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Abstract

Most conventional data-mining algorithms identify the relationships among transactions using binary values and find rules at a single concept level. Transactions with quantitative values and items with taxonomic relations are, however, commonly seen in real-world applications. Besides, the taxonomic structures may also be represented in a fuzzy way. This paper thus proposes a fuzzy multiple-level mining algorithm for extracting fuzzy association rules under given fuzzy taxonomic structures. The proposed algorithm adopts a top-down progressively deepening approach to finding large itemsets. It integrates fuzzy-set concepts, data-mining technologies and multiple-level fuzzy taxonomy to find fuzzy association rules from given transaction data sets. Each item uses only the linguistic term with the maximum cardinality in later mining processes, thus making the number of fuzzy regions to be processed the same as the number of the original items. The algorithm therefore focuses on the most important linguistic terms for reduced time complexity.

1. Introduction

Machine-learning and data-mining techniques have been developed to turn data into useful task-oriented knowledge [20]. Deriving association rules from transaction databases is most commonly seen in data mining [1][2][6][9][10][11][13][14][27][28]. It discovers relationships among items such that the presence of certain items in a transaction tends to imply the presence of certain other items.

Most previous studies have concentrated on showing how binary-valued transaction data may be handled. However, transaction data in real-world applications usually consist of quantitative values, so designing a sophisticated data-mining algorithm able to deal with quantitative data presents a challenge to workers in this research field.

In the past, Agrawal and his co-workers proposed several mining algorithms for finding association rules in transaction data based on the concept of large itemsets [1-2, 28]. They also proposed a method [27] for mining association rules from data sets using quantitative and categorical attributes. Their proposed method first determines the number of partitions for each quantitative attribute, and then maps all possible values of each attribute onto a set of consecutive integers. Other methods have also been proposed to handle numeric attributes and to derive association rules. Fukuda et al. introduced the optimized association-rule problem and permitted association rules to contain single uninstantiated conditions on the left-hand side [11]. They also proposed schemes for determining conditions under which rule confidence or support values are maximized. However, their schemes were suitable only for single optimal regions. Rastogi and Shim extended the approach to more than one optimal region, and showed that the problem was NP-hard even for cases involving one uninstantiated numeric attribute [23][24].

Fuzzy set theory is being used more and more frequently in intelligent systems because of its simplicity and similarity to human reasoning [18]. Several fuzzy learning algorithms for inducing rules from given sets of data have been designed and used to good effect with specific domains [3-4, 8, 12, 15-17, 25, 29-30]. Strategies based on decision trees were proposed in [7, 21-22, 25, 31, 33], and Wang et al. proposed a fuzzy version space learning strategy for managing vague information [29]. Hong et al. also proposed a fuzzy mining algorithm for managing quantitative data [14].

In [18], we proposed a data-mining algorithm able to deal with quantitative data under a crisp taxonomic structure. In that approach, each item definitely belongs to only one ancestor in the taxonomic structure. The taxonomic structures may, however, not be crisp in real-world applications. An item may belong to different classes in different views. This paper thus proposes a new fuzzy data-mining algorithm for extracting fuzzy multiple-level association rules under given fuzzy taxonomic structures. The proposed algorithm adopts a top-down progressively deepening approach to finding large itemsets.

2. Review of Fuzzy Set Concepts

Fuzzy set theory was first proposed by Zadeh in 1965 [34]. Fuzzy set theory is primarily concerned with quantifying and reasoning using natural language in which words can have ambiguous meanings. This can be thought of as an extension of traditional crisp sets, in which each element must either be in or not be in a set.

Formally, the process by which individuals from a universal set X are determined to be either members or non-members of a crisp set can be defined by a

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characteristic or discrimination function [34]. For a given crisp set A, this function assigns a value $\mu_A(x)$ to every $x \in X$ such that

$$\mu_A(x) = \begin{cases} 1 & \text{if and only if } x \in A \\ 0 & \text{if and only if } x \notin A. \end{cases}$$

Thus, the function maps elements of the universal set to the set containing 0 and 1. This function can be generalized such that the values assigned to the elements of the universal set fall within specified ranges, referred to as the membership grades of these elements in the set. Larger values denote higher degrees of set membership. Such a function is called the membership function, $\mu_A(x)$, by which a fuzzy set A is usually defined. This function is represented by

$$\mu_A : X \rightarrow [0,1],$$

where [0, 1] denotes the interval of real numbers from 0 to 1, inclusive. The function can also be generalized to any real interval instead of [0, 1].

