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Possibility Approach o Newsboy Problem

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POSSIBILITY APPROACH TO NEWSBOY PROBLEM

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1. ABSTRACT

The newsboy problem, also known as news-vendor or the single-period problem is a well-known inventory management problem. Interest in such a problem has increased over the past 40 years partially because the increased dominance of service industrial for which newsboy problem is very applicable in both retailing and service organization. Also, the reduction in product life cycles makes newsboy problem more relevant. Many extensions have been made in last decade, such as different objects and utility function, different supplier pricing policies, different new-vendor pricing policies [2][3][4]. However, almost all of extensions have been made in the probabilistic framework, that is, the uncertainty of demand and supply is characterized by the probability distribution, and the objective function is used to maximizing the expected profit or probability measure of achieving a target profit. There are still some problems left. The one is for life-cycle short products, such as fashion goods, season presents, there is no data to be used for statistical analysis to predict the coming demand. The other is newsboy problem is a typical one-shoot decision problem so that maximizing the expected profit or probability measure seems less meaningful. It seems that possibility theory-based method is another alternative to deal with such kind of decision problem. In this paper the plausible information of demand is dharacterized by the possibility distribution and the optimal order is determined according to the possibilistic decision criteria.

2. POSSIBILITY THEORY

Possibility theory has been proposed by Zadeh [6] and advanced by Dubois and Prade [1] as one of several formal mathematical systems to characterize and analyze the uncertainty in the real world. In the same way as probabilities can be interpreted in different ways (e.g., frequentist view and subjective view), possibility can be explained from several semantic aspects. First is the idea of feasibility, such as ease of achievement. Second is plausibility, referring to the propensity of events to occur. Third is logical consistency with available information. The last is the preference, referring to the willingness of agent to make a decision. Necessity is dually related to possibility with the relation that it is not possible that "not A" means A is necessary. A semantic analysis can be done parallel with possibility, referring belief, acceptance and priority. The decision analysis based on possibility theory has been researched in the papers [5].

Definition 1. Given a function

$$p : \Omega \to [0,1] \tag{1}$$

if

$$\sup_{x\in\Omega} p(x) = 1, \tag{2}$$

then the function $p(x)$ is called the possibility distribution.

The possibility distribution characterizes the unique possibility and necessity measure via the following formulas

$$Pos(A) = \sup_{x \in A \subseteq \Omega} \pi(x) \, , \tag{3}$$

$$Nec(A) = 1 - \sup_{x \in A^c \subseteq \Omega} \pi(x) \, , \tag{4}$$

$$\pi(x) = Pos(\{x\}) \quad x \in \Omega \, . \tag{5}$$

The possibility theory-based decision approaches seem realistic and suitable for the cases that information is plausible and decision will not be repeated. The possibility distribution $\pi(x)$ is used to characterize the plausible information of $x \in S$ where S is the set of states. The function $u(x, a)$ characterizes the utility obtained by taking the decision $a \in A$ when the state is x, where A is a set of decision. The evaluation of the decision a can be made by the following criteria, provided that a commensurability assumption between plausibility and preference is made.

(I) The optimistic criterion

$$v^*(a) = \sup_{x \in S} \min(\pi(x), u(x, a)) \tag{6}$$

where $v^*(a)$ is the optimistic evaluation of the decision a.

(II) The pessimistic criterion

$$v_*(a) = \inf_{x \in S} \max(1 - \pi(x), u(x, a)) \, , \tag{7}$$

where $v_*(a)$ is the pessimistic evaluation of the decision a.

The possibility theory-based optimal decision is the one, which makes $v_*(a)$ or $v^*(a)$ maximize, that is,

$$a^* = \arg\max_{a \in A} \nu^*(a) \quad \text{or} \quad a^* = \arg\max_{a \in A} \nu_*(a) \, . \tag{8}$$

It is clear the possibility theory based-approach makes some decision to balance plausibility and satisfaction. The optimistic criterion will give a higher evaluation of some decision if this action can lead to a higher utility with a high possibility. On the other hand, the pessimistic criterion will give a lower evaluation of some action if this action can lead to a lower utility with a high possibility.

3. NEWSBOY MODEL

Consider a retailer who sells a short life cycle, or single-period new product. The retailer orders q units before the season at the unit wholesale price W. Then demand d is observed, and the retailer sell units (limited by the supply q and the demand d) at unit revenue R (R>W). Any excess units can be salvaged at the unit salvage value S_o. Prior to the selling season, demand is uncertain. However the retailer can know plausible information of demand represented by a possibility distribution.

The profit function of the retailer is as follows

$$r = R \min(d, q) + S_o (q - d)^+ - Wq \, , \tag{9}$$

where

$$a^+ = \begin{cases} a \, ; a > 0 \\ 0 \, ; a \le 0 \end{cases}$$

where q is a decision variable and d is governed by a possibility distribution $\pi(d)$ given by experts to reflect plausible information on the demand.

