COMPOSING OFFER SETS TO MAXIMIZE EXPECTED PAYOFFS

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Recommended Citation
Atahan, Pelin; Johar, Monica; and Sarkar, Sumit, "COMPOSING OFFER SETS TO MAXIMIZE EXPECTED PAYOFFS" (2016). Research Papers. 111.
http://aisel.aisnet.org/ecis2016_rp/111
COMPOSING OFFER SETS TO MAXIMIZE EXPECTED PAYOFFS

Research

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Abstract

Firms are increasingly using clickstream and transactional data to tailor product offerings to visitors at their site. Ecommerce websites have the opportunity, at each interaction, to offer multiple items (referred to as an offer set) that might be of interest to a visitor. We consider a scenario where a firm is interested in maximizing the expected payoff when composing an offer set. We develop a methodology that considers possible future offer sets based on the current choices of the user and identifies an offer set that will maximize expected payoffs for an entire session. Our framework considers both the items viewed and purchased by a visitor and models the probability of an item being viewed and purchased separately when calculating expected payoffs. The possibility of a user backtracking and viewing a previously offered item is also explicitly modelled. We show that identifying the optimal offer set is a difficult problem when the number of candidate items is large and the offer set consists of several items even for short time horizons. We develop an efficient heuristic for the one period look-ahead case and show that even by considering such a short horizon the approach is much superior to alternative benchmark approaches. Proposed methodology demonstrates how the appropriate use of information technologies can help e-commerce sites improve their profitability.

Keywords: Ecommerce, data analytics, personalization, probability theory, target marketing
1. Introduction

The World Wide Web has brought a major change in the retail and financial sectors by enabling consumers to make purchases and carry out financial transactions over the Internet. The buying and selling of products and services over the Web (commonly referred to as ecommerce) is now a part of everyday life for millions of people. Consequently, ecommerce has become a critical component of the strategic plans of retail organizations (Mulpuru et al. 2011). An important byproduct of ecommerce activities is the amount of detailed data that is routinely captured by firms when conducting business over the internet. Two important kinds of such data are customer order data and web clickstreams of customers. These data are a rich source of knowledge about product preferences and customer buying behavior. The drastic reduction in the costs of information technology coupled with advancements in database technologies has significantly changed the economics of collection, storage, and processing of data. Retailers are now focused on understanding and leveraging such datasets in order to effectively target customers, improve repeat purchase, and drive cross-sell (Kalakota 2011). Kauffman et al. (2012) observe that the success of ecommerce operations will depend on data and information, and how they are used to optimize operations, drive sales and marketing, and grow the business.

In recent years there has been a considerable amount of activity by firms to leverage clickstream and other types of data. For example, some firms are using analytics to track which products are moving well and which are not, and determining what are the drivers for such performances (Angel 2012). These kinds of analysis help identify how to enable a smooth transaction conversion process. The goal in such applications is to determine ways to make a site engaging to a customer by providing the customer the information needed to complete a purchase as easily as possible.

Given the fine granularity of data available to ecommerce firms, it is now possible for firms to use business analytics to assist customers at the individual level. To do that, firms must understand a customer’s behavior at their site, and identify ways to improve the customer’s experience and thereby increase the probability of a sale. While the nature of the firm’s actions may be similar across customers, the specific action itself should be tailored to each customer’s actions. Therefore, the firm needs to make real-time decisions on what information to provide to a customer by taking into account the customer’s actions. The decisions need to consider a variety of factors that impact purchase behavior such as product popularities, product choices, product prices and margins, etc. While the benefits accrued from each individual decision may be small, given the repetitive nature of these decisions, a firm can considerably improve its bottom line by using analytics to improve such decisions. In this paper, we discuss in the context of a customer’s session at a site, a particular scenario that is repetitive in nature and that can be positively impacted by the appropriate use of analytics.

When a user visits a firm’s website, the visitor’s session usually consists of several actions within the site. The specific actions include, for example, clicking on an offered link to view additional information on an item, placing an item in a shopping cart, using a search tool, using a back button to revisit a page, etc. The firm (site) responds to each action of the visitor by presenting information as requested by the visitor. When the visitor clicks on a link, in addition to providing the information desired by the visitor the site can provide on the delivered page (that we subsequently refer to as an interaction) one or more additional links to items that the site would like to offer to the visitor. The visitor then has a choice to either examine (view) one of the recommended items, use the back button to examine an item offered in a previous interaction, conduct a search, or just quit the site. This process is iterative in nature, and repeats with each action (click) of the visitor. As long as the visitor stays on the site, the firm has the opportunity to repeatedly offer items based on the actions of the visitor.

It is usually the case that a site will display links to multiple items in each interaction (we refer to the set of such items as an offer set). This is because there is usually enough space on a web page to provide links to multiple items without severely constraining the amount of information displayed about the item currently being examined (or purchased) by the visitor. By displaying links to multiple items, the site hopes to increase the chance that the visitor will find at least one of those items to be
interesting, thereby increasing the probability the visitor will make a purchase. We examine the problem of composing an offer set that maximizes the expected payoff to a firm in the course of the entire session. Our analytics uses probability calculus as that enables us to do a rigorous cost benefit analysis.