A special notation is often used in the literature to represent fuzzy sets. Assume that x_1 to x_n are the elements in fuzzy set A, and μ_1 to μ_n are, respectively, their grades of membership in A. A is then represented as follows:

$$A = \mu_1/x_1 + \mu_2/x_2 + \ldots + \mu_n/x_n.$$

An α*-cut* of a fuzzy set A is a crisp set A_α that contains all elements in the universal set X with membership grades in A greater than or equal to a specified value α. This definition can be written as:

$$A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}.$$

The *scalar cardinality* of a fuzzy set A defined on a finite universal set X is the summation of the membership grades of all the elements of X in A. Thus,

$$|A| = \sum_{x \in X} \mu_A(x).$$

Among operations on fuzzy sets are the basic and commonly used *complementation*, *union* and *intersection,* as proposed by Zadeh.

The complementation of a fuzzy set A is denoted by $\neg A$, and the membership function of $\neg A$ is given by:

$$\mu_{\neg A}(x) = 1 - \mu_A(x), \quad \forall x \in X.$$

The intersection of two fuzzy sets A and B is denoted by $A \cap B$, and the membership function of $A \cap B$ is given by:

$$\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}, \quad \forall x \in X.$$

The union of fuzzy sets A and B is denoted by $A \cup B$, and the membership function of $A \cup B$ is given by:

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}, \quad \forall x \in X.$$

The above concepts will be used in our proposed algorithm to mine a set of fuzzy interesting association rules under fuzzy taxonomic structures.

3. Data Mining with a Fuzzy Taxonomic Structure

Previous studies on data mining focused on finding association rules on a single-concept level. Mining multiple-concept-level rules may, however, lead to discovery of more general and important knowledge from data. Relevant taxonomies of data items are thus usually predefined in real-world applications. An item may, however, belong to different classes in different views. When taxonomic structures are not crisp, hierarchical graphs can be used to represent them. Terminal nodes on the hierarchical graphs represent the items actually appearing in transactions; internal nodes represent classes or concepts formed by lower-level nodes. A simple example is given in Figure 1.

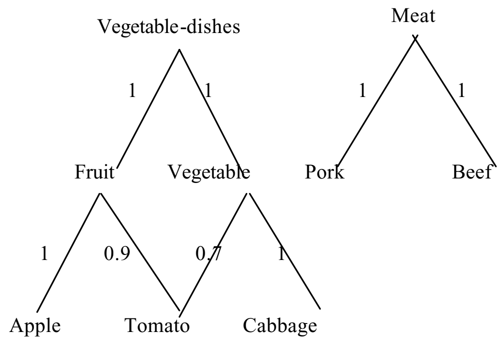

Figure 1. An example of fuzzy taxonomic structures

In this example, vegetable dishes fall into two classes: fruit and vegetable. Fruit can be further classified into apple and tomato. Similarly, assume vegetable is divided into tomato and cabbage. Note that tomato belongs to both fruit and vegetable with different membership degrees. It is thought of as fruit with 0.9 membership value and as vegetable with 0.7. The membership value of tomato belonging to vegetable dishes can be calculated using the max-min product combination. Since both fruit and vegetable belong to vegetable dishes with membership value 1, the membership value of tomato belonging to vegetable dishes is then *max(min(1, 0.9), min(1, 0.7))=0.9*. Only the terminal items (apple, tomato, cabbage, pork and beef) can appear in transactions. The membership degrees of ancestors for each terminal node are shown in Table 1.

Wei and Chen proposed a method to find generalized association rules under fuzzy taxonomic structures [32]. The items to be processed in their approach are binary. Their mining process first calculated the membership values of ancestors for each terminal node in the manner mentioned above. Han and Fu also proposed a method for finding level-crossing association rules at multiple levels [13]. In that method,

Table 1. The membership degrees of ancestors for each terminal node in this example

Terminal node	Membership values of ancestors
Apple	1/Fruit, 1/Vegetable-dishes
Tomato	0.9/Fruit, 0.7/Vegetable, 0.9/Vegetable-dishes
Cabbage	1/Vegetable, 1/Vegetable-dishes
Pork	1/Meat
Beef	1/Meat

each item only belongs to one ancestor in the preceding generation. A top-down progressively deepening search approach is used. Their method finds flexible association rules not confined to strict, pre-arranged conceptual hierarchies and exploration of "level-crossing" association relationships is allowed. Candidate itemsets on certain levels may thus contain other-level items. For example, candidate 2-itemsets on level 2 are not limited to containing only pairs of large items on level 2. Instead, large items on level 2 may be paired with large items on level 1 to form candidate 2-itemsets on level 2.

