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Ming-Hung Sung
Da-Yeh Institute of Technology

Kai-I Huang
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Hsin-Ginn Hwang
Northeast Louisiana University

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A FUZZY OBJECT-ORIENTED TOOL SELECTION SYSTEM

MING-HUNG SUNG, Associate Professor, Department of Industrial Engineering
Da-Yeh Institute of Technology, Taiwan, 51505, R.O.C.

KAI-I HUANG, Head and Associate Professor, Department of Industrial Engineering
Da-Yeh Institute of Technology, Taiwan, 51505, R.O.C.

HSIN-GINN HWANG, Associate Professor, Department of Computer and Office Information System
Northeast Louisiana University, Monroe, Louisiana, 71209-0120, USA

ABSTRACT

In the process of tool selection in a manufacturing environment, more than one attribute should be considered simultaneously. The process planner should consider all the factors, and each factor may not have the same weight. It is very difficult for the process planner to consider all the attributes concurrently, and in most instances, a decision is based upon only an economic aspect. A computer program, Fuzzy Object-Oriented Tool Selection System (FOOTSS), was developed to overcome these deficiencies by considering fuzzy multi-attribute, productivity, cost, and quality. In addition, a modified ranking index that provides easier computation is discussed.

INTRODUCTION

The function of process planning is to convert design specifications into manufacturing instructions. A survey of 23 companies [Eversheim et al. 1981] reveals that the selection of cutting tools is one of the most important tasks in process planning activities.

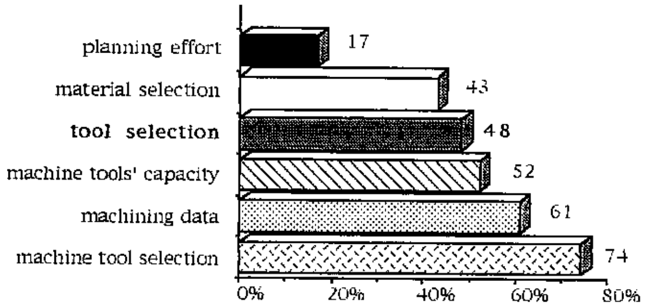


Figure 1. In this survey, 48% of the respondents regard tool selection as an important process planning task.

A similar survey [Chang 1990] confirms that cutting tool selection is one of the most important functions for process planning systems. This leads to the conclusion that cutting tool selection is becoming an important issue in the current manufacturing environment.

However, tool selection in process planning is highly interactive and complicated because there are a large number of alternatives available. For example, the number of cutting tools in a medium size Flexible Manufacturing System (FMS) can easily run into thousands [ElMaraghy 1985]. Similarly, a typical manufacturing system may have twenty machining centers and thousands of individual cutting tools consisting of about twenty different types [Choi 1982]. Therefore, the selection of the best tool among thousands becomes an overwhelming task in an integrated manufacturing environment.

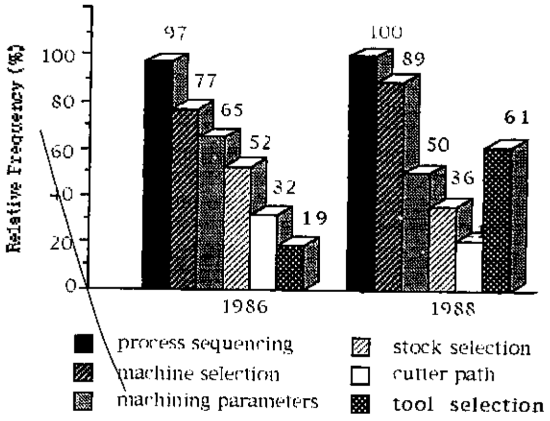


Figure 2. In another survey of important process planning functions, the response for tool selection increased from 19% in 1986 to 61% in 1988.

In today's manufacturing industries, optimizing one attribute may not be appropriate in complex production situations. In order to adapt the dynamic nature of manufacturing processes in reality, and to economically improve productivity and quality, all aspects related to the different attributes must be considered.

However, when multi-attribute optimization is applied, the different attributes and their relative importance in decision making may change dynamically due to unforeseen circumstances, such as maintenance, loading with high priority jobs, or tool breakage. Therefore, multiple-attribute optimization is still a key issue in the tool selection problem.

