packed simultaneously. The more general kind of KP is the Multi-Dimensional Knapsack Problem (MDKP), also known as Multi-Constrained Knapsack Problem, where there are more than one limited resources, i.e. the resources are multi-dimensional for the knapsack. And the MDMKP is the combination of MDKP and MKP. All Knapsack problems belong to the NP-hard family (see [6]), therefore it is impossible that polynomial time algorithms can be devised for them.

There are different algorithms for solving variants of KPs (see [14][15][16][17][11]). Generally, there are two methods of finding their solutions: one is a method for finding exact solutions and the other is heuristic. Finding exact solutions is NP hard. Using the branch and bound with linear programming technique, Kolesar ([9]), Shih ([20]), Nauss ([12]) and Khan ([7]) presented exact algorithms for KP, MDKP, MCKP and MDKP respectively. A greedy approach has been proposed by Khan et al ([7]), Martello and Toh ([11]) and Toyoda ([21]) to find near optimal solutions of KPs.

3.1 Heuristic Algorithm with Effective Gradient

MDMKP (M1) is different with all kinds of KPs mentioned in the literature and it needs to develop efficient algorithms for it. In this section, we give an efficient heuristic algorithm with the effective gradient for it.

Let $\overline{v}_{tkdj} = v_{tkdj} / V_{dj}$, $\overline{w}_{tkdj} = w_{tkdj} / W_{dj}$, then we can rewritten (M1) as follows

\[
\text{(M2): } \text{Maximize } z = \sum_{t=1}^{T} \sum_{k \in \tilde{K}} \sum_{d \in \{1, t + 1, \cdots, \tau_k\}} v_{tkdj} x_{tkdj} \tag{7}
\]

subject to

\[
\sum_{k \in \tilde{K}} \sum_{d \in \{1, t + 1, \cdots, \tau_k\}} v_{tkdj} x_{tkdj} \leq E_d \quad \forall d \in \tilde{T} \tag{8}
\]

\[
\sum_{k \in \tilde{K}} \sum_{d \in \{1, t + 1, \cdots, \tau_k\}} \overline{v}_{tkdj} x_{tkdj} \leq 1 \quad \forall d \in \tilde{T}, \quad j \in \tilde{J} \tag{9}
\]

\[
\sum_{l=1}^{d} \sum_{k \in \tilde{K}} \sum_{\tau_l \geq d} \overline{v}_{lkdj} x_{lkdj} \leq 1 \quad \forall d \in \tilde{T}, \quad j \in \tilde{J} \tag{10}
\]

\[
\sum_{d=1}^{T} x_{tkdj} \leq 1 \quad \forall t \in \tilde{T}, \quad k \in \tilde{K} \tag{11}
\]

\[
x_{tkdj} \in \{0, 1\} \quad \forall t \in \tilde{T}, k \in \tilde{K}, t \leq d \leq \tau_k \tag{12}
\]

When each of the candidate items (i.e. cargoes) involves only one limited resource, it is easy to pick up more profitable items. When there exists more than one limited resource in the candidate items, it is difficult to evaluate the amount of limited resources used. Toyoda’s heuristic for MDKP starts with no items, and adds one item at a time iteratively as long as the solution is feasible. At any iteration, the heuristic picks the item which provides the maximum value per unit of aggregate resource or the effective gradient among the not-yet-picked items. Computational experiments show that this heuristic provides very good near optimal solutions to the MDKP using very short computation time, and it can be applied to large problem instances, such as problems with more than a thousand variables (see [21]).

In this section, we propose a heuristic algorithm HAEG to provide a near-optimal solution to MDMKP (M2) by means of concepts such as penalty vector and effective gradient in [21].

The heuristic algorithm for MDMKP can be presented as follows.

Algorithm HAEG

Step 1: Initialization.

Step 1.1: Let $K_U \leftarrow \emptyset$, where $K_U$ is the set of accepted items.