Wei and Chen's concepts of fuzzy taxonomic structures and Han and Fu's top-down progressively deepening search approach were used in our approach to mine fuzzy generalized association rules from quantitative transaction data. The rules mined are expressed in linguistic terms, which are more natural and understandable for human beings.

4. Notation

The following symbols are used in our proposed algorithm:

n: the number of transactions;

D_i: the i-th transaction, $1 \le i \le n$;

x: the number of levels in a given taxonomy.

m^k: the number of items (nodes) at level k, $1 \le k \le x$;

I_j^k : the j-th item on level k, $1 \le k \le x$; $1 \le j \le m^k$;

h_j^k : the number of fuzzy regions for I_j^k ;

R_{jl}^k : the l-th fuzzy region of I_j^k , $1 \le l \le h_j^k$;

v_{ij}^k : the quantitative value of I_j^k in D_i ;

f_{ij}^k : the fuzzy set converted from v_{ij}^k ;

f_{ijl}^k : the membership value of D_i in region R_{jl}^k;

$count_{jl}^k$: the summation of f_{ijl}^k , i=1 to n;

$max\text{-}count_j^k$: the maximum count value among $count_{jl}$ values, l=1 to h_j^k ;

$max\text{-}R_j^k$: the fuzzy region of I_i^k with $\max - count_j^k$;

α : the predefined minimum support value;

λ : the predefined minimum confidence value;

C_r^k : the set of candidate itemsets with r items on level k;

L_r^k : the set of large itemsets with r items on level k.

5. The Multiple-level Fuzzy Data-mining Algorithm

The proposed mining algorithm integrates fuzzy set concepts, data mining and multiple-level fuzzy taxonomy to find fuzzy association rules in a given transaction data set. The knowledge derived is represented by fuzzy linguistic terms, and thus easily understandable by human beings.

The proposed fuzzy mining algorithm first calculates the membership values of ancestors for each terminal node as Wei and Chen's method did. It then adopts a top-down progressively deepening approach to finding large itemsets. Each item uses only the linguistic term with the maximum cardinality in later mining processes, thus making the number of fuzzy regions to be processed the same as the number of original items. The algorithm therefore focuses on the most important linguistic terms, which reduces its time complexity. A mining process using fuzzy counts is performed to find fuzzy multiple-level association rules. Details of the proposed fuzzy mining algorithm are stated below.

The fuzzy mining algorithm using taxonomy:

INPUT: A body of n quantitative transaction data, a set of membership functions, predefined fuzzy taxonomic structures, a predefined minimum support value α, and a predefined confidence value λ.

OUTPUT: A set of fuzzy multiple-level association rules.

STEP 1: Set k=1,where k is used to store the level number being processed.

STEP 2: Calculate the membership values of ancestors of each terminal node from the given fuzzy taxonomic structure.

STEP 3: Calculate the quantitative value v_j^k of each ancestor item I_j^k in transaction datum D_i *(i=1 to n)* as:

$$v_{\mathrm{ij}}^{\mathrm{k}} = \sum_{v_{ij}^x \ in \ D_i} v_{\mathrm{ij}}^{\mathrm{x}} * \mu_{I_{ij}^k}(I_{ij}^x)$$

where I_{ij}^x is a terminal item appearing in D_i, v_{ij}^x is the quantitative value of I_{ij}^x , and $\mu_{I_{ij}^k}(I_{ij}^x)$ is the membership value of item I_{ij}^x belonging to ancestor I_{ij}^k .

STEP 4: Count the number of occurrences of each ancestor item I_j^k in the transactions and remove the each ancestor item I_j^k with their counts less than α.

STEP 5: Transform the quantitative value v_{ij}^k of each transaction datum D_i, (i=1 to n) for each I_j^k into a fuzzy set f_{ij}^k represented as:

$$\left(\frac{f_{ij1}^{k}}{R_{j1}^{k}} + \frac{f_{ij2}^{k}}{R_{j2}^{k}} + \dots + \frac{f_{ijh}^{k}}{R_{jh}^{k}} \right)$$

by mapping v_{ij}^{k} form the given membership functions, where h is the number of fuzzy regions for I_j^k, R_{jl}^{k} is the l-th fuzzy region of I_j^k, $1 \le l \le h$, and f_{ijl}^{k} is v_{ij}^{k}'s fuzzy membership value in region R_{jl}^{k}.

