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## Abstract

Model management is the task of selecting and applying appropriate analytical models by a Decision Support System. The selection of a model in current model management systems requires definitive boundaries or explicit rules and reasoning. Selection of a model, however, often requires examining choice attributes that have no clear definitions or that overlap in their applicability. The selection of a single model from among various alternative models may thus require components of fuzzy reasoning. In this paper, we examine the use of belief graphs, a fuzzy technique, to select an appropriate integer programming solution methodology based upon several problem formulation characteristics. Our prototype shows that belief graphs are effective at selecting model methodologies within the scope of the decision setting.

## Introduction

A model is an abstraction of reality. Often, models provide the means of analysis during problem solving. As such, models are central to the concept of Decision Support Systems (DSS) which strive to assist a decision maker in arriving at a solution to a problem. A DSS uses data, models and interactive dialogues to assist a decision maker in this process. The feature of selecting and applying models is known as the Model Management System (MMS) in the DSS architecture (Sprague and Carlson, 1982). In practice, the MMS must function as an expert in the selection and application of models, much as would a consultant in the decision sciences.

Model management is historically approached in the literature with two distinct categories. One category, which we will refer to as micro-model management, is concerned with assisting users through a single modeling technique from the formulation of a problem through interpretation of a solution. A sophisticated linear programming package or advanced spreadsheet might be considered an MMS in this domain. The other category, which we will call macro-model management, directs its attention to selecting an appropriate modeling technique or structure from among the wide variety of modeling techniques available. Blanning (1993) provides an excellent overview and survey of MMS research. Chang, Holsapple and Whinston (1993) extend the survey with the addition of issues and research directions, including a call for more intelligence to be included in the systems.

Most MMS in the literature use a formalism that is restrictive with respect to the mechanics of the model selection function. Logic-based, rule-based, data-based, and object-based systems must conform to the theory on which they were founded. Enhancements to theory enable researchers to mitigate some of the rigidity. Different reasoning forms in an expert system allow leaps across the gaps of missing relationships between model components (Liu, et al., 1990). Possibility measures can be incorporated into expert systems that are rule or logic based (Zahedi, 1993). Objects can assume many properties including possibilities, values, or distributions. However, only a couple of proposed methods explicitly consider models as being hazy entities, where selection of a particular model may be based on an accumulation of evidence (Klein, et al., 1985) or analogy (Liang and Konsynski, 1993).

We propose to turn to a branch of engineering that could be directly incorporated into most, if not all, of the suggested approaches to MMS: Fuzzy Mathematics. Fuzzy mathematics is an area of study that allows expression of relationships, values, logic, and memberships that are not clearly defined (Dubois and Prade, 1987a; Klir and Folger, 1988; Negoita, 1985). Fuzzy studies sprang from interval analysis and includes ideas as fuzzy sets (Zadeh, 1965; Zimmerman, 1987), fuzzy logic (Viot, 1993), fuzzy numbers (Kaufmann and Gupta, 1991), and belief functions (Shafer, 1987). They are incorporated into expert systems (Dubois and Prade, 1987b; Negoita, 1985) and have many scientific and engineering applications (Bezdek, 1987).

Several fuzzy tools exist that could be incorporated into different model management structures. Belief functions are used in this exploration to build a simple prototype. The problem domain is integer programming. The model methodologies include several general integer programming optimization algorithms. The selection criteria contain characteristics of integer programs. A prototype is developed in FuziCalc, a spreadsheet which allows specification of a belief function in graph form as a cell value (FuziWare, Inc. 1993).

**Scope of the Model Base**

A decision maker has formulated a mixed- or pure integer programming problem, perhaps a difficult, large-scale scheduling or manpower/manufacturing planning problem. A solution to this mathematical program is required. What is the best general mixed-integer problem solver for the current formulation? The answer of course will vary depending upon a number of characteristics of the mixed-integer program. A good subset of general solvers includes Branch-and-Bound, Implicit Enumeration, Cutting Planes, Decomposition, and Lagrangian Relaxation. For detailed material on mixed- and pure integer programming problems (MILPs) and their solution procedures, see Salkin (1975), Garfinkel and Nemhauser (1972) and Nemhauser and Wolsey (1988).

The methods listed above are tried and true, effective integer programming techniques. In addition, there have been many advances in solving MILPs with substructures such as networks, though we do not deal explicitly with special structures in our fuzzy model management system. A fast and robust implementation that incorporates the five methodologies we consider, with a powerful linear programming solver, will generally be quite sufficient for solving many real-world, mixed-integer problems that do not exhibit special structure.

The characteristics of a mixed-integer program affecting selection of a solution methodology would include the size of the problem, measured in terms of 1) the number of variables and 2) the number of constraints; 3) the density of the constraint coefficient matrix; and 4) the fraction of pure integer variables. Based on these characteristics, an MMS must select the optimization methodology for the formulated program. Selection of an improper technique will result in unnecessary delays and for certain Decision Support Systems (DSS), response time is critical (e.g., for real-world scheduling problems, pricing problems, and stock, commodity or currency trading problems).

