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A Situation of Economic Management in NTU Cooperative Fuzzy Games

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Abstract: In economic management, we often use some (divisible) private resources to cooperative. Fuzzy coalitions always be used to describe this situation in cooperative fuzzy games. In this paper, we proposed two new solution concepts in NTU cooperative fuzzy games, and discussed their properties.

Keywords: Bargaining set; NTU; Cooperative Fuzzy game

1. INTRODUCTION

In economic management, businessmen often possess some (divisible) private resources such as commodities, time, money, and have to decide about the amount to be invested in the joint project. Cooperative games with fuzzy coalitions are introduced in [1] and [2] to model the above situations where businessmen have the possibility to cooperate with different participation levels, varying from non-cooperation to full cooperation, and where the obtained reward depends on the levels of participation.

In this paper, we proposed more reasonable concepts of the bargaining sets which are very important solution concepts in game theory underlying NTU cooperative fuzzy game, and prove the inclusion relation between them. It is necessary to claim that the results of this paper were based on the work of Tijs[3] Muto[4] and Zhang Fengrong[5] .

2. PRELIMINARIES

Let N={1,2,…,n} be a finite player set, and a fuzzy coalition is a vector $s\in[0,1]^N$. The i-th coordinate s_i of s is the participation level of player i in the fuzzy coalition s. Instead of $[0,1]^N$ we will also write Φ^N for the set of fuzzy coalitions on player set N.

For $s\in\Phi^N$ we define the carrier of s by car(s)={i∈N| s_i >0} and call s a proper fuzzy coalition if car(s)≠N. The set of proper fuzzy coalitions on player set N is denoted by $\Pi\Phi^N$, and the set of non-empty proper fuzzy coalitions on player set N by $\Pi\Phi_0^N$. For s, t∈Φ^N we use the notation s≤t expresses $s_i \le t_i$ for each i∈N. We define $s\wedge t=(\min(s_1, t_1),\ldots, \min(s_n, t_n))$ and $s\vee t=(\max(s_1, t_1),\ldots, \max(s_n, t_n))$.

Let $R^{car(s)}$ indicates $|car(s)|$-dimension Euclid space. Denoted $x\le y$ means $x_i\le y_i$ for each $i\in car(s)$; and $x<<y$ means $x_i< y_i$ for each $i\in car(s)$ for any $x, y\in R^{car(s)}$.

An NTU fuzzy cooperative game is a pair (N, V), where $N=\{1,2,\ldots,n\}$ is a non-empty finite set, and V, the characteristic function, assigns to every fuzzy coalition s a $V(s)$ which is a subset of $R^{car(s)}$, and satisfies the follow properties:

(1) $V(0)=\varnothing$, where 0=(0,0,…,0).

(2) For each $s\in\Phi^N$, $V(s)$ is a non-empty subset of $R^{car(s)}$, and it is closed and comprehensiveness. (that is to say, if $x\in V(s)$, $y\in R^{car(s)}$, and $y\leq x$, then $y\in V(s)$, too).

(3) For each $x\in R^{car(s)}$, $V(s)\cap(x+R_+^{car(s)})$ is bounded.

(4) $V(s)$ is nonlevelness, *i.e.*, if $x,y\in V^0(s)$, and $x\geq y$, then $x=y$, where, $V^0(s)=\{x\in V(s)|$there is no $y\in V(s)$, such that $x<<y\}$ is boundary of $V(s)$.

For any two disjoint fuzzy coalitions s and t, and two vectors $x\in R^{car(s)}$, $y\in R^{car(t)}$, we denote by (x, y) the vector in $R^{car(s)\cup car(t)}$ whose $car(s)$-components are as in x and $car(t)$-components are as in y. The game (N, V) is superadditive if for every two fuzzy coalitions s and t with $car(s)\cap car(t)=\varnothing$, and any two vectors $x\in V(s)$, $y\in V(t)$, we have $(x, y)\in V(s\vee t)$.

The preimputation set of NTU fuzzy game (N, V) is $I^0=V^0(N)$, imputation set is $I=V^0(N)\cap R_+^N$

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3. BARGAINING SET FOR COOPERATIVE FUZZY GAME WITH NON-TRANSFERABLE UTILITY

Wenbo Yang *etc.*[6] firstly extended the concept of bargaining set which was proposed by Davis-Machler[7] to TU fuzzy game. Here we will redefine the concept of bargaining set from the view point of Davis-Machler to NTU cooperative fuzzy game.

Definition 3.1 Let x be an imputation of the NTU fuzzy game (N, V), then an objection of player k against player l with respect to x is a pair (y, s), if $s_i y_i > s_i x_i$, for all i∈car(s), y∈V(s), where s∈N with car(s)≠0 and N, $s_k>0$, $s_l=0$, and y is a vector whose indices are the members of car(s).