We identify the nature of data and associated analytics needed to accomplish this objective. We identify three kinds of parameters associated with each item as central to this analysis. One is the profit margin of each item. In addition, we consider two probability parameters for each item, the viewing probability and the purchase probability based on a customer’s history (that includes both items viewed as well as items purchased). We consider these probability parameters separately because they correspond to distinct actions of customers, and the former action does not imply the latter; while there is a considerable amount of work on determining offer sets with a view to maximize sales (Breese et al. 1998, Ansari et al. 2000, Zaiane 2002, Ariely et al. 2004, Deshpande and Karypis 2004, Adomavicius and Tuzhilin 2005, Sandvig et al. 2007, Bodapati 2008), this body of research does not consider synthesizing the knowledge from past viewing as well as purchase behavior. A product, that is not as popular for viewing as other alternative products, may have a relatively higher conversion rate once it has been viewed. For example, for a customer searching for a camera, if a very specialized Hasselblad camera is offered along with a Canon camera, while the vast majority of customers may examine the Canon camera closely, the customers who examine the Hasselblad camera may be more predisposed to purchase it compared to the other customers’ likelihoods of purchasing the Canon camera.

In order to determine the full impact of an offer set, it is necessary to consider not only the likelihood of the customer purchasing one of the offered items, but to also evaluate how it may impact the customer’s choices at future interactions. For instance, if a customer chooses to examine an item from a current offer set by clicking on the corresponding link, the site would provide another offer set along with the information on the focal item. The site can pre-compute the best offer set for each selection at every feasible future interaction. By pre-computing such offer sets for future interactions the site can determine which offer set in the current interaction has the best long-term benefits. To our knowledge, extant research has not considered the expected payoff from future interactions with a customer when determining the current offer set.

Despite the amount of data typically available to a site, and the improvements in processing speed, the problem of determining an optimal offer set is nevertheless a very difficult one. First, it is usually not possible to estimate all the probability parameters needed for a comprehensive analysis of this problem that explicitly considers every feasible action a customer may take. This is because of the exponentially large number of feasible actions (item history of choices) of a customer as well as the possible combinations of items that could be offered by the site in the current and future offer sets. It is unlikely that a firm would have enough available data to robustly estimate each probability parameter for every feasible item in an offer set for all feasible item histories. Second, the potential payoff from including an item in the offer set depends not only on the properties of the item itself, but also on the other items included in an offer set. We show that an item that is part of the optimal offer set of a specified cardinality (say $r$) is not guaranteed to be part of the optimal offer set of a greater cardinality (e.g., $r + 1$). Finally, the number of feasible offer sets for future interactions also grows exponentially with the number of items a firm has in stock. This makes the exhaustive computation of the long-term payoff computationally intractable for real-time feedback.

To circumvent these constraints, we make certain assumptions to enable the composition of offer sets in real time. We show that by making appropriate independence and conditional independence assumptions a site can obtain a parsimonious set of the probability parameters that will enable it to efficiently calculate the expected payoff directly from items included in a feasible offer set. To reduce the computational burden of estimating the long term implications of an offer set (based on future viable offer sets and associated interactions), we restrict our attention to a single period look-ahead model. The proposed heuristic is compared with two benchmark approaches by simulating consumer behavior using data from a real ecommerce site. We find that relative to the benchmarks, the proposed approach can improve profitability by over 33% while at the same time reducing the search effort (i.e., number of clicks leading to a purchase) by over 40%.
2. Problem Description

Many firms provide targeted recommendations to visitors in an attempt to improve their visitors’ experiences and achieve higher conversion rates. Firms analyze user behaviors such as a visitor’s browsing and purchase history to infer user preferences (Koren et al. 2009). This can be particularly useful in determining a visitor’s interests for the current session. By examining the set of items previously viewed or purchased by a visitor (that we refer to as the visitor’s item history, or IH in short), and analyzing purchase patterns of other visitors who have the same (or similar) item histories, a firm can identify items that are more likely to be of interest to the focal visitor. In our work, we also consider using items viewed or purchased by a visitor within a session when selecting items for recommendation.

In this paper, we study how firms can maximize their expected payoffs from product recommendations they make to their customers. We first develop a framework that firms can use to evaluate the expected payoff from their recommended offer set to a visitor. A unique feature of our framework is that we model the viewing and purchasing probabilities separately when we evaluate the expected payoff from the offer set.

If a firm could look forward and calculate the expected payoffs from future actions of a user, the firm could guide the user towards potentially more profitable paths by carefully selecting the options to make available to the user at the current interaction. As the choices a user makes may depend on the user’s previous choices, modeling future actions of a user requires careful modeling of the user’s current choice. We extend our framework to capture expected payoffs from multiple periods. An interesting feature when modeling the choice of a visitor is the potential impact an item has on the probability of the visitor selecting other items in the offer set. In a web environment a user has the option of clicking on the back button to evaluate items offered in previous interactions. Therefore, the choice model should not only include the currently offered items, but also previously offered items. Items a user was exposed to in previous interactions may also impact the probability of the visitor selecting other items in the offer set.

A variety of data mining techniques are being deployed for making product recommendations; popular techniques include collaborative filtering, association rule mining, Bayesian networks, and decision trees, among others (Jannach et al. 2011). The proposed framework for evaluating expected payoffs is general and can be used with any of the above mentioned techniques as long as they enable the firm to estimate the relevant probability parameters. In this paper, we use association rules to illustrate how a visitor’s item history can be used to provide personalized offer sets. Association rules are convenient for several reasons. These rules explicitly capture important associations between items, and can be used to directly provide the probability a visitor may view (or purchase) a related target item given the visitor’s item history. The rules can be easily pre-computed from clickstream data using standard available techniques (e.g., Apriori or its extensions) to provide the probability parameters needed to calculate expected payoffs.

In Section 2.1, we show how expected payoff from an offer set can be calculated with a parsimonious set of parameters. In order to evaluate expected payoffs from multiple periods we need to model user choice. Section 2.2 presents how a user’s choice can be modeled with and without considering the user backtracking to an item in a previous offer set. Then, in Section 2.3, we show how expected payoffs can be calculated considering future interactions.