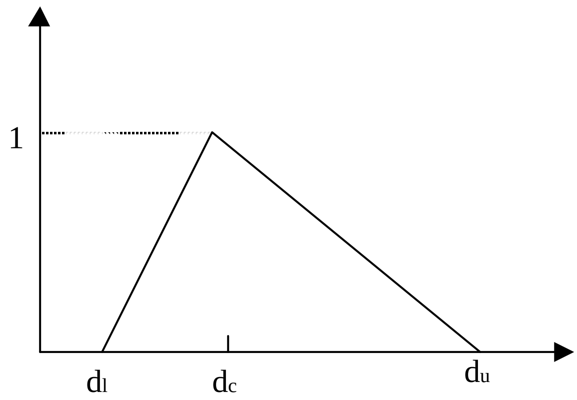

Fig.1 Triangular possibility distribution

Suppose that the possibility distribution for demand is characterized by the following triangular function

$$\pi(d) = \begin{cases} 1 - \dfrac{d_c - d}{d_c - d_l} & ; & d < d_c \\ 1 & ; & d = d_c \\ 1 - \dfrac{d - d_c}{d_u - d_c} & ; & d > d_c \\ 0 & ; & otherwise \end{cases} \, . \tag{10}$$

This triangular possibility distribution is simply denoted as $(d_l, d_c, d_u)_T$ and shown in Fig.1, where d_l and d_u are the lower and upper bounds of demand, respectively, d_c is the most possible amount of demand. Because the demand is inside the interval $[d_l, d_u]$, a reasonable supply also should lie inside this region. For the same supply q, the profit of retailer will change with demand d as shown in formula (9). If $d < q$, then $r = Rd + S(q-d) - Wq$, else $r = (R-W)q$. It means that given a supply q, the retailer possibly can earn from the lowest $(R-S)d_l - (W-S)q$ to the highest $(R-W)q$. The lowest and highest bounds for retailer's profit are $(R-S)d_l - (W-S)d_u$ and $(R-W)d_u$, respectively.

Definition 2. For a given supply q, the utility function $u(d,q)$ is given as follows:

$$u(d,q) = \begin{cases} \left(\frac{q}{d_u} - C(q)\right)\frac{d-d_l}{q-d_l} + (1+e)C(q); & d \le q \\ \frac{q}{d_u} + eC(q); & d > q \end{cases}, \quad (11)$$

$$C(q) = \frac{(R-S)d_l - (W-S)q}{(R-W)d_u} \quad (12)$$

$u(d,q)$ is used to represent the utility obtained by making a decision of supply q in the case of the demand being d, which is illustrated in Fig. 2 where the three points A, B and C correspond to the utilities obtained in the three cases, that is when supply is q and the demand are d_l, q and d_u, respectively. The coordinates of A, B and C are $A = (d_l, (1+e)C(q))$, $B = (q, \frac{q}{d_u} + eC(q))$ and $C = (d_u, \frac{q}{d_u} + eC(q))$. Parameter e makes the worst case of retailer's profit be considered.

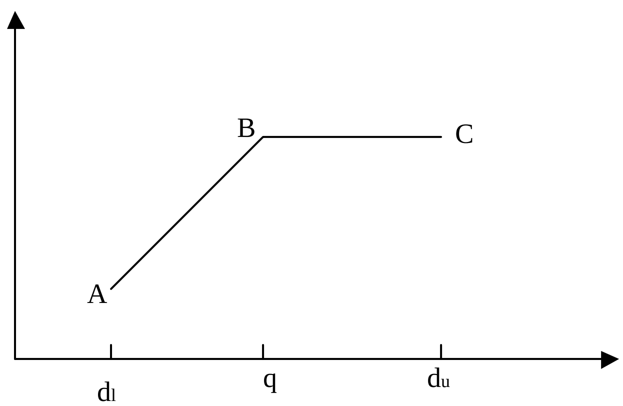

Fig.2 Utility function

Theorem 1. Optimal supply q_o^* based on the optimistic criterion (I) is

$$q_o^* = \frac{B2}{B1}, \quad (13)$$

where

$B1 = (2d_u - d_c)(R-W) - e(d_u - d_c)(W-S)$,

$B2 = d_u^2(R-W) - e(d_u - d_c)d_l(R-S)$,

$0 \le e < \frac{R-W}{W-S}$.

Theorem 2. Optimal supply q_p^* based on the pessimistic criterion (II) is

$$q_p^* = \frac{B4}{B3}, \quad (14)$$

where

$B3 = (d_u + d_c - d_l)(R-W) + e(d_l - d_c)(W-S)$,

$B4 = d_u d_c(R-W) - ed_l(d_c - d_l)(R-S)$,

$0 \le e < \frac{R-W}{W-S}$.

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