In this paper, we focus on the development of an automatic cutting tool selection system that takes multiple attributes into account. In the next section, a brief review of the previous research is given. Section 3 discusses the evaluation of alternatives using fuzzy set . Also, the attributes applied in the research are discussed. Some basic fuzzy set concepts and backgrounds are illustrated. Section 4 presents a scenario that shows how the proposed system can select an optimal tool. This will be followed by a description of the proposed system and a presentation of its advantages. Finally, Section 6 provides a summary and a discussion of some extensions of the paper. In Appendix A, we revise a ranking index method used by previous research and demonstrate its ease of use by examples.

REVIEW OF PREVIOUS RESEARCH

A literature review conducted in this study finds many misconceptions regarding the nature, the use, and the effectiveness of the optimization methodology. For example, most tool selection systems, such as Turbo-CAPP [Wang and Wysk 1987], do not apply any objective function; rather they take into account only geometrical or technological constraints. In other words, most tool selection systems select a cutting tool when it meets the requirements of a set of constraints instead of determining the optimal alternative. If there is more than one appropriate tool to consider, these systems would be unable to select the best one.

Cost or time related attributes may not be the only or even the most important factors to consider. The danger of using only one set of attributes is that the selected tool may be the best for one factor and may not be the best when all the factors are considered simultaneously. For example, the least expensive cutting tool does not mean the cost of the process will be minimized. Suppose that tool A is cheaper than tool B. On the other hand, tool A may have a shorter tool life and may produce worse surface quality. By applying the minimum tool cost attribute, such as SIPS [Nau and Luce 1987], tool A will be chosen, but in actuality, tool B may be preferred. Some other examples using single attribute are PRICAPP [Pande and Walvekar 1990], which minimizes machining cost and ROUND [van Houten 1986], which minimizes tool management, machining, tool changing and tool costs.

Some other approaches use more than one attribute, such as CUTTECH [Barkocy and Zdeblick 1984], which applies the attributes of Material Removal Rate (MRR), cost, cutting time, and rigidity; STOPP [Choi and Barash 1985], which chooses MRR and tools that have been selected; QTC [Chang 1990], which utilizes cost, tool life, and tool location; ATS [Chen et al. 1989], which selects machining cost and the least number of tools used. In addition, Lewis et al. [1982] use cutting time, the least number of tools used, and power requirements. It should be noted that tool life is difficult to predict and the rigidity is difficult to define and measure. The selection of the minimum number of required tools should be conducted with other important process planning functions, such as machine selection, fixture selection, and machining parameter selection.

Bard and Feo [1989] minimize batch cutting time and tool setup time while CUSPS [Bala and Chang 1988] minimizes machining time. However, Merchant [1977] observes that the actual machining time constitutes a small portion (1.5%) of the total time to produce a part. For this reason, applying the minimum machining time attribute may not result in the least overall production time. In this regard, unless the non-productive time and the loading/unloading time can be reduced, it makes less sense to minimize the machining time. On the other hand, if a machine is broken and this results in a bottleneck, then this attribute may need to be applied to achieve the maximum production rate.

Lastly, in a roughing operation, the principle of "the least number of cutting tools" should be applied, for example, CAPROT [Domaze 1990]. This will decrease the number of required tool changes and, consequently, reduce time and cost. On the other hand, in finishing operations, quality should be emphasized and incorporated into the attributes.

EVALUATION OF ALTERNATIVES USING FUZZY SET

Zimmer [1983] shows that the verbal expression of uncertainty is more accurate than numerical values in estimating the frequency of multiple attributes. Also, people are often notoriously unwilling to give precise numerical estimates of outcomes [Szolvits and Pauker 1982]. Hence to evaluate alternatives, a fuzzy set is considered to be more accurate and natural than point estimates.

The core of these attributes is central to the Productivity, Cost, and Quality (PCQ) Principle [Sung 1992]. In this study, it is assumed that all the candidate cutting tools satisfy all the geometrical and technological constraints, the route of these tools is optimal, and the least number of cutting tools is also determined. To be an optimal tool, the productivity and quality should be high, and the cost should be low. Higher productivity means one tool is able to produce more units in a given time over other selected tools. Lower cost means the costs of cutting time, tool changing time and idle time must be lower than other selected tools. Higher quality means the accuracy and precision are greater than other selected tools.