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1 In some circumstances, it may be possible for a firm to estimate such probabilities (conditioned on the visitor’s item history) directly from available data. However, if the number of possible items to consider is large, and the amount of data available is voluminous, it will not be feasible to obtain all the needed probability estimates in real time. Regardless of whether these probabilities are pre-computed or estimated in real time, and regardless of how they are obtained (e.g., using Bayesian techniques or collaborative filtering or association rules), the methodology to compose offer sets will not be affected.
2.1. Expected Payoff from an Offer Set

We first illustrate how to evaluate the expected payoff from an offer set using a simple example. We discuss the difficulty associated with estimating all of the parameters needed to calculate the expected payoffs for different feasible offer sets in general. We then show how, by making certain conditional independence assumptions, it is possible to compute the expected payoffs for any feasible offer set.

Consider the case where a firm offers a visitor, with item history IH, links to two items \( i_1 \) and \( i_2 \). For a given offer set \( O = \{i_1, i_2\} \), the site could determine the probabilities of the visitor clicking on one or both of the offered links, the probabilities of the visitor subsequently purchasing the associated items, and the probability of the visitor ignoring the offer set. Given the offer set, let \( P(v_1,v_2|IH,O) \), \( P(v_1,\overline{v}_2|IH,O) \), \( P(\overline{v}_1,v_2|IH,O) \), and \( P(\overline{v}_1,\overline{v}_2|IH,O) \) correspond to the probabilities associated with the visitor viewing information on both the items, viewing item \( i_1 \) but not \( i_2 \), viewing item \( i_2 \) but not \( i_1 \), and viewing neither item, respectively. Further, let \( P(s_i|v_1,v_2, IH,O) \) and \( P(s_i|v_1,\overline{v}_2, IH,O) \) be the probabilities associated with the visitor purchasing item \( i_1 \) after viewing both the items given the offer set, and after viewing item \( i_1 \) but not \( i_2 \) given the offer set, respectively (we assume that a visitor cannot purchase an item without first clicking on the associated link to view information on that item). Similarly, \( P(s_i|v_1,\overline{v}_2, IH,O) \) and \( P(s_i|\overline{v}_1,v_2, IH,O) \) are the probabilities associated with the visitor purchasing item \( i_2 \) after viewing both the items given the offer set, and after viewing item \( i_2 \) but not \( i_1 \) given the offer set, respectively. Finally, let \( \omega_i \) be the profit realized from the sale of item \( i_j \). The expected payoff from the offer set can be written as

\[
EP(O) = [P(s_i|v_1,v_2, IH,O) P(v_1,v_2|IH,O) + P(s_i|v_1,\overline{v}_2, IH,O) P(v_1,\overline{v}_2|IH,O)] \omega_1
+ [P(s_i|v_1,\overline{v}_2, IH,O) P(v_1,\overline{v}_2|IH,O) + P(s_i|\overline{v}_1,v_2, IH,O) P(\overline{v}_1,v_2|IH,O)] \omega_2.
\]

When offer sets of size two are considered, and each possible offer set of size two have been offered many times, it may be possible for a firm to estimate all the desired probabilities from the sites log files. However, when offer sets of larger cardinality are considered, estimation of all the desired probabilities is non-trivial. First, the number of probability parameters that the site would need to estimate grows exponentially even for a single offer set. More importantly, it is unlikely that every feasible combination of items of the desired cardinality would have been offered in the past; the number of such combinations also grows exponentially with the cardinality of the offer set. In principle, each such offer set should have been offered multiple times to obtain reliable estimates of the desired parameters. For instance, if there were a thousand candidate items, and offer sets of cardinality five were considered, the number of possible combinations would be \( 8.25 \times 10^{12} \). As a result, it will usually not be possible for a firm to explicitly estimate all the necessary probabilities from their log files to determine the expected payoffs from an offer set, and then to compare different offer sets.

Given such practical considerations, we examine how to capture the essence of the problem with as parsimonious a set of probability parameters as possible. A parsimonious set of parameters, which may still enable meaningful evaluation of the expected payoff is to consider parameters associated with individual items (and disregard higher order distributions), i.e., consider parameters of the form \( P(v_j|IH) \) and \( P(s_i|v_j, IH) \). These parameters can be more easily estimated from a site’s log files. These probabilities are conditioned on the items previously viewed or purchased and capture the important associations between items.

We illustrate how association rules could be used to obtain these probabilities. For example, if a visitor has viewed items \( i_1 \) and \( i_2 \), and there exists a mined rule of the form \( \{v_1,v_2\} \rightarrow v_i \), this rule would provide the conditional probability \( P(v_i|v_1,v_2) \) which may be more contextually relevant than considering the marginal probability \( P(v_i) \) in the expected payoff calculations. Corresponding rules could then be mined to obtain contextualized purchase probabilities. For instance, given the above rule, a rule of the form \( \{v_1,v_2,v_j\} \rightarrow s_j \) would yield the probability estimate that item \( i_j \) is purchased by a visitor who has viewed the items \( i_1, i_2, \) and \( i_j \), i.e., \( P(s_j|v_1, v_2, v_j) \).

Then, the expected payoff from offer set \( O = \{i_1, i_2\} \) can be computed in the following manner:

\[
EP(O) = P(s_1|IH, v_1) P(v_1|IH) \omega_1 + P(s_2|IH, v_2) P(v_2|IH) \omega_2.
\]
This formulation captures the expected payoffs from the items displayed in the given offer set for the user’s session, i.e., this accounts for the visitor purchasing one or more of those items during that session.

The general equation for calculating expected payoffs from an offer set with multiple items is:

\[ P(O|IH) = \sum_{i \in O} P(s_i|v_j, IH)P(v_j|IH)\alpha_j \]  

(1)

2.2. Modelling user’s choice

Items viewed by a visitor determine the navigational path of the visitor, so it is important to carefully model the likelihood of a visitor viewing an item when presented with a set of choices. We first show how a user’s choice can be modeled without considering the possibility of the user backtracking to an item that was offered in a previous interaction. Then, we show how backtracking to a previously offered item can be incorporated in this model.

