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# INVENTORY MANAGEMENT OF PLATELETS IN HOSPITALS: OPTIMAL INVENTORY POLICY FOR PERISHABLE PRODUCTS WITH EMERGENCY REPLENISHMENTS

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## Abstract

Platelets are short-life blood components used in hospital blood transfusion centers. Excluding time for transportation, testing, and arrangement, clinically transfusable platelets have a mere three-day life-span. This paper analyzes a periodic review inventory system for such perishable products under two replenishment modes. Regular orders are placed at the beginning of a cycle. Within the cycle, the manager has the option of placing an emergency order, characterized by an order-up-to level policy. We prove the existence and uniqueness of an optimal policy that minimizes the expected cost. We then derive the necessary and sufficient conditions for the policy, based on which a heuristic algorithm is developed. A numerical illustration and a sensitivity analysis are provided, along with managerial insights. The numerical results show that, unlike typical inventory problems, the total expected cost is sensitive to the regular order policy. It also shows that the optimal policy is sensitive to changes in the expected demand, suggesting to decision makers the significance of having an accurate demand forecast.

**Key words:** Perishable items, Platelet inventory management, Emergency orders, Stochastic dynamic programming

## Introduction

Platelets are blood cells used by the body in the process of blood clotting. Platelet transfusion is often a vital part of treating patients with leukemia and other types of cancer, patients undergoing open heart surgery, and those in need of bone marrow and organ transplants. Currently, blood platelets must be stored at room temperature using environmental chambers because refrigeration changes their density and structure. Platelets are expensive and perishable. Typically, they can only be used for transfusion within five days after donation. After deducting the time spent in transportation, testing, and arrangement, clinically transfusable platelets

have a mere three-day life-span in hospitals. Due to the short life-span and their medical importance, a critical issue to the decision makers of local hospitals is how to manage platelet inventory in a manner that minimizes outdates and satisfies the demand.

Local hospitals typically order blood products from a central blood center. A generally adopted method is a periodic review ordering policy with standing orders (regular order) at the beginning of the ordering cycle and ad hoc orders (emergency orders) during the cycle. The combined use of regular and emergency orders is considered an efficient way to reduce outdates and to lower the risk of shortages. A survey of 255 hospitals reports that 53% of them manage platelet supplies based on a combination of block ordering and ad hoc ordering, and that the rest use ad hoc orderings only[2].

Based on demand forecast, the manager issues a regular order to a central blood center. Regular orders are usually issued at the end of the day, and are fulfilled the next morning. Because of the short life-span of platelets, it is unlikely that the manager would consider regular ordering cycles of more than three days. For hospitals that do not have critical needs of platelets, it is common that regular orders are placed every other day. Between arrival of regular orders (the days without regular orders), the manager examines the platelet inventory level. If the level is judged to be unable to satisfy the anticipated demand before the next regular order, the manager issues an emergency order, which is usually received within an hour.

In practice, hospitals design their inventory policies based on experience. There are often complaints about frequent emergency orders, large amounts of outdates, or serious shortages. To have an effective policy, the decision maker must balance the critical costs of outdates, shortages, and emergency orders. However, determination of an optimal regular-emergency ordering policy to minimize the total inventory cost is a complex endeavor.

This research problem can be classified as a fixed lifetime perishable inventory problem with

stochastic demand and emergency ordering. There is a large body of literature on perishable product inventory problems ([7]; [13]; [16]). For fixed-life perishable problems with general stochastic demand, the determination of optimal regular order policies was first examined in [14], in which a product life of two periods was addressed and the convexity of the corresponding cost functions was characterized. The two-period problem was later extended to  $m$  periods, and an algorithm to obtain the optimal ordering policies was designed ([11]; [12]). Similarly, bounds were developed for a near-myopic heuristic which searches for a desirable inventory policy ([15]). A combination of dynamic programming and simulation was applied to explore the “near optimal” inventory policies for platelets ([8]). In general, because of the complexity of tracking the states of  $m$ -period perishable products, analytical results have not been explored extensively.

For fixed-life perishable problem with Poisson demand, queueing properties have been utilized to examine the optimal inventory policy. In [6], the perishable inventory system was formulated as a two-stage queueing model (one holding fresh items and the other holding older items), where the system’s performance was approximated using simulation. A continuous review perishable system was analyzed in [9], where the expected inventory cost was characterized based on renewal demand and instantaneous replenishments. Assuming both the arrival of items and the demand are Poisson processes, [17] developed ergodic limits for the average inventory and outdates. Utilizing the steady state properties of a Markov chain, [4] obtained bounds on the limiting distribution of the number of outdates in a period.

The consideration of emergency order as a major aspect of an inventory policy for perishable products has not been addressed in the literature. For inventory research on nonperishable product, emergency order was featured as a complementary ordering policy in [3] and [18]. In the former, multiple emergency orders were allowed within a regular review cycle and were managed under a continuous review policy, where a complex control policy was derived based on a stochastic dynamic programming approach. In the latter, emergency orders were restricted to specific review times, and a simple and time-invariant emergency order-up-to level was shown to be unique and was approximated using a heuristic algorithm.

In this paper, we address a platelet inventory problem characterized by a fixed-lifespan of three days, general stochastic demand, and a combined use of regular and emergency orderings, where regular orders are placed at the beginning of a cycle and emergency orders are periodically reviewed within the cycle according to an order-up-to level policy.