Definition 3.2 Let x be an imputation of the NTU fuzzy game (N, V), and let (y, s) be an objection of player k against player l with respect to x. Then (z, t) is a counter objection of player l to the objection (y, s) of player k, if

$$\begin{gathered} t_i(z_i-x_i)\geq s_i(y_i-x_i),\quad \textit{for all } i\in car(s)\cap car(t) \\ t_i z_i\geq t_i x_i,\quad \textit{for all } i\in car(t)\setminus car(s) \\ z\in V(t), \end{gathered}$$

where $t\in\Phi^N$ with $t_k=0$, $t_l>0$, and $t_i\geq s_i$, for all $i\in car(s)\cap car(t)$, and z is a vector whose indices are the members of $car(t)$.

Definition 3.3 An imputation x belongs to the bargaining set $M_F^{(i)}(V)$ for a NTU fuzzy cooperative game, if for every objection, there exists a counter objection to it.

Mos-Colell[8](1989) proposed another bargaining set, now we will redefine the concept to NTU fuzzy game as follows.

Definition 3.4 Let x be a preimputation of the NTU fuzzy game (N, V), for a fuzzy coalition $s\in\Phi^N$, if

$car(s) \neq 0$ and N, an objection on fuzzy coalition s with respect to x is a pair (y, s), where $y \in R^{car(s)}$ and satisfying:

$$y \in V(s),$$
$$s_i y_i \geq s_i x_i, i \in car(s),$$

and at least one of the inequalities above is strict.

Definition 3.5 Let x be a preimputation of the NTU fuzzy game (N, V), and let (y, s) be an objection to fuzzy coalition s with respect to x. Then (z, t) is a counter objection on fuzzy coalition $t \in \Phi^N$ with respect to x, here $t_i \geq s_i$, and for all $i \in car(s) \cap car(t)$, $z \in R^{car(t)}$ satisfying

$$z \in V(t)$$
$$t_i(z_i - x_i) \geq s_i(y_i - x_i), \textit{ for all } i \in car(s) \cap car(t)$$
$$t_i z_i \geq t_i x_i, \textit{ for all } i \in car(t) \setminus car(s)$$

and at least one of the inequalities above is strict.

An objection is called justified, if there is no counter objection.

Definition 3.6 A preimputation x belongs to the bargaining set $MB_F(V)$ of NTU fuzzy cooperative game if there is no justified objections with respect to x.

Theorem 3.1 Let (N, V) be a supperadditive NTU fuzzy game, then

$$M_F^{\{i\}}(V) \subseteq MB_F(V).$$

Proof. Suppose, for contradiction, that $x \in M_F^{\{i\}}(V) \setminus MB_F(V)$. Let (y, c) be a justified objection in the sense of $MB_F(V)$ at x. Chosen c such that $c \geq s$, where s is an any fuzzy coalition which can cause a justified objection in $MB_F(V)$ at x. Let $c_k > 0$, and $c_k y_k > c_k x_k$. From the concept of the bargaining set $MB_F(V)$, we have $car(s) \neq N$. Let $l \in N \setminus car(c)$.

By the comprehensiveness and nonlevelness of $V(c)$, we can modify y, and y is still in $V(c)$. Let the k-component of y will decrease, but is still great then x_k, and the other components of y will increase. So, we obtain a vector $\tilde{y} \in V(c)$ satisfying

$$c_k \tilde{y}_k > c_k x_k$$
$$c_i \tilde{y}_i > c_i y_i, i \in car(c) \setminus \{k\}$$

Clearly, $(\tilde{y}, c)$ is an objection of k against l at x. Since $x \in M_F^{\{i\}}(V)$, there exists a counter objection (z, d) to $(\tilde{y}, c)$ satisfying $d_i \geq c_i, i \in car(d) \cap car(c)$, $d_k = 0$, $d_l > 0$. So we have

$$d_i(z_i - x_i) \geq c_i(\tilde{y}_i - x_i) > c_i(y_i - x_i), i \in car(d) \cap car(c) \tag{1}$$

$$d_i z_i \geq d_i x_i, i \in car(d) \setminus car(c) \tag{2}$$

Thus, (z, d) satisfies almost all the requirements to be a counter objection to (y, c) in the sense of $MB_F(V)$, except one condition, *i.e.*, at least one strict inequality in (1) and (2) is true. Since (y, c) is justified, and in the inequality (1), $car(c) \cap car(d)$ must be an empty set. But from the property of surperadditivity, we know that $((y, z), c \vee d)$ is a justified objection in the sense of $MB_F(V)$ at x, so $c < (c \vee d)$, contradicting our assumption. The conclusion is true.

4. CONCLUSIONS

NTU cooperative game can be converted to TU cooperative game under specific conditions. In the general sense, NTU cooperative game can be viewed as the extension of TU cooperative game. But, many properties of NTU cooperative game can help to study the properties of the TU cooperative games. In this paper we extended two concepts of the bargaining set for TU cooperative game to NTU cooperative fuzzy game, and proved that the inclusion relation between these two bargaining sets is still satisfied. The problem of the bargaining sets how can be used in economic management effectively is still open[10]. We will work on it in future.

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