Step 1.2: Assign all items to $K_D = K - K_U$, where $K_D$ is the set of items not in $K_U$ and

\[
K = \{k : k \in \tilde{K}, t \in \tilde{T}\}.
\]

Step 1.3: Let $A_U^d \leftarrow (0, 0)$, where $A_U^d$ is the total quantity vector of accepted items shipping to
Let the objective value be zero, i.e., $z \leftarrow 0$.

Step 1.5: Let $x_{k \times 1\times d_{j\times 1}} \leftarrow 0$, for
\[ \forall t \in \bar{T}, k \in \bar{K}, d \in \{t, t+1, \cdots, \tau_k\} \].

Step 2: Let $K_C \leftarrow \{k: k \in K_D, \exists d \leq \tau_k \}
\text{s.t.} (\bar{v}_{k \times 1\times d_{j\times 1}}, \bar{w}_{k \times 1\times d_{j\times 1}}) \leq (1, 1) - A_{d_{j\times 1}}^{d_{j\times 1}},
\quad v_{k \times 1\times d_{j\times 1}} \leq E_d - \sum_{k \in K_C} v_{k \times 1\times d_{j\times 1}} \}
\text{where } K_C \text{ is the set of candidate items.}

Step 3: Check $K_C$. If $K_C$ is empty, the procedure terminates. Otherwise, proceed to the next step.

Step 4: Let $\bar{K}_C = \{(k, d): k \in K_D, (\bar{v}_{k \times 1\times d_{j\times 1}}, \bar{w}_{k \times 1\times d_{j\times 1}}) \leq (1, 1) - A_{d_{j\times 1}}^{d_{j\times 1}},
\quad v_{k \times 1\times d_{j\times 1}} \leq E_d - \sum_{k \in K_C} v_{k \times 1\times d_{j\times 1}} \}$.

Compute effective gradients for the items in $K_C$ as follows.

Step 4.1: If $A_{d_{j\times 1}}^{d_{j\times 1}}$ is a zero vector, then we set
\[ G_{k \times d_{j\times 1}} \leftarrow \sqrt{v_{k \times 1\times d_{j\times 1}}} / \bar{v}_{k \times 1\times d_{j\times 1}} \text{ for } (k, d) \in \bar{K}_C \]

Step 4.2: Otherwise, we set
\[ G_{k \times d_{j\times 1}} \leftarrow \frac{v_{k \times 1\times d_{j\times 1}}}{\bar{v}_{k \times 1\times d_{j\times 1}}} \left( \sum_{k \in K_C} \bar{v}_{k \times 1\times d_{j\times 1}} \right)^2 + \left( \sum_{k \in K_C} \bar{w}_{k \times 1\times d_{j\times 1}} \right)^2 \],
for $(k, d) \in \bar{K}_C$, where
\[ K''_C = \{k': k' \in K_U, x_{k' \times 1\times d_{j\times 1}} = 1\}.

Step 5: Find that item $k$ whose effective gradient is the largest in a period, i.e.,
\[ G_{k \times d_{j\times 1}} \leftarrow \max \{G_{k \times d_{j\times 1}}: (k', d') \in \bar{K}_C \} \]

Step 6: Accept $k$. Let $K_U \leftarrow K_U + \{k\}$,
\[ A_{d_{j\times 1}}^{d_{j\times 1}} \leftarrow A_{d_{j\times 1}}^{d_{j\times 1}} + (\bar{v}_{k \times 1\times d_{j\times 1}}, \bar{w}_{k \times 1\times d_{j\times 1}}), \]
\[ z \leftarrow z + v_{k \times 1\times d_{j\times 1}} \hat{r}_{k \times 1\times d_{j\times 1}}, \quad K_D \leftarrow K_D - \{k\}, \]
\[ x_{k \times d_{j\times 1}} \leftarrow 1. \text{ Then, goto step 2.} \]

We may notice that the solution of HAEG is a feasible solution of MDMKP (M2) and the objective function value is a lower bound of the optimal solution value of it. The numerical experiments in Section 4 show that the solution of HAEG is very close to the optimal solution of MDMKP (M2).