There has been limited research that analytically models the perishable inventory problem beyond two periods for general stochastic demand, since the number of states explodes exponentially with the number of lifetime periods.

For perishable inventory with emergency orders, the convexity of the cost function for a general demand distribution is no longer assured. However, by first showing the existence of a unique interior stationary point and then proving the strict convexity at the point’s neighborhood, we are able to prove the uniqueness of a decision policy doublet that minimizes the total expected cost. We also derive the necessary and sufficient conditions for the optimal doublet, based on which a heuristic algorithm is designed.

Our numerical analysis shows that the total expected cost is sensitive to changes in the optimal regular order but not in the order-up-to level. Also, while both the optimal regular order and the order-up-to level are not sensitive to changes in cost parameters, both are sensitive to changes in the expected demand. It shows that the accuracy in estimating the expected demand is by far the most important aspect for the decision maker, as the error could result in a significant amount of extra costs.

In the ensuing sections, we first discuss the assumptions, followed by the development of the one cycle problem. Then the model is extended to a multi-cycle problem, and the optimal policy structure is examined. We derive the necessary and sufficient conditions for the optimal policy, based on which a heuristic algorithm is designed. Numerical illustrations and a comprehensive sensitivity analysis are provided, along with managerial insights.

## The Modeling Framework

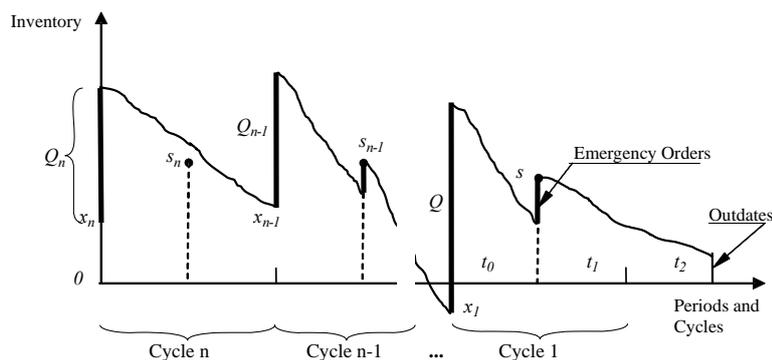
We consider a single-item periodic review inventory system that employs certain order amount for its regular order and an order-up-to level for its emergency order. Throughout the paper, we use *period* as a time unit (day), and *cycle* to express the interval of regular orders. To manage platelets with emergency ordering, the regular order cycles are limited to either two days or three days, because a one-day cycle would rule out emergency orders and platelets have a three-day lifetime. Since it is not uncommon for hospitals to place regular orders every other day, and as a start, we assume the regular order cycle to be two periods.

The decisions for this problem are regular order quantities  $Q_n$  and the emergency order-up-to level  $s_n$  (Figure 1). Here, we assume that all orders are placed at the start of a period and received instantaneously. This is a reasonable real-life approximation, since emergency orders are usually delivered promptly and regular orders arrive

overnight. Also, unsatisfied demand of platelets is assumed to be backlogged, because of the critical nature of their medical significance. Moreover, we assume that the period demands are independent and identically distributed, and are satisfied according to a FIFO policy.

Another important assumption has to do with the vintage of the platelets. Regular ordered platelets are usually fresh. Since emergency platelets are expected to be used right away, blood

banks may have the option to deliver platelets that are fresh, one-period old, or two-period old. Depending on the availability and policy between the blood bank and local hospitals, all three types of vintage are plausible emergency supplies. Here, we start our exposition with the assumption that emergency platelets are one-period old, because it offers certain modeling simplifications. At a later section, we will explore the scenarios where the emergency platelets are fresh or two-period old.



**Figure 1. Evolution of Inventory Position in a Periodic Review (Q, s) System of Perishable Products with Emergency Replenishments**

**Notions:**

- $t_j =$  demand in period  $j$  ( $j=0, 1, 2$ ) with distribution
- $F$  and density function  $f: f(t) > 0$  for  $t > 0$ ;  
 $f(t)=0$  for  $t \leq 0$
- $\mu =$  mean demand per period,  $E[t]$
- $y_e =$  emergency order amount
- $L_i =$  shortage at period  $i$  of a cycle ( $i=1, 2$ )
- $x_n =$  initial inventory of cycle  $n$  ( $x$  for one-cycle problem)

- $z =$  outdates of platelets ordered in this cycle
- $s_n =$  emergency order-up-to level in cycle  $n$  ( $s$  for one-cycle problem)
- $Q_n =$  regular order quantity in cycle  $n$  ( $Q$  for one-cycle problem)
- $c_e =$  emergency cost per unit
- $c_p =$  shortage cost per unit.
- $c_r =$  outdate cost per unit.

**One Cycle Problem**

In this section we build expressions for costs of replenishing, shortages, and carrying for the one cycle problem. The expected cost of the whole cycle is the summation of these three types of costs. For replenishing cost, we focus on emergency cost since it is significantly more expensive than the regular cost. For carrying cost, we focus on the cost of outdates, which is the predominant part of the carrying cost.

The inventory level at the end of the first period is  $Q - (t_0 - x)^+$ , where  $t_0$  is the demand in the first period. The plus function in this expression ensures that all initial inventories will deteriorate if they are not depleted at the end of the first period.

