5-26-2012

Shannon Wavelet Chaotic Neural Network with Nonlinear Self-feedback

Yaoqun XU
Institution of System Engineering, Harbin University of Commerce, Harbin, 150028, China, xuyaoqun@sina.com

Jian LIU
Institution of System Engineering, Harbin University of Commerce, Harbin, 150028, China

Shannon Wavelet Chaotic Neural Network with Nonlinear Self-feedback

Yaoqun XU[1*], Jian LIU
[1]Institution of System Engineering, Harbin University of Commerce, Harbin, 150028, China

Abstract: Shannon wavelet chaotic neural network is a kind of chaotic neural network with non-monotonous activation function composed by Sigmoid and Wavelet. In this paper, wavelet chaotic neural network models with different nonlinear self-feedbacks are proposed and the effects of the different self-feedbacks on simulated annealing are analyzed respectively. Then the proposed models are applied to the 10-city traveling salesman problem (TSP) and by comparison the performance of the model with wavelet self-feedback is superior to that of the rest others presented in this paper. Moreover, the performance of the model with wavelet self-feedback is improved by the scale index and the location index of the wavelet. Finally, the dynamics of an internal state of the model for the 10-city TSP is researched, including chaotic area distribution, the largest Lyapunov exponents and the effects of the chaotic distribution on the performance of the network for 10-city TSP. The numerical simulations show that the models can converge to the global minimum or approximate solutions more efficiently than the Hopfield network, and the performance of the model with wavelet self-feedback is superior to that of the others.

Key words: Shannon wavelet chaotic neural network; Nonlinear self-feedback; Optimization; Chaotic area distribution;

1 INTRODUCTION

The Hopfield network, proposed by Hopfield and Tank [1, 2], has been extensively applied to many fields in the past years. The Hopfield neural network converges to a stable equilibrium point due to its gradient decent dynamics. However, it causes severe local-minimum problems whenever it is applied to optimization problems. Chaotic neural network has abundant dynamic property to move chaotically over fractal structure in the state space so that it can suffer from the local minima. Chen and Aihara[3], by introducing a simulated annealing to chaotic neural network, effectively improved the performance of chaotic neural network in solving combinational optimization problems. From then on, more and more research results [4~8] turned out in chaotic neural network. Wavelet is a strong mathematic tool in various science and technology fields, and the combination of wavelet and neural network has been researched numerously because of the excellent properties of wavelet such as multilayer feed-forward wavelet neural networks [9, 10]. Wavelet chaotic neural network with non-monotonous activation function composed by sigmoid and wavelet functions has been used to solve 10-city TSP effectively [11, 12]. In this paper, we introduce two categories of the simplest nonlinear functions into wavelet chaotic neural network as self-feedback and use the model to solve 10-city TSP. The model is described in Section2. In Section 3, the effects of the nonlinear self-feedbacks on simulated annealing are discussed. The model with nonlinear self-feedbacks is applied to 10-city TSP and used to compare with other models in Section 4. In Section 5, the chaotic dynamics of an internal state of the model is analyzed in solving 10-city TSP, including chaotic area distribution and the largest Lyapunov exponents.

2 SHANNON WAVELET NETWORK WITH NONLINEAR SELF-FEEDBACK

The model of Shannon wavelet chaotic neural network is described as follows.

$$x_i(t) = f(y_i(t)) \tag{1}$$

* Corresponding author. Email: xuyaoqun@sina.com (Yaoqun Xu)

$$y_i(\mathrm{t}+1) = ky_i(\mathrm{t}) + \alpha\left[\sum_{\substack{j=1 \\ j\neq i}}^{n} w_{ij}x_j(\mathrm{t}) + I_i\right] - z_i(\mathrm{t})g[(x_i(\mathrm{t}) - I_0)] \tag{2}$$

$$f(\mu) = \mathrm{Sigmoid}(\mu, \varepsilon_1) + coef \cdot \mathrm{Shannon}(\mu, \varepsilon_2) \tag{3}$$

$$\mathrm{Sigmoid}(\mu, \varepsilon_1) = \frac{1}{1 + \exp(\mu / \varepsilon_1)} \tag{4}$$

$$\mathrm{Shannon}(\mu, \varepsilon_2) = \frac{\sin\pi(\mu/\varepsilon_2 - \frac{1}{2}) - \sin 2\pi(\mu/\varepsilon_2 - \frac{1}{2})}{\pi(\mu/\varepsilon_2 - \frac{1}{2})} \tag{5}$$