2.2.1. User’s choice without backtracking

We illustrate how a visitor’s choice can be modeled with a simple example. Consider the situation where a visitor with item history \( IH \) is presented with links to two items \( i_1 \) and \( i_2 \) (offer set \( O = \{i_1, i_2\} \)). If we ignore the possibility of the visitor backtracking to a previously offered item, the visitor has the following options: view item \( i_1 \), view item \( i_2 \), and not view any of the items, i.e., ignore the offer set. When estimating a visitor’s likelihood of selecting an item to view from an offer set, we consider the relative interestingness of items in the offer set – i.e., we factor the other items in the offer set when estimating the probability that the visitor would find an item \( i_j \) in the offer set to be interesting.

Let a visitor’s likelihood of viewing information on item \( i_1 \) when it is a part of the offer set \( O = \{i_1, i_2\} \) be denoted by \( P(v_1|IH, O) \). To determine this probability, we first determine the probability the visitor will view item \( i_1 \) conditional on the event that the visitor views one of the offered items (that we refer to as the event \( V \)).

\[ P(v_1|IH, O, V) = \frac{P(v_1|IH)}{P(v_1|IH)P(v_2|IH)} \]  

(2)

The conditioning event \( V \) in the above expression captures the assumption that at least one of the offered items is found interesting (and viewed). \( P(v_2|IH, O, V) \) can be calculated in a similar manner. The unconditional probability that a visitor will view item \( i_1 \) (i.e., without assuming the visitor views an offered item) is then obtained as

\[ P(V|IH, O) = P(v_1|IH, O, V)P(V|IH, O) \]  

(3)

Where \( P(V|IH, O) \) is the probability the visitor will view at least one of the items in the offer set \( O \). This term can be estimated by considering its negation, which is the situation where the visitor does not view either item. The probability the visitor ignores all the items is interpreted as the probability that the visitor is not interested in any of the items. We refer to this as the ignoring probability. The ignoring probability, denoted by \( P(\emptyset|IH, O) \), can be computed as

\[ P(\emptyset|IH, O) = 1 - P(v_1|IH) - P(v_2|IH) - P(v_1, v_2|IH) \]

When the offer sets consist of more items, the number of joint probabilities and therefore the number of rules needed to determine \( P(\emptyset|IH, O) \) would grow exponentially. In such cases, for tractability considerations the site may assume the probability a visitor finds item \( i_1 \) to be interesting to be conditionally independent of the probability the visitor finds item \( i_2 \) to be interesting given the item history. The above joint probability can then be computed as product of the marginal probabilities, i.e.,

\[ P(v_1, v_2|IH) = P(v_1|IH)P(v_2|IH) \]

Then the ignoring probability can be written as

\[ P(\emptyset|IH, O) = \prod_{i\in O}(1 - P(v_i|IH)) \]  

(4)

While the conditional independence assumption may appear to be limiting, the assumption has been widely used and been found to be robust for obtaining probability estimates in many applications (Domingos and Pazzani 1997). This assumption impacts a visitor’s likelihood of viewing at least one of the items from the offer set. Therefore, the exact likelihood of viewing an item may be higher or
lower for all the items, but the relative interestingness of the items is unaffected by this assumption. As a result, the externality an item has on the other offered items is still captured.

2.2.2. User’s choice with backtracking

For a more comprehensive choice model, we incorporate a user’s ability to backtrack to a previously offered item. In our formulation, the items that were offered in previous offer sets \( O^k \), but not viewed by the user are treated as *virtual links* in the current interaction. Here, the superscript \( k \) stands for how many offer sets back the item was offered. Technically, the user can click on the back button to go back as many times as he or she has traversed so far. However, as a user keeps browsing and is presented with more and more offer sets, it may be difficult for the user to remember all the previously offered items. As a result, it will be unlikely that a user remembers the offers presented many clicks back and tracks back several clicks. So when considering the back button, the site may employ a rolling window approach in order to decide offer sets from how far back in a user’s browsing session should be accounted for in the user’s choice problem. For instance, if a site is considering a window size of three, then the site may consider links from only the three most recent offer sets as virtual links when modeling the user’s next choice.

We refer to the list of items that a visitor can view at a given interaction as the choice set \( (CS) \). The choice set includes the items in the current offer set and the items that were not viewed in the previous offer sets within the chosen window.

Remembering the previously offered items and comparing them with the newly offered items requires additional cognitive effort by the user. Additionally, clicking the back button to go back to the previous offer sets requires further effort by the user. Therefore, the likelihood of clicking on a virtual link, i.e., likelihood of going back and viewing an item, is discounted by reducing the odds ratio of clicking the virtual link by a *forget factor* of \( \delta^k \), where \( 0 < \delta^k \leq 1 \) and \( \delta^k \leq \delta^j \) for \( \forall k \). Note that the forget factor depends on how many offer sets back the item was offered. If, an item has appeared in multiple previous offer sets, the most recent appearance in the chosen window can be considered.