An emergency order is issued when  $Q - (t_0 - x)^+$  is smaller than  $s$ . We can express the emergency order as:

$$y_e = (s - [Q - (t_0 - x)^+])^+ \tag{1}$$

Shortages may occur in both periods of the cycle. For the first period, shortage occurs when demand  $t_0$  is higher than the sum of the regular order quantity and initial inventory:

$$L_1 = (t_0 - Q - x)^+ \tag{2}$$

For the second period, shortage depends on the emergency order quantity and the inventory level at the end of the first period:

$$L_2 = (t_1 - y_e - [Q - (t_0 - x)^+] )^+ \tag{3}$$

$$= (t_1 - (s - [Q - (t_0 - x)^+] )^+ - [Q - (t_0 - x)^+] )^+$$

For assessing the outdate cost, we need to consider the lag effect where outdates for the amount ordered in the current cycle occur only in subsequent cycles (see also [14] and [15]). Because the emergency orders are assumed to be one-period old, all the inventories at the beginning of the second period have the same age. Therefore, the outdate is

$$z = (Q + y_e - [t_2 + t_1 + (t_0 - x)^+] )^+ \tag{4}$$

$$= (Q + (s - [Q - (t_0 - x)^+] )^+ - [t_2 + t_1 + (t_0 - x)^+] )^+$$

where  $[t_2 + t_1 + (t_0 - x)^+]$  is the total demand to be satisfied by platelets ordered in the cycle.

**Optimal Policy with Initial Inventory ( $x \geq 0$ )**

We first address the optimal policy when there is initial inventory ( $x \geq 0$ ). The situation when there is backlog ( $x < 0$ ) is addressed in the next subsection.

**Expected Emergency Cost**

If  $Q \leq s$  and from (1),  $Q - (t_0 - x)^+ \leq s$ .

Thus,

$$y_e = \begin{cases} s - Q, & t_0 \leq x; \\ s + t_0 - Q - x, & t_0 \geq x. \end{cases}$$

The expected emergency cost is

$$E[y_e] = c_e \int_{t_0=0}^x (s - Q) dF(t_0) + c_e \int_{t_0=x}^{\infty} (s + t_0 - Q - x) dF(t_0)$$

$$= c_e (s - Q + \mu - x + \int_0^x F(t_0) dt_0)$$

where  $c_e$  is the emergency cost per unit. Here, we assume that the emergency cost is in terms of cost per unit, and that the fixed cost is negligible. The same assumption is adopted in [3] and [18].

If  $Q \geq s$ , then  $y_e > 0$  if and only if

$$s + (t_0 - x)^+ > Q. \text{ Thus, } y_e = \begin{cases} s + t_0 - Q - x, & t_0 \geq Q + x - s; \\ 0, & \text{otherwise.} \end{cases}$$

Then the expected emergency cost would be

$$E[y_e] = c_e \int_{t_0=Q+x-s}^{\infty} (s + t_0 - Q - x) dF(t_0)$$

$$= c_e (s - Q + \mu - x + \int_0^{Q+x-s} F(t_0) dt_0).$$

Note that when  $Q = s$ , the two expressions of  $E[y_e]$  converge. This means the expected emergency cost is continuous at the point. Similarly, it can be shown that expected shortage and outdate are continuous at this point as well.

**Expected Shortage Cost**

For the first period,

$$E[L_1] = c_p \int_{t_0=Q+x}^{\infty} (t_0 - Q - x) dF(t_0)$$

$$= c_p (\mu - Q - x + \int_0^{Q+x} F(t_0) dt_0),$$

For the second period,

$$\text{If } Q \leq s, L_2 = \begin{cases} t_1 - s, & t_1 \geq s; \\ 0, & \text{otherwise.} \end{cases}$$

and the expected shortage is

$$E[L_2] = c_p \int_{t_0=s}^{\infty} (t_1 - s) dF(t_1) = c_p (\mu - s + \int_0^s F(t_0) dt_0).$$

If  $Q \geq s$ ,

$$L_2 = \begin{cases} (t_1 - Q)^+, & t_0 \in [0, x]; \\ (t_1 - Q - x + t_0)^+, & t_0 \in [x, x + Q - s]; \\ (t_1 - s)^+, & t_0 \in [x + Q - s, \infty). \end{cases}$$

Then the expected shortage is

$$E[L_2] = c_p (\int_{t_0=0}^x \int_{t_1=Q}^{\infty} (t_1 - Q) dF(t_1) dF(t_0) + \int_{t_0=x}^{x+Q-s} \int_{t_1=x+Q-t_0}^{\infty} (t_1 - Q - x + t_0) dF(t_1) dF(t_0) + \int_{t_0=x+Q-s}^{\infty} \int_{t_1=s}^{\infty} (t_1 - s) dF(t_1) dF(t_0)).$$

By integrating by parts twice, this expression could be simplified as

$$E[L_2] = c_p (\mu - s + \int_0^s F(t_1) dt_1 - \int_x^{x+Q-s} F(t_0) dt_0 + \int_x^{x+Q-s} F(t_0) F(x + Q - t_0) dt_0).$$

**Expected Outdate Cost**

If  $Q \leq s$ , the inventory level will be replenished to  $s$  at the beginning of the second period. Therefore, outdates only depend on emergency order level and demand of later periods. From (4), we have  $z = (s - t_1 - t_2)^+$ . Thus the expected outdate cost is

$$E[z] = c_r (\int_{t_1=0}^s \int_{t_2=0}^{s-t_1} (s - t_1 - t_2) dF(t_2) dF(t_1))$$

$$= c_r \int_0^s F(t_1) F(s - t_1) dt_1,$$

If  $Q \geq s$ , the outdate amount would depend on both  $Q$  and  $s$ , as well as demands  $t_0, t_1$ , and  $t_2$ .