### 3.2 A Simple Example

Let $\hat{r}_{k \times 1\times d_{j\times 1}} = v_{k \times 1\times d_{j\times 1}} / \bar{r}_{k \times 1\times d_{j\times 1}}$, $\bar{T} = \{1, 2\}$, $\vec{J} = \{1, 2\}$
\[ \vec{K}_1 = \{1, 2, 3, 4, 5\}, \quad \vec{K}_2 = \{6, 7\}; \quad j_1 = 1, \text{ for } k = 1, 3, 5, 6; \quad j_k = 2, \quad j_k = 4, 7; \quad \tau_1 = 1, \tau_k = 2, \text{ for } 2 \leq k \leq 7; \}
\[ (E_1, E_2) = (40, 32), \quad k = (V_{12}, V_{22}) = (13, 21), (V_{11}, V_{21}) = (39, 30), \]
\[ k = (W_{11}, W_{21}) = (35, 25), (W_{12}, W_{22}) = (12, 38), \]
\[ (w_{1111}, w_{1521}, w_{1261}) = (10, 13, 5, 9), \]
\[ (w_{1222}, w_{1432}, w_{2722}) = (13, 12, 8) ; \]
\[ (\hat{r}_{1111}, \hat{r}_{1311}, \hat{r}_{1511}) = (340, 660, 60), \]
\[ (\hat{r}_{1212}, \hat{r}_{1412}) = (182, 77), \]
\[ (\hat{r}_{1312}, \hat{r}_{1521}, \hat{r}_{2621}) = (506, 70, 170), \]
\[ (\hat{r}_{1212}, \hat{r}_{1412}, \hat{r}_{2722}) = (70, 231, 200). \]

We solve it by the heuristic algorithm with effective gradient HAEG given in Section 3.1. the procedure is briefly given as follows.

Since $v_{k \times 1\times d_{j\times 1}} / V_{d_{j\times 1}}$, $\bar{w}_{k \times 1\times d_{j\times 1}} = w_{k \times 1\times d_{j\times 1}} / W_{d_{j\times 1}}$, we have
\[ (\bar{v}_{1111}, \bar{v}_{1311}, \bar{v}_{1511}) = (0.4359, 0.5641, 0.1282), \]
\[ (\bar{w}_{1222}, \bar{w}_{1422}, \bar{w}_{2722}) = (0.3333, 0.5238, 0.8095), \]
\[ (\bar{w}_{1111}, \bar{w}_{1311}, \bar{w}_{1511}) = (0.2857, 0.3714, 0.1429), \]
\[ (\bar{w}_{1212}, \bar{w}_{1412}) = (1.0833, 1), \]
\[ (\bar{w}_{1222}, \bar{w}_{1422}, \bar{w}_{2722}) = (0.3421, 0.3158, 0.2105), \]
\[ (\bar{w}_{1212}, \bar{w}_{1412}) = (0.5385, 0.8462), \]
\[ (\bar{w}_{1312}, \bar{w}_{1521}, \bar{w}_{2621}) = (0.7333, 0.1667, 0.3333), \]
\[ (\bar{w}_{1312}, \bar{w}_{1521}, \bar{w}_{2621}) = (0.52, 0.2, 0.36). \]

Step 1. Let $K_U \leftarrow \phi$, all items be assigned to
\[ K_D = K - K_U, \quad z \leftarrow 0, \quad A_{d_{j\times 1}}^{d_{j\times 1}} \leftarrow 0, \quad x_{k \times 1\times d_{j\times 1}} \leftarrow 0, \forall t \in \bar{T}, k \in \bar{K}, t \leq d \leq \tau_k \].

Step 2. Let $K_C \leftarrow \{k: k \in K_D, \exists d \leq \tau_k \}
\text{s.t.} (\bar{v}_{k \times 1\times d_{j\times 1}}, \bar{w}_{k \times 1\times d_{j\times 1}}) \leq (1, 1) - A_{d_{j\times 1}}^{d_{j\times 1}},
\quad v_{k \times 1\times d_{j\times 1}} \leq E_d - \sum_{k \in K_C} v_{k \times 1\times d_{j\times 1}} \}
\]

Step 3. Since $K_C \neq \phi$, proceed to the next step.