The Fuzzy Weight Index (FWI) is defined as

$$\mathrm{FWI}(ij) = Pj * W_i + Ci * W_i + Qj * W_i \tag{1}$$

FWI(ij) denotes the overall weights of a feasible cutting tool (i) in a certain situation (j). For a given feasible tool (i), Wi is the fuzzy set that represents weight of the feasible tool (i) based on productivity, cost, and quality attribute, respectively. In addition, Pj, Cj, and Qj are fuzzy sets denoting the weight in the situation (j). Then, the Fuzzy Weighted Average (FWA) [Dong and Wong 1987] is defined as follows.

$$R_j = \mathrm{FWI}_{ij} / \Sigma W_i \tag{2}$$

The relevant merits of the various tools are judged by comparing and ranking the final ratings $R_1, R_2, \ldots R_m$. Therefore, the preferred alternative is to select the alternative that has the highest final rating.

The membership function is also referred to the compatibility function. Usually, the membership function is determined by opinion pools, statistical data, expert's judgment, or interactive selection [Sung 1992]. Another approach is by the use of back-propagation neural networks to construct membership functions [Sung and Kuo 1992]. It should be noted that the determination of the membership functions depends on the nature of the base variable and on the context of the application. Fuzzy numbers with triangular membership functions are used to simplify the calculation. The membership functions corresponding to each linguistic rating are shown below. They are

$$\mu_{\text{fair}}(x) \quad \begin{array}{ll} = 0 & x \le 0 \\ = x & 0 \le x \le 1 \\ = -0.5x + 1.5 & 1 \le x \le 3 \end{array} \tag{3}$$

$$\mu_{\text{good}}(x) \quad \begin{array}{ll} = 0 & x \le 2 \\ = x - 2 & 2 \le x \le 3 \\ = x - 4 & 3 \le x \le 4 \\ = 0 & x \ge 4 \end{array} \tag{4}$$

$$\mu_{\text{excellent}}(x) \quad \begin{array}{ll} = 0 & x \le 3 \\ = .5x - 1.5 & 3 \le x \le 5 \\ = -x + 6 & 5 \le x \le 6 \\ = 0 & x \ge 6 \end{array} \tag{5}$$

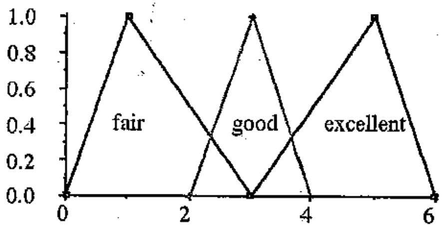


Figure 3. The ratings, *fair*, *good*, and *excellent*, are based on equation 3, 4 and 5, respectively.

The weights of the PCQ attributes are defined as follows They are

$$\mu_{\text{unimportant}}(x) \quad \begin{array}{ll} = 0 & x \le 0 \\ = x & 0 \le x \le 1 \\ = -0.5x + 1.5 & 1 \le x \le 3 \end{array} \tag{6}$$

$$\mu_{\text{important}}(x) \quad \begin{array}{ll} = 0 & x \le 1 \\ = .5x - .5 & 1 \le x \le 3 \\ = -.5x + 2.5 & 3 \le x \le 5 \\ = 0 & x \ge 5 \end{array} \tag{7}$$

$$\mu_{\text{extremely important}}(x) = \begin{cases} 0 & x \le 3 \\ .5x - 1.5 & 3 \le x \le 5 \\ -x + 6 & 5 \le x \le 6. \\ 0 & x \ge 6 \end{cases} \qquad (8)$$

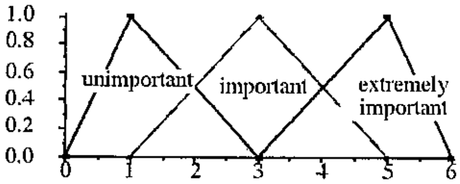


Figure 4. The weights, *unimportant*, *important*, and *extremely important*, of PCQ principle are based on equation 6, 7, and 8, respectively.