$$z = \begin{cases} (Q - t_1 - t_2)^+, & t_0 \in [0, x]; \\ (Q + x - t_0 - t_1 - t_2)^+, & t_0 \in [x, x + Q - s]; \\ (s - t_1 - t_2)^+, & t_0 \in [x + Q - s, \infty). \end{cases}$$

The expected outdate cost would be

$$E[z] = c_r \left\{ \int_{t_0=0}^x \int_{t_1=0}^Q \int_{t_2=0}^{Q-t_1} (Q-t_1-t_2) dF(t_2) dF(t_1) dF(t_0) \right. \\ + \int_{t_0=x}^{x+Q-s} \int_{t_1=0}^{Q+x-t_0} \int_{t_2=0}^{Q+x-t_0-t_1} (Q+x-t_0-t_1-t_2) dF(t_2) dF(t_1) dF(t_0) \\ \left. + \int_{t_0=x+Q-s}^{\infty} \int_{t_1=0}^s \int_{t_2=0}^{s-t_1} (s-t_1-t_2) dF(t_2) dF(t_1) dF(t_0) \right\}.$$

After integrating by parts three times, it is simplified as

$$E[z] = c_r \left( \int_0^s F(t_1) F(s-t_1) dt_1 \right. \\ \left. + \int_{t_0=x}^{Q+x-s} F(t_0) F^{(2)}(x+Q-t_0) dt_0 \right),$$

where  $F^{(2)}(t)$  is a convolution of  $F(t)$  defined by  $F^{(2)}(t) = \int_0^t F(t-t') dF(t')$ . Since a convolution of CDF functions is still a CDF function, we define its density function as  $f^{(2)}(t)$ . It is easy to check that  $f^{(2)}(t) = \int_0^t f(t-t') dF(t')$  based on positive demand functions.

Combining all the expressions above, the one cycle total expected cost becomes:

When  $Q \leq s$ ,

$$EC(Q, s) = c_e (s - Q + \mu - x + \int_0^x F(t_0) dt_0) \\ + c_p (\mu - Q - x + \int_0^{Q+x} F(t_0) dt_0) \\ + c_p (\mu - s + \int_0^s F(t_0) dt_0) \\ + c_r \int_0^s F(t_1) F(s-t_1) dt_1. \tag{5}$$

When  $Q \geq s$ ,

$$EC(Q, s) = c_e \left[ s - Q + \mu - x + \int_0^{Q+x-s} F(t_0) dt_0 \right] \\ + c_p \left[ \int_x^{x+Q-s} F(t_0) F(x+Q-t_0) dt_0 \right. \\ \left. + 2\mu - Q - x - s + \int_0^{Q+x} F(t_0) dt_0 \right. \\ \left. + \int_0^s F(t_1) dt_1 - \int_x^{x+Q-s} F(t_0) dt_0 \right] \\ + c_r \left[ \int_0^s F(t_1) F(s-t_1) dt_1 \right. \\ \left. + \int_{t_0=x}^{Q+x-s} F(t_0) F^{(2)}(x+Q-t_0) dt_0 \right]. \tag{6}$$

At first sight, it seems that we have to discuss optimal policy for both cases listed in (5) and (6). However, the following lemma guarantees that we only need to examine the case  $Q \geq s$  for optimal policies.

**Lemma 1.** 1. The expected cost function is continuous in  $Q$  and  $s$ .

2. When  $Q \leq s$ , the expected cost will reach its minimum at the boundary where  $Q = s$ .

The proof is given in the Appendix. From the first statement of this Lemma, we know that the boundary solution is covered in the case  $Q \geq s$ , which means it is the only case we need to consider. Also, at the minimum, the optimal regular ordering quantity  $Q$  will never be smaller than the emergency stock level  $s$ . We know that if the optimal policy doublet is an interior point of the feasible set, it must necessarily satisfy the first-order conditions

$$\partial EC / \partial Q = 0 \quad \text{and} \quad \partial EC / \partial s = 0.$$

**Theorem 1.** For the one cycle inventory problem, there exists a unique interior global minimum for the expected cost function. The optimal ordering policies  $(Q^*, s^*)$  are determined by the following two equations:

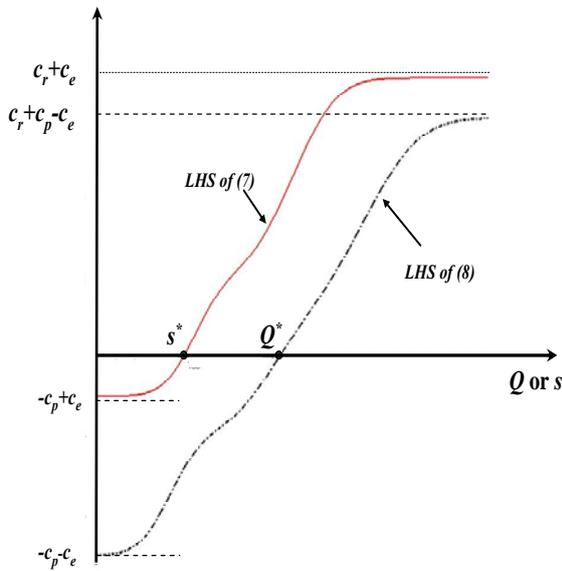
$$c_e - c_p + c_p F(s^*) + c_r F^{(2)}(s^*) = 0, \tag{7}$$

$$-c_e - c_p + c_p F(Q^* + x) \\ + \int_{s^*}^{Q^*} F(Q^* + x - t_0) (c_p f(t_0) \\ + c_r f^{(2)}(t_0)) dt_0 = 0 \tag{8}$$