STEP 6: Calculate the scalar cardinality of each fuzzy region R_{jl}^{k} in the transaction data:

$$count_{jl}^{k} = \sum_{i=1}^{n} f_{ijl}^{k}.$$

STEP 7: Find $max - count_j^k = \overset{h_j^k}{\underset{l=1}{MAX}} \left(count_{jl}^{k}\right)$, for j= 1 to m^k, where h_j^k is the number of fuzzy regions for I_j^k and m^k is the number of items (nodes) on level k. Let $max - R_j^k$ be the region with $max\text{-}count_j^k$ for item I_j^k, which will be used to represent the fuzzy characteristic of item I_j^k in later mining processes.

STEP 8: Check whether the value $max\text{-}count_j^k$ of a region $max\text{-}R_j^k$, j=1 to m^k, is larger than or equal to the predefined minimum support value α. If a region $max\text{-}R_j^k$ is equal to or greater than the minimum support value, put it in the large 1-itemsets (L_1^k) at level k. That is,

$$L_1^k = \left\{ \max - R_j^k \middle| \max - count_j^k \ge \alpha,\ 1 \le j \le m^k \right\}.$$

STEP 9: If L_1^k is null, then set $k = k + 1$ and go to STEP 4; otherwise, do the next step.

STEP 10: Generate the candidate set C_2^k from L_1^1, L_1^2, …, L_1^k to find "level-crossing" large itemsets. Each 2-itemset in C_2^k must contain at least one item in L_1^k and the other item may not be its ancestor in the taxonomy. All possible 2-itemsets are collected in C_2^k.

STEP 11: For each newly formed candidate 2-itemset s with items (s_1, s_2) in C_2^k:

(a) Calculate the fuzzy value of s in each transaction datum D_i as $f_{is} = f_{is_1} \Lambda f_{is_2}$, where f_{is_j} is the membership value of D_i in region s_j. If the minimum operator is used for the intersection, then $f_{is} = \min(f_{is1}, f_{is2})$.

(b) Calculate the scalar cardinality of s in the transaction data as:

$$count_s = \sum_{i=1}^{n} f_{is}.$$

(c) If $count_s$ is larger than or equal to the predefined minimum support value α, put s in L_2^k.

STEP 12: Set r=2, where r is used to represent the number of items stored in the current large itemsets.

STEP 13: If L_r^k or C_{r+1}^k is null, then set k=k+1 and go to STEP 3; otherwise, do the next step.

STEP 14: Generate the candidate set C_{r+1}^k from L_r^k in a way similar to that in the *apriori* algorithm. That is, the algorithm first joins L_r^k and L_r^k, assuming that r-1 items in the two itemsets are the same and the other one is different. The different items must also not have a hierarchical relationship. That is, items may not be ancestors or descendants of one another. Store in C_{r+1}^k all itemsets having all their sub-r- itemsets in L_r^k.

STEP 15: For each newly formed $(r+1)$-itemset s with items $(s_1, s_2, \dots, s_{r+1})$ in C_{r+1}^k:

(a) Calculate the fuzzy value of s in each transaction datum D_i as

$$f_{is} = f_{is_1} \Lambda f_{is_2} \Lambda \dots \Lambda f_{is_r},$$

where f_{is_j} is the membership value of D_i in region s_j. If the minimum operator is used for the intersection, then:

$$f_{is} = \overset{r}{\underset{j=1}{Min}}\ f_{is_j}.$$

(b) Calculate the scalar cardinality of s in the transaction data as:

$$count_s = \sum_{i=1}^{n} f_{is}.$$

(c) If $count_s$ is larger than or equal to the predefined minimum support value α, put s in L_{r+1}^k.

STEP 16: If L_{r+1}^k is null and k reaches the level number of the fuzzy taxonomic structures, then do the next step to find association rules; otherwise, set r=r+1 and go to STEP 13.

STEP 17: Construct the fuzzy association rules for all large q-itemset s containing items $\left(s_1, s_2, \dots, s_q\right)$, $q \ge 2$, as follows.

(a) Form all possible association rules as:

$$s_1 \Lambda \dots \Lambda s_{k-1} \Lambda s_{k+1} \Lambda \dots \Lambda s_q \rightarrow s_k,$$

k=1 to q.