AN EXAMPLE OF TOOL SELECTION USING THE FOOTSS

In order to illustrate how fuzzy set can be applied to multi-attribute cutting tool selection, an example of tool selection using the FOOTSS is presented. Three candidate cutting tools are presented in this scenario. In this particular example, if another machine is broken, the productivity of this machine, to which the candidate tool is mounted, is important to balance the production line. Also, the cost of machining is unimportant or contributes only a small amount of overall cost. The quality of this product is important since this is the final finishing operation. According to past records or by the assessment of an expert, the weight of each tool can be obtained. Based on the above scenario, the input information is shown in Table I. The calculation of each FWA is shown in Appendix A.

Table I. Different ratings and weights are in turn input in this table. The first step is to assess the importance of these attributes, and the second step is to rate each individual tool based on these attributes.

Attribute	Productivity	Cost	Quality
Importance	important	unimportant	important
tool$_1$	good	fair	good
tool$_2$	fair	good	fair
tool$_3$	excellent	good	fair

After calculation, the FWA shows that Tool$_3$ has the highest value. In other words, Tool$_3$ is more favorable than Tool$_1$ and Tool$_2$. However, if Tool$_3$ is unavailable at the time of machining, Tool$_1$ can be utilized since the difference of the enclosed area and the mode between Tool$_1$ and Tool$_3$ may not be significant.

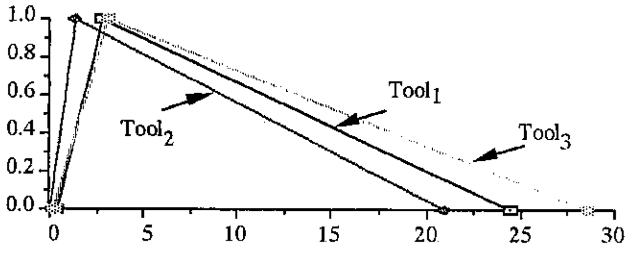
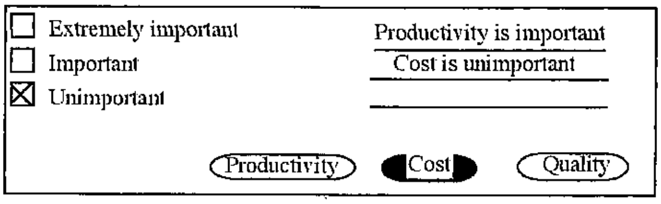


Figure 5. The final result of three alternatives shows that Tool$_3$ is the best selection. This is because Tool$_3$ has a larger mode than Tool$_1$ and Tool$_2$. Also, the area enclosed in Tool$_3$ is shifted more to the right than that of Tool$_1$ and Tool$_2$.

FOOTSS PROGRAM

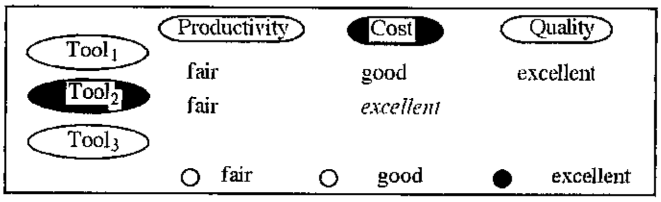
The Fuzzy Object-Oriented Tool Selection System (FOOTSS) program was written in an Object-Oriented Programming (OOP) language, HyperTalk, and implemented on the Macintosh microcomputer. This program acquires information from the user through a set of buttons. A button provides a mechanism for manipulating information by moving the mouse, then clicking a button. The input, intermittent, and output information are held in "Fields." Currently, each button or field contains from ten to forty lines of source code.

During the process of tool selection, the buttons of three primary attributes, productivity, cost, and quality, as well as three alternative tools, tool$_1$, tool$_2$, and tool$_3$, are highlighted in turn to provide an instant response as shown in Figures 6 and 7. The input information of selected weights for each attribute and tool are shown automatically to provide feedback for the user. In addition, the user is provided with a help facility for obtaining guidance during the selection process.


☐ Extremely important
☐ Important
☒ Unimportant

Productivity is important
Cost is unimportant

Figure 6. The user assesses the weights of the PCQ. This window shows that the "Cost" button is highlighted to prompt the user assessing the weight of "Cost." When the user clicks the "Unimportant" box, a check mark is placed on it.



	Productivity	Cost	Quality
Tool$_1$	fair	good	excellent
Tool$_2$	fair	*excellent*	

○ fair ○ good ● excellent

Figure 7. The user rates alternatives based on the PCQ. The "Cost" button is highlighted to prompt the user rating "Tool$_2$," which is also highlighted. When the user clicks the "excellent" button, the button is highlighted and *excellent* appears to provide an instant response.