The proof is given in the Appendix. Equations (7) and (8) are the necessary and sufficient conditions for the optimal policy. By first showing the existence of a unique interior stationary point and then proving the strict convexity at the point's neighborhood, we prove the uniqueness of a decision policy doublet that minimizes the total expected cost. Note that, throughout the proof, the convexity of the cost function is not required for the determination of the optimal solution. In fact, the total cost function (6) is not convex in general (see also Figure 2), largely due to the presence of emergency ordering. This represents a departure from the convex cost function in traditional regular-order perishable inventory system.

From (7), note also that  $s^*$  is independent of the initial inventory. This is true when the probability of an emergency order is non-zero. However, when the initial inventory is large enough to cover the demand during the first period for sure, (7) no longer holds and  $s^*$  is no longer relevant (see A.1 in the Appendix).

Figure 2 is a typical plot of the Left Hand Side (LHS) of (7) and (8), with the periodic demand following a normal distribution and with given values of  $c_e$ ,  $c_p$ ,  $c_r$ , and  $x$ . LHS of (7) is bounded between  $-c_p + c_e$  and  $c_r + c_e$ , while LHS of (8) is bounded between  $-c_p - c_e$  and  $c_r + c_p - c_e$ . The optimal policies  $Q^*$  and  $s^*$  are therefore the points where these two respective curves intersect with the horizontal axis.



**Figure 2. Finding the Optimal Policy Doublet  $(Q^*, s^*)$**

**Optimal Policy with Backlog  $(x \leq 0)$**

In previous discussions, we assumed the initial inventory to be  $x \geq 0$ . For  $x \leq 0$ , the formulation of the cost function as well as the optimum solution is very similar. For the sake of brevity, we list the major results as follows.

When  $Q+x \leq s$ ,

$$EC(Q, s) = c_e(s - Q + \mu - x) + c_p(\mu - Q - x + \int_0^{Q+x} F(t_0)dt_0) + c_p(\mu - s + \int_0^s F(t_0)dt_0) + c_r \int_0^s F(t_1)F(s - t_1)dt_1 \tag{9}$$

When  $Q+x \geq s$ ,

$$EC(Q, s) = c_e \left[ s - Q + \mu - x + \int_0^{Q+x-s} F(t_0)dt_0 \right] + c_p \left[ \int_0^{x+Q-s} F(t_0)F(x+Q-t_0)dt_0 + 2\mu - Q - x - s + \int_0^{Q+x} F(t_0)dt_0 + \int_0^s F(t_1)dt_1 - \int_0^{x+Q-s} F(t_0)dt_0 \right] + c_r \left[ \int_0^s F(t_1)F(s-t_1)dt_1 + \int_0^{Q+x-s} F(t_0)F^{(2)}(x+Q-t_0)dt_0 \right] \tag{10}$$

For the same reason as stated in Lemma 1, we only need to consider (10) to determine the

optimal policy. Similar to (6), this cost function (10) has the boundary  $Q + x = s$ .

**Corollary 1.** For any initial inventory  $x \leq 0$ :

1) The optimal order policy doublet  $(Q^*, s^*)$  given by the following two equations is the unique and global optimum solution for the one cycle expected cost function.

$$c_e - c_p + c_p F(s^*) + c_r F^{(2)}(s^*) = 0, \tag{11}$$

$$-c_e - c_p + c_p F(Q^* + x) + \int_{s^*}^{Q^*+x} F(Q^* + x - t_0)(c_p f(t_0) + c_r f^{(2)}(t_0))dt_0 = 0 \tag{12}$$

2) At the optimum,

$$Q^*(x) = Q^*(0) + |x|. \tag{13}$$

The proof is given in the Appendix. Equation (13) of Corollary 1 shows that the optimal regular order quantity is the sum of two parts, the optimal order with zero initial inventory and the backlog. Note that  $Q^*$  at zero initial inventory is already covered in (6).

**The Dynamic Approach for the Multi-cycle Problem**

In this section, we extend the single cycle model to a multi-cycle model, where a sequence of ordering decisions is made. As is common with periodic review inventory problems, our mode of analysis is the functional equation approach of dynamic programming.

First, we need to formulate the inventory level at the end of a cycle. This is given by the following lemma. Note that we label the cycles backward, starting with the final cycle as cycle 1.