(b) Calculate the confidence values of all association rules using the formula:

$$\frac{\sum_{i=1}^{n} f_{is}}{\sum_{i=1}^{n}(f_{is_1} \Lambda \ldots \Lambda\ f_{is_{K-1}},\ f_{is_{K+1}} \Lambda \ldots \Lambda f_{is_q})}.$$

STEP 18: Keep the rules with confidence values larger than or equal to the predefined confidence threshold λ. Output the rules with their support measures and confidence measures larger than or equal to the predefined λ and α threshold to users as generalized association rules.

The rules output from Steps 18 can then serve as meta-knowledge concerning the given transactions.

6. An Example

In this section, an example is given to illustrate the proposed data-mining algorithm. This is a simple example to show how the proposed algorithm generates association rules from quantitative transactions using fuzzy taxonomic structures. The data set includes the six transactions shown in Table 2.

Table 2. Six transactions in this example

Transaction ID	Items
1	(Apple, 3) (Tomato, 4) (Beef, 2)
2	(Tomato, 7) (Cabbage, 7) (Beef, 7)
3	(Tomato, 2) (Cabbage, 10) (Beef, 5)
4	(Cabbage, 9) (Beef, 10)
5	(Apple, 7) (Pork, 9)
6	(Apple, 2) (Tomato, 8)

Each transaction includes a transaction ID and some purchased items. Each item is represented by a tuple (item name, item amount). For example, the fourth transaction consists of nine units of cabbage and ten units of beef. Assume the predefined fuzzy taxonomic structures are as shown in Figure 2. For convenience, the simple symbols in Table 3 are used to represent the items and groups.

Table 3. Items and groups are represented by simple symbols for convenience

Items	Symbol	Groups	Symbol
Apple	A	Fruit	T_1
Tomato	B	Vegetable	T_2
Cabbage	C	Vegetable dishes	T_3
Pork	D	Meat	T_4
Beef	E		

Also assume that the fuzzy membership functions are the same for all the items and are as shown in Figure 3.

In this example, amounts are represented by three fuzzy regions: *Low*, *Middle* and *High*. Thus, three fuzzy

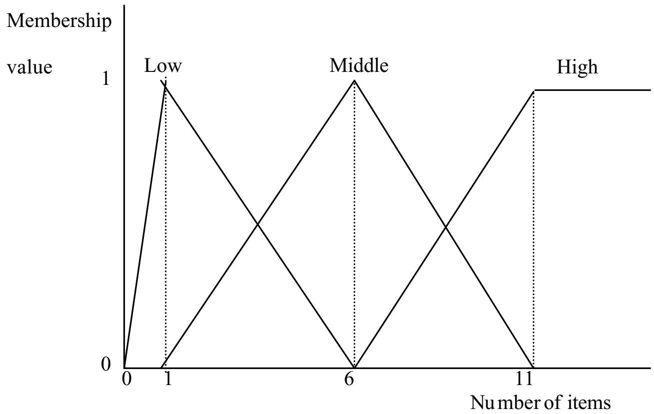

Figure 3. The membership functions used in this example

membership values are produced for each item amount according to the predefined membership functions. Assume in this example, the support threshold α is set at 1.5 and the confidence threshold λ is set at 0.7. For the transaction data in Table 2, the following three rules are mined from the proposed algorithm:

1. If E = *Middle,* then T_2 = *High,* with a support value of 1.8 and a confidence value of 0.9;
2. If E = *Middle,* then T_3 = *High,* with a support value of 1.92 and a confidence value of 0.96;
3. If T_4 = *Middle,* then T_3 = *High,* with a support value of 2.12 and a confidence value of 0.82.

These there rules can then serve as meta-knowledge concerning the given transactions.

7. Discussion and Conclusions

In this paper, we have proposed a fuzzy multiple-level data-mining algorithm that can process transaction data with quantitative values among them. The rules thus mined exhibit quantitative regularity on multiple levels and can be used to provide suggestions to appropriate supervisors.

Although the proposed method works well in data mining for quantitative values, it is just a beginning. There is still much work to be done in this field. Our method also assumes that membership functions are known in advance. In [15-17], we proposed some fuzzy learning methods to automatically derive membership functions. We will therefore attempt to dynamically adjust the membership functions in the proposed mining algorithm to avoid the bottleneck of membership function acquisition. We will also attempt to design specific data-mining models for various problem domains.

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