HyperTalk was chosen because it has some advantages as follows. First, Object-Oriented modularization helps isolate errors within the data type and eases the maintenance and modification because of its modularity. Second, during the selection processes, unnecessary information will not be input into the system, and needed information will not be ignored. This not only reduces the chances to make errors but also increases the efficiency of the selection processes. Last, the use of Graphical User Interface (GUI), such as icons, windows, mouse, and dialog boxes, enhances the user friendly environment. Interface between the FOOTSS and the user is simple and fast. This feature is especially important in a job shop environment.

CONCLUSIONS

In this paper, previous research works in cutting tool selection have been investigated. The problem with existing tool selection is that many of the other selection methods take only economical aspects, mainly costs into account. In order to solve this problem, an Object-Oriented computer program, Fuzzy Object-Oriented Tool Selection System (FOOTSS), which applies fuzzy set concept to rank three primary attributes, productivity, cost, and quality, has been introduced and presented by one example. In addition, the advantages of an Object-Oriented approach have been illustrated. However, the current FOOTSS provides only three sets of ranking alternatives, and this may or may not be enough to satisfy the user's need. Therefore, an extension of this research will be to develop a flexible number of alternatives set by the user to enhance the FOOTSS.

Finally, the modified ranking index is simplified by adding minimum, mode, and maximum together. The modified ranking index has been demonstrated as an easier computation by comparing Juang's method. Currently, in this study only the triangular membership functions are assumed and applied. Further research will incorporate different membership functions and justify the use of this membership function as well as test the prototype FOOTSS.

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Appendix A. Calculation of Example

The main concern here is to rank the alternatives; therefore we are showing four fuzzy number operations to obtain the final ranking index. The first step is to obtain the overall weights in the denominator. The second step is to calculate each rating by multiplying the weight associated with it. There are nine possible situations listed. The third step is to sum the results from the second step for all three attributes and divide by the result from the first step. The last step is to rank three alternatives by comparing their ranking indexes.

Step 1. the fuzzy summation:

① ΣW_i = important + unimportant + important

$$\begin{array}{ll} = 0 & x \leq 2 \\ = .5x - 1.5 & 2 \leq x \leq 7 \\ = -x + 6 & 7 \leq x \leq 13 \\ = 0 & x \geq 13 \end{array}$$

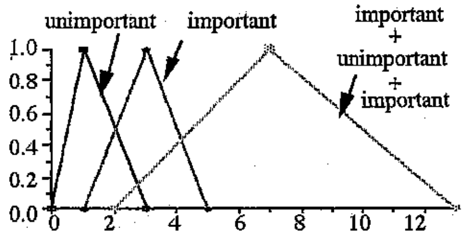


Figure 8. The fuzzy number, ΣW_i, on the right is obtained from the summation of these three *weights important*, *unimportant*, and *important*.

Step 2. the fuzzy multiplication

Table II. The Fuzzy Multiplication

*	fair	good	excellent
unimportant	❶	❷	❸
important	❹	❺	❻
extremely important	❼	❽	❾

❶ fair * unimportant

$$\begin{array}{ll} = 0 & x \le 0 \\ = x & 0 \le x \le 1 \\ = -(1/8)x + (9/8) & 1 \le x \le 9 \\ = 0 & x \ge 9 \end{array}$$

❷ good * unimportant

$$\begin{array}{ll} = 0 & x \le 0 \\ = (1/3)x & 0 \le x \le 3 \\ = -(1/9)x + (4/3) & 3 \le x \le 12 \\ = 0 & x \ge 12 \end{array}$$

❸ excellent * unimportant

$$\begin{array}{ll} = 0 & x \le 0 \\ = (1/5)x & 0 \le x \le 5 \\ = -(1/13)x + (18/13) & 5 \le x \le 18 \\ = 0 & x \ge 18 \end{array}$$

❹ fair * important

$$\begin{array}{ll} = 0 & x \le 0 \\ = (1/3)x & 0 \le x \le 3 \\ = -(1/12)x + (5/4) & 3 \le x \le 15 \\ = 0 & x \ge 15 \end{array}$$