**Lemma 2.** If the inventory level at the beginning of cycle  $n$  is  $x_n$ , the ordering policies are  $Q_n$  for the regular order and  $s_n$  for the emergency order-up-to level, and the demands for the two periods of the cycle are  $t_0$  and  $t_1$ , the inventory level at the end of the cycle would be

$$x_{n-1}(x_n, Q_n, s_n, t_1, t_0) = \begin{cases} Q_n - t_1, & t_0 \in [0, x_n]; \\ Q_n + x_n - t_0 - t_1, & t_0 \in [x_n, x_n + Q_n - s_n]; \\ s_n - t_1, & t_0 \in [x_n + Q_n - s_n, \infty). \end{cases} \tag{14}$$

Proof is obvious and is omitted. Define  $C_n(x_n)$  as the minimum expected discounted cost of the last  $n$  cycles where  $x_n$  is initial inventory of the cycle  $n$ . Similar to our discussions in the one-cycle problem, each cycle contains exactly two periods and all fresh inventory will perish in three periods. We define

$$B_n(x_n, Q_n, s_n) = EC(x_n, Q_n, s_n) + \alpha \int \int C_{n-1}[x_{n-1}(x_n, Q_n, s_n, t_1, t_0)]dF(t_1)dF(t_0). \tag{15}$$

Then the principle of optimality for this model takes the form  $C_n(x_n) = \inf_{Q_n \geq 0, s_n \geq 0} \{B_n(x_n, Q_n, s_n)\}$  for  $n \geq 1$ . In (15),  $EC(x_n, Q_n, s_n)$  is the one-cycle cost function whose properties are analyzed in the previous section and  $\alpha$  is the discount factor ( $\alpha \in [0,1]$ ). In the ensuing theorem, following a similar approach as in the one-cycle problem, we demonstrate inductively that the existence and uniqueness of the optimal policy for the last  $n-1$  cycles is preserved in the last  $n$  cycles.

**Theorem 2.** For the last  $n$  cycles with initial inventory  $x_n$ , we have the following properties:  
 1) For any given  $x_n$ , there is a unique stationary point  $(Q_n^*, s_n^*)$  given by the solution to the following expressions,

$$s_n^* = G_{n-1}^{-1}(c_p - c_e), \tag{16}$$

$$-c_e - c_p + c_p F(Q_n^* + x_n) + \int_{s_n^*}^{Q_n^*} F(Q_n + x_n - t_0)g_{n-1}(t_0)dt_0 = 0, \tag{17}$$

$$Q_n^*(x_n) = Q_n^*(0) + |x_n|, \text{ if } x_n \leq 0, \tag{18}$$

where

$$g_n(x) = c_p f(x) + c_r f^{(2)}(x) + \alpha \int_0^x C_n''(x-t)f(t)dt,$$

and  $G_n(x) = \int_0^x g_n(t)dt$  for  $x > 0$  and  $n > 0$ .

2) In the neighborhood of the stationary point  $(Q_n^*, s_n^*)$ ,  $B_n(x_n, Q_n, s_n)$  is strictly convex for any given  $x_n$ .

The proof is given in the Appendix. Recall that in the single-cycle model, it is shown that for the optimal policy,  $s^*$  must be less than  $Q^*$ , that the optimal policy with initial backlog is to order the sum of optimal quantity with zero initial inventory and the backlogged amount, that the optimal policy exists and is unique, and that the necessary and sufficient conditions for the optimal policy with non-negative inventory are provided in (7) and (8). For comparison, equations (7) and (8) can be respectively expressed as

$$s^* = G_0^{-1}(c_p - c_e),$$

$$-c_e - c_p + c_p F(Q^* + x) + \int_s^{Q^*} F(Q^* + x - t_0)g_1(t_0)dt_0 = 0$$

For the multi-cycle model, according to Theorem 2, the basic features of the one-cycle problem are preserved. That is,  $s_n^*$  is less than  $Q_n^*$ . The optimal regular order quantity can be expressed as  $Q_n^*(x_n) = Q_n^*(0) + |x_n|$ . The two properties of Theorem 2 state that there exist a unique stationary

point for the total cost function, and that the unique point  $(Q_n^*, s_n^*)$  is a global minimum solution. The necessary and sufficient conditions for the optimal policy are provided in (16)-(18).

Note that (16)-(18), similar to (7)-(8), indicate that the initial inventory of each cycle does not affect the emergency order-up-to level, as long as the initial inventory is not unreasonably high. This largely reflects the vintage condition of the initial inventory. Because of the assumption that emergency orders are one-period old, the initial inventory of the following cycle is then two-period old and will perish before the next emergency orders. Hence, for the scenario where the emergency order is one-period old, each cycle's initial inventory has no effect on the order-up-to level.

### A Heuristic Algorithm for the Multi-cycle Inventory Problem

In the multi-cycle problem, the optimal order policies for each cycle are determined sequentially. For the optimal policy doublet of cycle  $n$ , we need to know the explicit decision function of the cycle  $n-1$ , and so on. This is essentially a discrete-time, continuous-state, stochastic optimization problem. Based on the sufficient and necessary conditions for the optimal policies developed in Theorem 2, we provide a heuristic algorithm to approximate the optimal solution for the multi-cycle problem.

The heuristic algorithm is as follows. As an initial setup, for each cycle  $j$ , estimate the range of the initial platelet inventory. Discretize the non-negative range into  $m$  equal intervals depending on the precision desired. For negative range of  $x_j$ , let them be treated as if the initial inventory is zero. For the root-finding steps, we select the adjacent integer (to the root) that has the smallest residual value.

**Step 1.** For cycle 1, determine  $s_1^*$  by finding the root of equation (7) using a linear search.

**Step 2.** For each state of  $x_1$ , determine  $Q_1^*$  by substituting  $s_1^*$  into equation (8) and finding the root of (8) using a linear search.

**Step 3.** For cycles  $j = 2, \dots, n$ , determine  $s_j^*$  by finding the root of equation (16) using a linear search.