The *discounted* probability for a virtual link can be calculated by multiplying its odds ratio by the corresponding forget factor. For a virtual link \( i_v \), to calculate the discounted viewing probability with a forget factor \( \delta^2 \), we first calculate the modified odds ratio (MOR) in the following manner:

\[
MOR = \frac{P(v_i|IH)}{1-P(v_i|IH)} \times \delta^k
\]  

(5)

Then, the discounted probability of viewing the item \( P^d(v_i|IH) \) can be calculated as:

\[
P^d(v_i|IH) = \frac{MOR}{1+MOR}
\]  

(6)

Consider that a visitor, who has viewed item \( i_v \) when presented with items \( i_1 \) and \( i_2 \) in the previous interaction \( O^1 = \{i_1,i_2\} \), is now presented with offer set \( O = \{i_3,i_4\} \). The choice set for this user consists of the current offers and previously offered items that were not viewed and therefore \( (CS = \{i_2,i_3,i_4\}) \). This visitor may view one of the items in the current offer set, or backtrack and view item \( i_2 \) from the previous offer set or ignore the choice set. We may determine the probability that the user will backtrack and view item \( i_2 \) given the choices available to the user \( P(v_2|IH,CS) \), by again first conditioning it on the event that the user will view at least one of the items in the choice set.

\[
P(v_2|IH,CS,V) = \frac{P^d(v_2|IH)}{P^d(v_2|IH) + P(v_2|IH) + P(v_4|IH)}
\]

In this formulation, the discounted probability of viewing item \( i_2 \) will be used. The probability of ignoring the choice set can be calculated as follows:

\[
P(\emptyset|IH,O) = (1 - P^d(v_2|IH))(1 - P(v_2|IH))(1 - P(v_4|IH))
\]

Then, the unconditional probability of viewing item \( i_2 \) can be calculated as before

\[
P(v_2|IH,CS) = P(v_2|IH,CS,V)P(V|IH,CS),
\]

where \( P(V|IH,CS) = 1 - P(\emptyset|IH,O) \).

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\( ^2 \) This parameter can be estimated from log files by evaluating how many times visitors click on a link in the current offer set, how many times they backtrack one offer set, backtrack two offer sets, and so on.
The probability of viewing $i_3$ and $i_4$, given the current choice set can be calculated in a similar fashion.

Links not viewed in the previous offer sets can be offered again as part of the new offer set. Reoffering a previously offered item may have a positive impact on a user’s likelihood of viewing the item. This impact is similar to displaying advertising where the likelihood of reacting to an ad depends upon the number of times the ad was presented to the user. Therefore in our analysis, the likelihood of viewing an offered item that was also offered in a previous interaction is reinforced by increasing the odds ratio of viewing such an item by a factor of $\gamma^l$, where $\gamma^l \geq 1$ and $\gamma^{l+1} \geq \gamma^l$, for $\forall l.$ Here, the superscript $l$ denotes how many times the item appeared in the previous offer sets within the session.

The reinforced probability for a reoffered link can be obtained by following a procedure similar to how the discounted probability was obtained. We first multiply the odds ratio of the reoffered link with the appropriate reinforcement factor, and then calculate the reinforced probability of viewing based on this modified odds ratio.

Consider that in the previous example, the visitor is offered $O = \{i_2, i_3\}$, after having viewed $i_1$ in the first interaction. The choice set available to the visitor then is $CS = \{i_2, i_3\}$. We can calculate the probability the visitor will view the reoffered item $i_2$ again by first conditioning it on the event that the visitor will view at least one of the items in the choice set.

$$P(v_2|IH, CS, V) = \frac{P^r(v_2|IH)}{P^r(v_2|IH) + P(v_2|IH)}$$

This time, since item $i_2$ is reoffered, the reinforced probability of viewing item $i_2$, $P^r(v_2|IH)$, will be used. The other probability calculations will remain the same.

### 2.3. Expected Payoff considering future interactions

We formulate the expected payoff over multiple periods in the following manner.

$$EP(CS|IH) = \sum_{i \in CS} P(v_i|IH, CS) \left( P(s_j|IH, v_j) \alpha_j + EP(CS^{+1}|IH^{+1}) \right)$$

(7)

The first term in the above equation is the likelihood of viewing an item in a choice set. How to determine this probability was presented in Section 2.2. Then given an item is viewed from a choice set, the firm may realize some payoff if the user purchases the item, and the firm will have some expected payoffs from the next period choice set ($CS^{+1}$). The next period expected payoff $EP(CS^{+1}|IH^{+1})$, will also include a corresponding expected payoff term to capture the payoffs from the following period, and the item history in that period will now include the item user has viewed in the previous period.

### 3. Determining Good Offer Sets

Consider a firm attempting to maximize the expected payoffs from an offer set consisting of $n$ items. A straightforward way to identify the optimal offer set would be to evaluate the expected payoffs from all possible offer sets. Exhaustive evaluation of all possible offer sets would require evaluating all possible combinations over all the periods considered, which will usually not be feasible in real time. To obtain the optimal offer set without exhaustive enumeration, we ideally need a way to identify local properties of items or intermediate solutions that characterize optimal solutions. We find that the problem of identifying the optimal offer set is difficult even for the first period, where we assume the firm knows expected payoffs from all future offer sets. Therefore we consider a one-step look ahead problem and develop a heuristic approach to solve that problem.

### 3.1. Difficulty of the Problem

The expected payoff from a choice set in a given period depends not only on the subsequent choice set, but also on the previous choice set because of the dependence of probability parameters on item history and the possibility of the visitor backtracking to previously offered items. Therefore, the offer

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3 This parameter can be estimated from log files by evaluating how many times visitors view an item after it is presented to them once, how many times they view it after being presented twice, and so on.
set in any given period cannot be determined in isolation. Consequently, the number of combinations that needs to be examined will grow exponentially with the number of periods considered.

Proposition 1 below shows how growing an optimal offer set incrementally does not guarantee optimality for the eventual offer set even when future payoffs are ignored.

**Proposition 1:** An item that is part of the optimal offer set of cardinality \( r \) is not guaranteed to be part of the optimal offer set of cardinality \( r + 1 \).