Step 4. Since $A_{d_{j\times 1}}^{d_{j\times 1}} = (0, 0)$, the effective gradients are calculated by formula:
\[ G_{k \times d_{j\times 1}} = \frac{\hat{r}_{k \times 1\times d_{j\times 1}} \sqrt{2}}{\bar{v}_{k \times 1\times d_{j\times 1}} + \bar{w}_{k \times 1\times d_{j\times 1}}} \text{ for } (k, d) \in \bar{K}_C \]

we have $G_{11} = 666.33$, $G_{31} = 997.7$, $G_{51} = 313.04$, $G_{32} = 666.33$, $G_{52} = 269.99$. 

Step 5. The largest effective gradient is $G_{31} = 997.7$.

Step 6. Accept item $3$. Let $K_U \leftarrow K_U \cup \{3\}$, 
where $G_{d3} = 346.75 \ , \ G_{41} = 58.98 \ , \ G_{22} = 146.56 \ , \ G_{42} = 389.09 \ , \ G_{72} = 277.28$.

Step 5. The largest effective gradient is $G_{31} = 997.7$.

Step 6. Accept item $3$. Let $K_U \leftarrow K_U \cup \{3\}$, 
where $G_{d3} = 346.75 \ , \ G_{41} = 58.98 \ , \ G_{22} = 146.56 \ , \ G_{42} = 389.09 \ , \ G_{72} = 277.28$.

3.3 Modified Heuristic Algorithm

In this section, we modify the effective gradient $G_j$ which introduced in Section 3.1 to increase accuracy of the heuristic algorithm.

**Algorithm MHA**

Step 1: Initialization.

Step 1.1: Let $l \leftarrow 0$, $\hat{z} \leftarrow 0$, $\hat{x}_{dij} \leftarrow 0$, 
where $\forall \ t \in \bar{T}, k \in \bar{K}_t, t \leq d \leq \tau_k$.

Step 1.2: Let $K_U \leftarrow \phi$, where $K_U$ is the set of accepted items.

Step 1.3: Assign all items to $K_D = K - K_U$, where $K_D$ is the set of items not in $K_U$ and $K = \{k : k \in \bar{K}_t, t \in \bar{T}\}$.

Step 1.4: Let $A_{dj}^d \leftarrow (0, 0)$, where $A_{U}^d$ is the total quantity vector of accepted items shipping to port $j$ in period $d$.

Step 1.5: Let the objective value be zero, i.e., $z \leftarrow 0$.

Step 1.6: Let $x_{dij} \leftarrow 0, \forall \ t \in \bar{T}, \ k \in \bar{K}_t, \ t \leq d \leq \tau_k$.

Step 2: Let $K_C \leftarrow \{k : k \in K_D, \exists d \leq \tau_k \ 
\begin{align*}
&\text{s.t.} \ (\bar{v}_{dij}, \bar{w}_{dij}) \leq (1, 1) - A_{U}^d, \\
&\bar{v}_{dij} \leq E_d - \sum_{k \in K(t), \tau_k \geq d} v_{dij},
\end{align*}
where $K_C$ is the set of candidate items.

Step 3: Check $K_C$. If $K_C$ is empty, goto step 7. Otherwise, proceed to the next step.

Step 4: Let $K_C = \{(k, d) : k \in K_D, (\bar{v}_{dij}, \bar{w}_{dij}) \ 
\begin{align*}
&\leq (1, 1) - A_{U}^d, \ 
&v_{dij} \leq E_d - \sum_{k \in K(t), \tau_k \geq d} v_{dij} \}.
\end{align*}

\begin{align*}
&\text{Compute effective gradients for the items in } K_C \text{ as follows.}
\end{align*}

Step 4.1: If $A_{U}^d$ is a zero vector, then we set 
$G_{kd} \leftarrow \sqrt{2} v_{dij} r_{dij}, \text{ for } (k, d) \in K_C$

Step 4.2: Otherwise, let 
$K_U = \{k' : k' \in K_U, x_{dij} = 1\}, \text{ for } l = 0, 1, 2$.

for $l = 0, 1, 2$;

Step 5: Find that item $k$ whose effective gradient is the largest in a period, i.e., 
$G_{kd} = \max \{G_{kd'} : (k', d') \in K_C\}$.