**Step 4.** For each state of  $x_j$ , determine  $Q_j^*$  by substituting  $s_j^*$  into equation (A.12) in the Appendix and by finding the root

of (A.12) using a linear search.  
**Step 5.** Calculate the total expected cost  $C_n$  using the expression (A.8) in the Appendix.

	$x_2 = 200$	$s_1^* = 124$	$Q_2^* = 151$
<b>Cycle 3</b>	$x_3 = 30$	$s_1^* = 124$	$Q_3^* = 225$
<i>Total Cost = 15,266</i>			
<i>Total Computation Time: 1.402 sec.</i>			

**Numerical Illustration and Sensitivity Analysis**

Consider a local hospital making platelet inventory decisions daily based on a weekly forecast, approximately 3 cycles. Demands (in terms of pools) are assumed to be normally distributed and within the range [1, 200]; the estimated cost parameters and the initial inventory are shown in Table 1. These are realistic estimates of real-life data (Haijema et al., 2007). Using the heuristic algorithm developed in the preceding section, we now examine the model’s behavior with several sets of numerical investigations. The algorithm is coded in Matlab 2007b (a copy of the main code is shown in the Appendix). We have run a number of instances, each of which takes less than a few seconds on a typical personal computer.

**Table 1. Parameters of the Numerical Example**

Number of Cycles	$c_e$ (US\$100/pool)	$c_p$ (US\$100/pool)	$c_r$ (US\$100/pool)
3	5	30	10
Initial Inventory (pools)	Mean of the daily demand $\mu$ (pools)	Standard deviation $\sigma$ (pools)	
30	100	25	

Following the algorithm, a set of  $Q_1^*$  and  $s_1^*$  are first computed for a range of  $x_1$ . Then a set of  $Q_2^*$  and  $s_2^*$  are computed for a range of  $x_2$ . As shown in Table 2, the optimal ordering policy for an initial inventory of 30 units is  $Q_3^* = 225$ ,  $s_3^* = 124$ , and the optimal expected cost is US\$15,266. The breakdown of the optimal expected cost is: emergency cost (US\$2,016), shortage cost (US\$4,981), and outdate cost (US\$8,269).

**Table 2. Optimal Policies for the Numerical Example (Base Case)**

<b>Cycle 1</b>	$x_1 = 0$	$s_1^* = 124$	$Q_1^* = 254$
	$x_1 = 1$	$s_1^* = 124$	$Q_1^* = 252$
	$x_1 = 2$	$s_1^* = 124$	$Q_1^* = 251$
	...		...
	$x_1 = 200$		$Q_1^* = 150$
<b>Cycle 2</b>	$x_2 = 0$	$s_1^* = 124$	$Q_2^* = 256$
	$x_2 = 1$	$s_1^* = 124$	$Q_2^* = 255$
	...		...

Figure 3 is a plot of the corresponding total expected cost with respect to different values of  $Q_3$  and  $s_3$ . For this example problem, it is interesting to observe that the expected cost function is sensitive to changes in  $Q_3$ . For example, a 10% increase in  $Q_3$  results in an increase of around 15% in the total cost, while a 10% decrease in  $Q_3$  would increase the total cost by around 20%. It can be observed that underestimating  $Q_3^*$  would result in a bigger error in the total expected cost than overestimating  $Q_3^*$  by the same percentage. On the other hand, the total cost is quite insensitive to changes in  $s_3$ . A 10% increase or decrease in  $s_3$  results in less than 3% change in expected total cost.

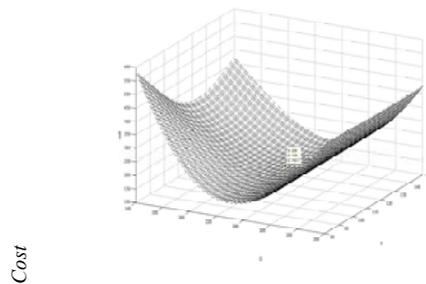
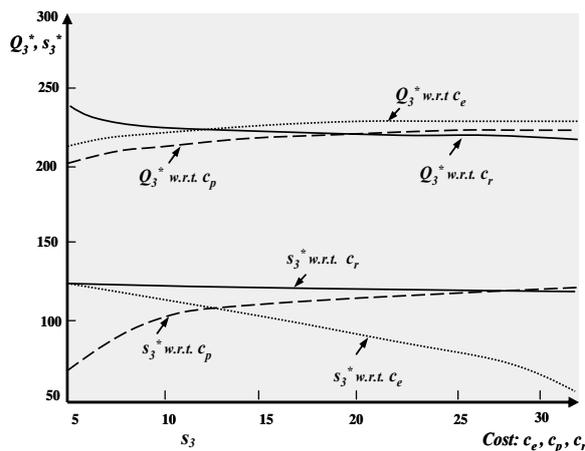


Figure 3. Plot of the Expected Total Cost with respect to different values of  $(Q_3, s_3)$

Next, we explore the change in the optimal policy  $(Q_3^*, s_3^*)$  with respect to the change in the cost parameters  $c_r, c_e$  and  $c_p$  (Figure 4). As expected, an increase in the emergency cost  $c_e$  decreases  $s_3^*$  while increases  $Q_3^*$ ; an increase in the outdate cost  $c_r$  decreases both  $Q_3^*$  and  $s_3^*$ ; and an increase in the shortage cost  $c_p$  increases both  $Q_3^*$  and  $s_3^*$ .