**Proof:** The proof is by counter example and for simplicity we just show it for \( r = 2 \). Consider a site recommending items to a visitor. Four items with the probability parameters shown in Table 1 are assumed to be available to the site. The profits \( \omega_j \) are assumed to be equal to one and expected payoff from next period expected is ignored for all candidate items.

| Items | \( P(v_j|IH) \) | \( P(s_j|v_j, IH) \) |
|-------|-----------------|-----------------|
| \( i_1 \)  | 0.70            | 0.32            |
| \( i_2 \)  | 0.60            | 0.35            |
| \( i_3 \)  | 0.45            | 0.39            |
| \( i_4 \)  | 0.52            | 0.37            |

*Table 1. Candidate items for Proposition 1 Example*

The optimal offer set of cardinality two is \( O = \{i_1, i_2\} \), with an expected payoff of 0.2938. The optimal offer set of cardinality three is \( O' = \{i_2, i_3, i_4\} \) with an expected payoff of 0.3292. Although item \( i_1 \) is part of the optimal offer set of cardinality two, it is not part of the optimal offer set of cardinality three.

This finding is due to the fact that the change in the expected payoff by adding an item to the offer set depends not only upon the probability parameters associated with the item itself, but also on the probability parameters of the other items already in the offer set. The expected payoff from an offer set \( O \) attributed to an item \( i \) in the offer set explicitly considers the interestingness of all the other items in the offer set. Thus, depending on which other items are included in the offer set, an item’s likelihood of being viewed and then purchased will also be different. For instance, if the overall viewing probability of the current offer set is low, then an item with a viewing probability higher than another item may be preferred even when the purchase probability is considerably lower than the latter. As more items are added to the offer set, the overall viewing probability may become higher and replacing such an item could lead to an increase in expected payoffs. This is, in fact, true regardless of whether the items are conditionally independent or not. Therefore, the problem of identifying the optimal offer set in a given period is difficult even for the case where the firm already knows expected payoffs from future offer sets. Another implication of this proposition is that greedy approaches that select the items with highest expected payoffs (individually) do not guarantee optimality.

### 3.2. Proposed Approach

We consider a one-step look ahead model, where we calculate expected payoffs for the second period using Equation 1. The one-step look ahead model we consider is:

\[
EP(CS|IH) = \sum_{i \in CS} P(v_j|IH, CS) \left( P(s_j|IH, v_j) \omega_j + EP(O^{+1}|IH^{+1}) \right)
\]

Using Equation 1, the second period offer set can be determined exhaustively as the contribution of an item to an offer set can be evaluated individually without considering combinations of items. However, as illustrated in Section 3.1, it is difficult to determine the first period offer set even for the case where the second period expected payoff is known. For efficiency considerations, adding items iteratively while growing the offer set to the target size is desirable.

The proposed algorithm first determines the optimal offer set for the second period for each candidate item in the current offer set assuming that the candidate item is clicked in the current period. Then, given the second period expected payoff for each candidate item, the first period offer set is
determined by adding items iteratively while growing the offer set to the target size. This approach calculates the expected payoffs for offer sets of intermediate cardinalities by considering each of the remaining items individually to add to the offer set and select the item whose addition leads to the highest expected payoff. This approach will, in each iteration, pick the locally optimal item, and continue in this manner until $n$ items are identified.

However, as discussed in Proposition 1, an item that is part of the optimal solution for a given cardinality may not be part of the optimal solution for a larger cardinality. At smaller cardinalities (earlier iterations), when the ignoring probability is high, items with higher viewing probabilities might lead to higher expected payoffs. These items might not be a part of the optimal offer set of a larger cardinality. In order to replace suboptimal items that might have been included in the earlier iterations, at each cardinality, the oldest item is removed and each of the remaining candidate items are considered as replacements. If such a replacement cannot improve on the expected payoff, the original item is brought back in the offer set. Otherwise, the replacement that leads to the highest expected payoff is included instead.

The complexity of the proposed algorithm for determining the offer sets in the second period is $O(K^2 \log(K))$ and for determining the offer set in the first period is $O((2^n-1)K) \sim O(nK)$, where $n$ is the cardinality of the offer set and $K$ is the total number of candidate items. The overall complexity will be in the order of $K^2$.

<table>
<thead>
<tr>
<th>Proposed Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inputs</strong></td>
</tr>
<tr>
<td>- List of $K$ candidate items and offer set cardinality $n$</td>
</tr>
<tr>
<td>- Item related parameters (i) $P(v_j</td>
</tr>
<tr>
<td>- $\delta$- forget factor and $\gamma$- reinforcement factor</td>
</tr>
<tr>
<td>- $O^1 \rightarrow HI$ - list of items that were not viewed by the user in the previous</td>
</tr>
<tr>
<td><strong>Steps</strong></td>
</tr>
<tr>
<td>1. Initialize the offer set to $\emptyset$, and choice set to $O^1 \rightarrow HI$ (items offered in the previous offer set, but not viewed)</td>
</tr>
<tr>
<td>2. Repeat for each item $i_j$ //Estimating the expected payoff from the next period</td>
</tr>
<tr>
<td>a. Assume that item $i_j$ was the last item viewed in the IH</td>
</tr>
<tr>
<td>b. Calculate expected payoffs for each remaining item if that item was the only offered item</td>
</tr>
<tr>
<td>c. Select top $n$ items with the highest expected payoffs and calculate the expected payoff from that offer set by summing the expected payoffs from each item</td>
</tr>
<tr>
<td>d. Store the expected payoff as next period expected payoff for that item</td>
</tr>
<tr>
<td>3. Repeat steps 4 and 5 until $n$ items are identified //This is where we start creating the offer set</td>
</tr>
<tr>
<td>4. For each item that is not part of the offer set,</td>
</tr>
<tr>
<td>a. Evaluate the expected payoff from the choice set if that item was added to the current offer set</td>
</tr>
<tr>
<td>b. Add to the offer set the item that leads to the highest expected payoff</td>
</tr>
<tr>
<td>5. If $</td>
</tr>
<tr>
<td>a) Remove the earliest added item from the offer set</td>
</tr>
<tr>
<td>b) For each remaining item, evaluate the expected payoff from the choice set if that item was added to the current offer set</td>
</tr>
<tr>
<td>c) Add to the offer set the item that leads to the highest expected payoff</td>
</tr>
</tbody>
</table>