Step 6: Accept $k$. Let $K_U \leftarrow K_U + \{k\}$.

Step 7: If $z > \hat{z}$, then $\hat{z} \leftarrow z$, $\hat{x}_{dij} \leftarrow x_{dij}$, 
for $t \in \bar{T}, \ k \in \bar{K}_t$, $d \in \{t, t+1, \cdots, \tau_k\}$.

Step 8: If $l = 8$, then the procedure terminates.

Otherwise, let $l \leftarrow l + 1$, goto step 1.1.

From the steps of the modified heuristic algorithm, the solution of MHA is closer to the optimal solution than HAEG, because we take the maximum value among
solutions given by nine methods including the method used in HAEG. We may easily notice that the computation time of MHA is more than four times of HAEG.

4. Numerical Experiments

In this section, we implement the modified heuristic algorithm MHA and give comparisons among heuristic solutions and optimal solutions or LP optimal solutions (as the up bound of optimal solutions). The algorithm has been coded in C++ and run under Windows 2000 Professional using a notebook computer (Pentium IV with 1794MHz and 256MB in RAM). CPU times were obtained through the C++ function clock(). To conduct our experiments we used randomly generated instances. For simplicity of implementation, we assumed that destination port \( j_k \) of each cargo \( k \) is decided by:

\[
j_k = j, \text{ if } \lfloor K/J \rfloor \times (j-1) \leq j_k < \lfloor K/J \rfloor \times j,
\]

for \( j = 1, 2, \ldots, J - 1 \);

\[
j_k = J, \text{ if } \lfloor K/J \rfloor \times (J-1) < j_k \leq J.
\]

For each set of parameters \( J, T \) and \( K \), we generated 10 random MDMKP instances. We tested heuristic solutions and optimal solutions or LP optimal solutions on all 10 instances, and tabulated the average relative gap and average computation time. In table 1, the relative gap between heuristic solution and optimal solution \( g_o \) is computed as

\[
\frac{\text{total profit of optimal solution} - \text{total profit of heuristic solution}}{\text{total profit of optimal solution}} \times 100\%
\]

In table 1 and table 2, the relative gap between heuristic solution and LP relaxation optimal solution \( g_L \) is computed as

\[
\frac{\text{total profit of LP relaxation optimal solution} - \text{total profit of heuristic solution}}{\text{total profit of LP relaxation optimal solution}} \times 100\%
\]

Table 1 shows the results obtained for a set of small test problems. Test problems 1-2 have 2 periods, 2 destination ports and 74, 114 items (cargoes) respectively; test problem 3 have 3 periods, 4

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<th>Number of constraints</th>
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destination ports and 27 items; test problem 4 have 4 periods, 8 destination ports and 23 items. For comparison, the optimal solution has been computed using CPLEX 8.0. As can be seen from table 1, the obtained results seem to be encouraging. The gap between the optimal solution and the heuristic solution is small and the computation time is very short. Table 2 shows the results obtained for a set of medium scale problems. Table 3 gives the computation times for a set of large scale problem. It show that the computation time of the heuristic algorithm is very short even for very large scale problems with tens of thousands of decision variables.

We believe that heuristic algorithm will be a very good candidate for solving MDMKP in time critical or real-time applications such as sea cargo mix problems where a near optimal solution is acceptable, and fast computation is more important than guaranteeing optimal value.

5. Conclusions

We formulated the multi-period sea cargo mix problem as the MDMKP model and presented an effective heuristic algorithm for it.

The heuristic algorithm gives fast and near optimal solution for the multi-period sea cargo mix problem. We presented experimental results to evaluate the algorithm by a wide range of problem instances. The heuristic algorithm is a very effective for time critical operations level decisions where a near optimal solution is acceptable, and fast computation is more important than guaranteeing optimal value.

References