Of course, many other factors could be considered when selecting an optimizer, including special structures, satisfaction with sub-optimality or epsilon optimality, computer resources available, application domain, and an evaluation of the combinatorial explosiveness of the formulation. However, the limited set of criteria presented will serve to demonstrate the use of belief functions for model selection, and serve in preliminary evaluation of the fuzzy technique when applied to a fairly general case.

**The System**

Belief functions have a body of surrounding theory (Shafer 1987). In a simplistic sense, they represent the fractional degree to which each value x in b(x) belongs to the category represented by the function. Belief functions need to be constructed for each cell in the FuziCalc spreadsheet where the intersection is the integer program method and the criteria of selection. For this example, there are five methods and four criteria, resulting in construction of twenty belief functions. Note that these twenty functions will also take all criteria interaction effects into account, something that simple rules or logic could not consider without implicit or explicit enumeration of all combinations.

Once all twenty belief functions are defined, they are entered into the FuziCalc system. A sample spreadsheet appears in Figure 1 (next page). Fuzzy cells are indicated with a triangle to the left of the cell and the centroid of the belief function as the numeric value. Graphs of the belief functions are viewed on demand. For this demonstration all values were scaled to be from zero to one. Selection of a model is made by entering the characteristics of the formulation on the spreadsheet input line (Line 10). The belief functions (Cells B3 to E7) are converted to values (Cells B13 to E17) by taking the entered characteristics and looking up their corresponding belief value. The beliefs for each optimization technique are summed resulting in a belief score (Cells F13 to F17). The highest score is then selected as the optimization methodology of choice. Competing or close scores may indicate multiple methods are appropriate.

The process of simple summation of belief is the most straightforward. Fuzzy procedures also exist for ranks, dot products, and other transforms of the belief functions that further research may find more appropriate in model management. Conceptually, the summation approach is similar to constructing value functions for each cell and using an additive multi-attribute value function to select the appropriate integer programming optimizer (Keeney and Raiffa 1976). The result for the values entered in Figure 1 are sensible, with Branch & Bound selected for a mixed problem with a large number of variables and moderate constraints and density. Many other cases were examined and results compared to those expected by the authors.

**Conclusion**

This paper describes a simple application of belief functions in a linearly driven Model Management System. The importance of this summary is that we have made an initial step in bringing fuzzy mathematics literature to bear upon model management. Each fuzzy technique has its own unique properties and the power to represent situations where the decisions cannot be clearly defined, as in the selection of an appropriate model. For the domain of the presentation, the fuzzy technique of belief functions proved to be robust, effective, and easy to manipulate.

Another unique feature of fuzzy mathematics is that the aspects of fuzziness can be incorporated directly into other model management techniques. Fuzzy expert systems also hold promise as a technique for model management systems. Fuzzy logic can be a way to incorporate possibility concepts into logic driven systems. Objects can contain fuzzy sets as well as crisp sets. Relational approaches may benefit from the work already done in fuzzy databases (Dubois, Prade and Sessa 1994). Newer approaches may be derived from the use of fuzzy neural networks (Gupta and Rao 1994).

Author's Note

A longer version of this paper including references is available from the second author.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | IP Class | Pure/Mixed/Linear | Number of vars | No. of Constraints | Matrix Density | | |
| 2 | | | | | | | |
| | | | | | | | |
| 3 | Cutting Plane | 0.15 | 0.50 | 0.52 | 0.07 | | |
| 4 | Implicit Enumeration | 0.00 | 0.29 | 0.78 | 0.53 | | |
| 5 | Lagrangian Multiplier | 0.43 | 0.93 | 0.07 | 0.50 | | |
| 6 | Decomposition | 0.32 | 0.30 | 0.50 | 0.43 | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 7 | Branch and Bound | 0.47 | 0.35 | 0.49 | 0.51 | | |
| 8 | | | | | | | |
| 9 | | Fraction(pure=0) | # of var(max 200) | # of con(max 200) | % dense | | |
| 10 | Enter Values | 0.40 | 100.00 | 50.00 | 0.40 | | |
| 11 | | | | | | | |
| 12 | Scale to zero-one | 0.40 | 0.50 | 0.25 | 0.40 | | |
| 13 | Belief for cut plane | 0.00 | 1.00 | 0.71 | 0.00 | 1.71 | Cut |
| 14 | Belief for Implicit | 0.00 | 0.50 | 0.00 | 0.39 | 0.89 | Implicit |
| 15 | Belief for Legrangian | 0.90 | 0.00 | 0.00 | 0.35 | 1.25 | Lagranian |
| 16 | Belief for Decomposition | 0.85 | 0.48 | 0.51 | 0.35 | 1.98 | Decomposition |
| 17 | Belief for B & B | 0.59 | 0.48 | 0.53 | 0.52 | 2.12 | B & B |

Figure 1 - Example of Fuzzy MMS Selection of the Branch & Bound Method