4. Experiments

We conducted simulated experiments to compare the performance of the proposed approaches with alternative benchmark approaches. We use clickstream data from an ecommerce site (Gazelle.com), which was a legwear and leg care web retailer. The data was made available by Blue Martini Software for the data mining competition KDD Cup 2000. We had access to about two weeks of clickstream data from April 14th 2000 to April 30th 2000. The data tracked user clickstreams at the page request level, which included the URL for each request, and it was organized by sessions. Parsing the URLs we were able to figure out what kind of page the user viewed. The types of pages include category
page, specific product page, and shopping cart to name a few. We did not have access to the actual order information. If the user visited a shopping cart page right after visiting a product page, we assumed the user added the product to the shopping cart and treated it as a purchase in our experiments.

We cleaned the data to keep only clicks related to product views and purchases in each session. We removed data from shopbots and single item view sessions. We ended up with 6848 sessions related to 302 unique products, 15025 product views and 2181 purchases. We used WEKA data mining software to mine the rules in order to obtain probabilities based on clickstream histories. We mined rules based on views to obtain probability parameters associated with views. Then for each of these probability parameters we directly estimated the probability of purchase of the corresponding item, given the items in the antecedent are viewed. For instance, if we obtained the rule $P(v_1|v_2)$ we also estimated $P(s_1|v_1,v_2)$ from the data and smoothed the probabilities using Laplace smoothing.

### 4.1. Experiment Design

We implemented the proposed one-step look ahead approach (referred to as Proposed) based on the algorithm presented in Section 3.2. The proposed approach in the experiments considers virtual links from one offer set back. We also implemented an approach that selects the optimal offer set based on the expected payoff calculation presented in Section 2.1 (referred to as Views&Purchases). The Views&Purchases approach considers the current offer set only, but models viewing and purchasing probabilities separately in the expected payoff calculation. It determines optimal offer set through exhaustive search.

We also considered two benchmark approaches: one that makes recommendations based on the purchase probabilities of the items (referred to as PurchasesOnly) and another one that makes recommendations based on viewing probabilities of the items (referred to as ViewsOnly). Both of these benchmarks utilize the item history of users similar to our approach. The benchmark approach PurchasesOnly ranks items based on their likelihood of being purchased given the current item history of the visitor and offers the top $n$ items to the visitor at each interaction. The benchmark ViewsOnly ranks items based on their likelihood of being viewed given the current item history of the visitor and offers the top $n$ items to the visitor at each interaction.

For tractability, all the approaches consider up to three items in the item history. If for an item there are no rules that have the most recent three items in the user’s item history in its antecedent, then we consider rules with the most recent two items in the history. If that is also not available we consider the most recent click, and if that is not available either, we consider the marginal probability for that item.

We simulate 5000 users and offer sets of cardinality 8 items. At each interaction, the approach that is being evaluated determines the offer set that will be made available to the visitor. Then the user’s click is simulated based on the choices available to the user – the current offer set and any offers that were not visited in the previous offer set. The simulation allows users to backtrack to the previous offer set. Each time the user views an item, the user’s purchase decision is simulated based on the relevant purchase probability. If the user purchases the product, the purchase is recorded. The user’s clickstream simulation is continued until the user ignores the choice set.

The profit for each item is set to one in our simulated experiments. The forget factor for virtual links ($\delta$) is set to 0.5 and the reinforcement factor for re-offered links ($\gamma$) is set to 2.$^5$

### 4.2. Experimental Results

Improving the profits is typically the primary goal for many e-commerce sites. Therefore, one of the metrics we use for comparing the performances of the individual approaches is the total profit. All the

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$^4$ There might be instances where a user may abandon the site without completing the purchase.

$^5$ We conducted experiments where we vary the forget factor and reinforcement factor and also offer set size to evaluate the sensitivity of results to these parameter. The results of these experiments (which are not presented for space considerations) are qualitatively similar.
approaches determine items of interest to a visitor by evaluating the item history of visitors. In the experiments, the users are assumed to be new users, where no item history is available and consequently all the users are offered the same initial offer set. Some approaches may be worse at generating the first click, but may help a user convert once the user views an item. Therefore, we use average profit per person among the visitors who have made a click as another performance metric. A third performance metric we use is the conversion speed, i.e., how quickly the different approaches lead a user to a purchase.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Total profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>ViewsOnly</td>
<td>176</td>
</tr>
<tr>
<td>PurchasesOnly</td>
<td>131</td>
</tr>
<tr>
<td>Views&amp;Purchases</td>
<td>205</td>
</tr>
<tr>
<td>Proposed</td>
<td>234</td>
</tr>
</tbody>
</table>

Table 2. Total Profit

Table 2 presents the total profit obtained based on each of the implemented approaches in these experiments. We find the total profit to be highest for the Proposed approach. The total profit for Proposed approach is 33% higher compared to ViewsOnly, 79% higher compared to PurchasesOnly, and 14% higher compared to the Views&Purchases approach. The Views&Purchases approach also does well in comparison to the benchmark approaches. Total profit for Views&Purchases is 16% higher compared to ViewsOnly and 56% higher compared to PurchasesOnly. Interestingly, the total profit for the ViewsOnly approach is higher (about 34%) than that for the PurchasesOnly approach.