❺ good * important

$$\begin{array}{ll} = 0 & x \le 2 \\ = (1/7)x - (2/7) & 2 \le x \le 9 \\ = -(9/11)x + (20/11) & 9 \le x \le 20 \\ = 0 & x \ge 20 \end{array}$$

❻ excellent * important

$$\begin{array}{ll} = 0 & x \le 3 \\ = (1/12)x - (1/4) & 3 \le x \le 15 \\ = -(1/15)x + 2 & 15 \le x \le 30 \\ = 0 & x \ge 30 \end{array}$$

❼ fair * extremely important

$$\begin{array}{ll} = 0 & x \le 0 \\ = (1/5)x & 0 \le x \le 5 \\ = -(1/13)x + (18/13) & 5 \le x \le 18 \\ = 0 & x \ge 18 \end{array}$$

❽ good * extremely important

$$\begin{array}{ll} = 0 & x \le 6 \\ = (1/9)x - (6/9) & 6 \le x \le 15 \\ = -(1/9)x + (8/3) & 15 \le x \le 24 \\ = 0 & x \ge 24 \end{array}$$

❾ excellent*extremely important

$$\begin{array}{ll} = 0 & x \le 9 \\ = (1/16)x - (9/16) & 9 \le x \le 25 \\ = -(1/11)x + (36/11) & 25 \le x \le 36 \\ = 0 & x \ge 36 \end{array}$$

Step 3. the fuzzy division : The FWA of $Tool_1$, $Tool_2$, and $Tool_3$ respectively is calculated as follows. The final shape is presented by the [minimum, mode, maximum].

$$\frac{(\text{good*important + fair*unimportant + good*important})}{(\text{important + unimportant + important})}$$

$$= \frac{\text{❺} + \text{❶} + \text{❺}}{\text{❿}} = \left(\frac{4}{13}, \frac{19}{7}, \frac{49}{2}\right)$$

$$\frac{(\text{fair*important +good*unimportant + fair*important})}{(\text{important + unimportant + important})}$$

$$= \frac{\text{❹} + \text{❷} + \text{❹}}{\text{❿}} = \left(0, \frac{9}{7}, 21\right)$$

$$\frac{(\text{excellent*important +good*unimportant + fair*important})}{(\text{important + unimportant + important})}$$

$$= \frac{\text{❻} + \text{❷} + \text{❹}}{\text{❿}} = \left(\frac{3}{13}, 3, \frac{57}{2}\right)$$

Step 4. the fuzzy number representation: The ranking index (RI) proposed by Juang [1987] is as follows:

$$RI = A_R - A_L + C \tag{9}$$

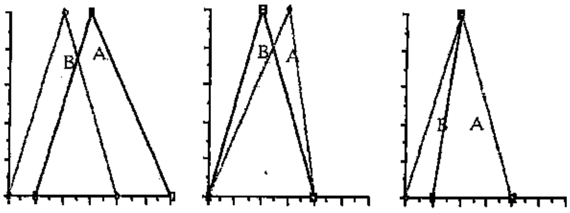
where

A_R: area enclosed to the right of the membership function,

A_L: area enclosed to the left of the membership function, and

C: the area enclosed by the universe of discourse.

When carefully inspecting this equation, one can see that (i) all final representing numbers have the same membership value (= 1) at the mode of each normalized triangular shape, (ii) the lower bound is monotonically increasing to the mode and then the mode is also monotonically decreasing to the upper bound. Hence, the actual ranking index can be simplified by adding minimum, mode, and maximum together. To illustrate this point, three different cases are presented in Figure 9.



Case I Case II Case III

Figure 9. The illustrations of Case I, Case II, and Case III represent all possible situations for two triangular memberships.

Although the results show that both approaches achieve the same answers as shown in Table III, that is, alternative A is better than alternative B, the revised method is much easier to implement than Juang's method.

Table III. The comparison of two ranking methods, the results of the above cases are calculated and compared using Juang's Ranking Index and the revised method. The numbers in brackets are the ranking values of A and B respectively, [A,B]. Note that Juang's method is to select the smallest number, and the revised method is to select the largest number.

Case	mode and bound	Juang's method	revised method
I	different mode, different bound	[2.75, 4]	[5, 3]
II	different mode, same bound	[3.5,4]	[3.5,3]
III	same mode, different bound	[3.75,4]	[3.5,3]