**Figure 4. Effects of Cost Parameters on the Optimal Policies**

Moreover, our numerical analysis shows that the optimal  $(Q_3^*, s_3^*)$  is insensitive to the changes of  $c_r$ ,  $c_e$  and  $c_p$ . Especially,  $s_3^*$  is very insensitive to the change of  $c_r$ ; and  $Q_3^*$  is very insensitive to the change of all the cost parameters. For example, compared with the base case, if  $c_e$  is underestimated by 40%, the incorrect  $(Q_3', s_3')$  would be (223, 131), only about 1% and 5% away from their respective optimal values. The corresponding expected total cost would be higher by roughly 1% from the optimal one.

Similarly, we explore the change in the optimal  $Q_3^*$  and  $s_3^*$  with respect to the change in demand parameters. We found that both the optimal  $Q_3^*$  and  $s_3^*$  are sensitive to changes of the expected demand (Table 3). For example, if expected demand is overestimated by 10% (from 100 to 110), the optimal policies change to (248, 134), which are about 10% and 8% away from the original values. The expected cost, using Figure 2, can be found as US\$17,700, which is about 16% higher than the optimal cost. From Table 3, it appears that both underestimating and overestimating the expected demand has significant impact on the total expected cost, with underestimating having a higher impact. With respect to changes in the standard deviation, there is little change in the total expected cost.

**Table 3. Change of  $Q_3^*$  and  $s_3^*$  w.r.t. Demand Parameters**

Normal distribution	$Q_3^*$	$s_3^*$	Cost
Base case, $\mu = 100; \sigma = 25$	225	124	152
Change $\mu = 110; \sigma = 25$	248	134	177

of $\mu$	$\mu = 125; \sigma = 25$	284	149	323
	$\mu = 90; \sigma = 25$	200	113	196
	$\mu = 75; \sigma = 25$	164	97	433
Change of $\sigma$	$\mu = 100; \sigma = 31.5$	228	128	153
	$\mu = 100; \sigma = 18.75$	221	118	155

Table 4 illustrates similar behavior with respect to a set of uniform demand distributions. We observe the similar impact of the expected demand on optimal decisions. Again, the variance has little impact. In Table 5, we provide comparisons of the several typical demand distributions. It can be observed that the distribution, *per se*, has little impact on the decisions.

**Table 4. Change of  $Q_3^*$  and  $s_3^*$  w.r.t. Demand Parameters with Uniform Distribution**

Uniform distribution		$Q_3^*$	$s_3^*$
Base case, $\mu = 100; \sigma = 25$		223	127
Change of $\mu$	$\mu = 110; \sigma = 25$	247	137
	$\mu = 125; \sigma = 25$	282	152
	$M = 90; \sigma = 25$	202	118
	$M = 75; \sigma = 25$	162	100
Change of $\sigma$	$\mu = 100; \sigma = 31.5$	226	133
	$\mu = 100; \sigma = 18.75$	219	120

**Table 5. Change of  $Q_3^*$  and  $s_3^*$  w.r.t. Different Distributions with Roughly the Same  $\mu$  and  $\sigma$**

$\mu = 100; \sigma = 9.5$	$Q_3^*$	$s_3^*$
Normal Distribution	214	108
Uniform Distribution ( $a=84, b=116$ )	219	110
Binomial Distribution ( $p=0.1, n=1000$ )	216	108
Poisson Distribution ( $\lambda=100$ )	217	108
Gamma Distribution ( $\theta=0.9, k=111$ )	217	108

The preceding sensitivity analysis shows that the accuracy in estimating the expected demand is by far the most important aspect for the platelet

inventory decision maker, as the error could result in a significant amount of extra cost. The analysis also points out that overestimating the expected demand results in a relatively smaller amount of error than underestimating it. However, changes in both the variance and the type of distribution have little impact on the optimal policy. In general, when there is a need to adjust the regular order, the decision maker should do so judiciously as the total expected cost is sensitive to the change in regular order. Such behavior is not observed regarding the emergency order-up-to level decisions.

For this one-period old emergency order scenario, the emergency order-up-to level is independent of the initial inventory of the cycle. Moreover, when there is backlogged demand in the beginning of the cycle, the manager can first refill the backlogged demand and then determine the regular order quantity for zero initial inventory.

### Conclusion and Future works

This paper presents an analysis of a periodic review inventory system of a perishable product with emergency replenishments, characterized by fresh regular orders and one-period-old emergency replenishments. We first formulate the problem as a one-period model and then as a multi-cycle model. For both cases, we prove the existence of a unique optimal solution and derive the necessary and sufficient conditions for the optimal policy. A heuristic algorithm is designed based on the optimality conditions. We perform an extensive sensitivity analysis which, unlike typical inventory problems, shows that the total expected cost is sensitive to the regular order policy. It also shows that the optimal policy is sensitive to changes in the expected demand, suggesting to decision makers the significance of having an accurate demand forecast.

We believe this paper merely scratches the surface of this complex problem of perishable products and combined use of regular order and emergency order. There are quite a number of scenarios involving different combinations of cycle length and vintage of emergency orders. While there are many possible scenarios, each of which could have different results, we believe the approach taken in this paper is applicable to other scenarios as well. The scenario with fresh emergency platelets should be investigated next. It might be interesting to generalize the models in this paper by incorporating the fixed replenishing costs of regular and emergency orders and the examination of lost demand, thus making the model applicable to other perishable products.

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### Appendix

Proofs of the Lemmas and Theorems are omitted here due to the limitation of the space, and are available on request.

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