Table 3 presents average profit per person among the users who have made a click along with some additional details on these experiments. The second column presents the probability of clicking on a link in the initial offer set. We see (not surprisingly) that a user is much more likely to make a click when presented with the initial offer set generated by the ViewsOnly approach (30%) in contrast to the offer set generated by the PurchasesOnly approach (14%). As a result, in our simulated experiments the number of users who made at least one click is 1454 and 718 for the ViewsOnly and PurchasesOnly approaches, respectively. Similarly, in these experiments, total clicks are also much higher for the ViewsOnly approach (13425) compared to PurchasesOnly approach (3184). Consequently, although the total profit is lower for the PurchasesOnly approach, the average profit per person among those who made a click is higher (0.18) for the PurchasesOnly approach compared to the ViewsOnly approach (0.14). Thus while the users may be less likely to click on a link in an offer set generated by the PurchasesOnly approach, they are much more likely to purchase an item once they make a click.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Initial Click Probability</th>
<th>Users who made a click</th>
<th>Total clicks</th>
<th>Average profit per person</th>
<th>Average clicks per person</th>
</tr>
</thead>
<tbody>
<tr>
<td>ViewsOnly</td>
<td>0.30</td>
<td>1454</td>
<td>13425</td>
<td>0.12</td>
<td>9.23</td>
</tr>
<tr>
<td>PurchasesOnly</td>
<td>0.14</td>
<td>718</td>
<td>3184</td>
<td>0.18</td>
<td>4.43</td>
</tr>
<tr>
<td>Views&amp;Purchases</td>
<td>0.27</td>
<td>1400</td>
<td>7447</td>
<td>0.15</td>
<td>5.32</td>
</tr>
<tr>
<td>Proposed</td>
<td>0.15</td>
<td>746</td>
<td>3735</td>
<td>0.31</td>
<td>5.01</td>
</tr>
</tbody>
</table>

Table 3. Average Clicks per Person

Interestingly, although the likelihood of an initial click probability is also low for the Proposed approach, the average profit per person and the total profit are substantially higher. The average profit per person is 0.31 for the Proposed approach, which is about 72% higher than that of the PurchasesOnly approach and 158% higher than the ViewsOnly approach.

Table 4 displays the average number of items a user viewed prior to the first purchase and how many users made a purchase. The fewest number of clicks (i.e., faster conversion) are made prior to the first purchase when the offer sets are composed by the Proposed approach. An interesting observation is that the average number of clicks to first purchase for the Proposed approach is even lower (by about 3%) than that for the PurchasesOnly approach. Not surprisingly, the largest number of
clicks required prior to first purchase (8.56, which is twice as many clicks as for the Proposed approach) occur when the offer sets are composed by the ViewsOnly approach.

<table>
<thead>
<tr>
<th></th>
<th>Average number of clicks to first purchase</th>
<th>Number of users who made a purchase</th>
</tr>
</thead>
<tbody>
<tr>
<td>ViewsOnly</td>
<td>8.56</td>
<td>163</td>
</tr>
<tr>
<td>PurchasesOnly</td>
<td>4.35</td>
<td>111</td>
</tr>
<tr>
<td>Views&amp;Purchases</td>
<td>5.46</td>
<td>181</td>
</tr>
<tr>
<td>Proposed</td>
<td>4.23</td>
<td>205</td>
</tr>
</tbody>
</table>

Table 4. Number of Clicks to First Purchase

Table 5 displays how many purchases were made in total and how many items were viewed prior to each purchase. The results are similar to the results for the initial purchase (note that there are not many multiple purchases in our experiments). Again the users view fewest items (4.04) prior to purchase when offer sets are based on the Proposed approach.

<table>
<thead>
<tr>
<th></th>
<th>Average number of clicks to each purchase</th>
<th>Number of purchases</th>
</tr>
</thead>
<tbody>
<tr>
<td>ViewsOnly</td>
<td>8.41</td>
<td>176</td>
</tr>
<tr>
<td>PurchasesOnly</td>
<td>4.20</td>
<td>131</td>
</tr>
<tr>
<td>Views&amp;Purchases</td>
<td>5.22</td>
<td>205</td>
</tr>
<tr>
<td>Proposed</td>
<td>4.04</td>
<td>234</td>
</tr>
</tbody>
</table>

Table 5. Number of Clicks to Each Purchase

5. Discussions and Conclusions

Our research has addressed an interesting and important problem in an ecommerce context – that of determining the composition of the offer set to provide to a visitor at each interaction. We show how data available to an ecommerce firm can be used to compare alternative offer sets in a decision-theoretic framework. The proposed approach considers both the items viewed as well as the items purchased by a visitor by modeling separately the probability of an item being viewed as well as the probability of an item being purchased when calculating expected payoffs. We extend our analysis to situations where a firm strategically composes an offer set by considering not only the payoff from the items offered in a current offer set, but also by taking into consideration the expected payoff from future interactions. We also consider the externality from previously offered items on the user’s choice.

We conduct simulated experiments using clickstream data available from an ecommerce site. Two variants of our approach are considered: the first variant considers only the use of probability estimates using viewing and purchase probabilities given a customer’s item history, while the second variant also looks ahead one-period to better incorporate the future implications of including an item in the current offer set. Benchmark approaches used for comparison include using either the viewing history or the purchase history for composing offer sets (as is typically done in extant literature). Both variants of our proposed approaches substantially outperform the benchmarks. The first variant demonstrates the improvements that can be achieved by synthesizing knowledge from historical viewing and from purchase behavior of customers, while the second variant demonstrates the additional value from strategically considering the implications of future offerings given a current offer set.
References