where

x_i = output of neuron i,

y_i = internal state of neuron i,

w_{ij} = connection weight from neuron j to neuron i,

I_i = input bias of neuron i,

g = nonlinear self-feedback,

α = positive scaling parameter for inputs,

k = damping factor of nerve membrane ($0 \le k \le 1$),

z_i = self-feedback connection weight,

ε_1, ε_2 = steepness parameters of the activation function,

Sigmoid = Sigmoid function,

Shannon = ShannonWavelet function,

c = a positive coefficient before W.

Chaotic searching is generated in chaotic neural network by the introduction of self-feedback, so different self-feedbacks can make chaotic neural network exhibit different chaotic dynamics behaviors. Besides the non-monotonous activation function, the model has a term of nonlinear self-feedback which is distinct from other models. Here, the form of the nonlinear self-feedback function $g(x)$ is chosen under the following considerations.

(1) In the region $x\in$ [-1,1], $|g(x)|<|x|$ for the whole region or $|g(x)|>|x|$ for $x\in[-x_0, x_0]$ and $|g(x)|<|x|$ for the other region where $x_0<1$ except the point 0 where $g(0)=0$. This represents that the point 0 is the fixed point of $g(x)$, and the other region in $x\in$ [-1,1] can be decreased or increased.

(2) In the region $x\in(0,1]$, $g(x)>0$; In the region $x\in[-1,0)$, $g(x)<0$; In the point $x=0$, $g(x)=0$. This stands for that $g(x)$ can not change the sign of x and $x=0$ is the fixed point of $g(x)$.

Based on the conditions given above, $g(x)$ is taken the following three functions in term of $|g(x)|$.

For $|g(x)|<|x|$,

$$g_1(x)=x\exp(-x^2/2),\ x\in[-1,1]. \tag{6}$$

$$g_2(x)=x(1-x^2),\ x\in[-1,1]. \tag{7}$$

For $|g(x)|>|x|$,

$$g_3(x) = 4x(1-x^2),\ x\in[-1,1]. \tag{8}$$

3 EFFECTS OF NONLINEAR SELF-FEEDBACK ON SIMULATE ANNEALING

Although the chaotic neural network is a promising technique for optimization problems, the converging process has not been satisfactorily solved in relation to chaotic dynamics. It is difficult to decide how to control the chaotic behavior in chaotic neural network for converging to chaotic dynamics. To make use of the chaotic dynamics, different forms of simulated annealing can be drawn on [3,16,17], which can introduce transiently chaotic dynamics to chaotic neural network. In order to control the chaotic dynamic regimes, the exponential simulated annealing strategy is also embedded into wavelet chaotic neural network. So the transiently wavelet chaotic neural network with nonlinear self-feedback can be described as equations (1) ~ (5) together with equation (9).

$$z_i(t+1) = (1-\beta) z_i(t) \tag{9}$$

where β is the simulated annealing parameter ($0 \le \beta \le 1$). The larger the parameter β is, the faster the decrement speed is, and the smaller β conduces to the network searching the globally optimal or approximate optimal solution. In addition, the initial temperature z_i has a subtle relation with the performance of transiently chaotic neural network. While a high temperature is required to investigate chaotic dynamics, a low temperature is preferred for combinational optimization application[5].However, the nonlinear self-feedback introduced in the Section 2 draws on a larger temperature as to $|g(x)|<|x|$ and a smaller temperature with respect to $|g(x)|>|x|$ without destructing the performance of the network but improving it. It is because that the effects of the nonlinear self-feedback on the network have two aspects. On the one hand, the nonlinear self-feedback can further affect the value of the searching states of transiently chaotic dynamics besides the exponential simulated annealing, that is, for $|g(x)|<|x|$ the nonlinear self-feedback can decrease the values of the searching states and for $|g(x)|>|x|$ the nonlinear self-feedback can increase the values of the searching states to some extent. Especially for $|g(x)|<|x|$, the appropriate nonlinear self-feedback can make the searching states converge earlier so that the network has an ability to search the globally optimal solution with smaller iterations.

4 APPLICATION TO TRAVELING SALESMAN PROBLEM

A computational energy function which is to minimize the total tour length while simultaneously satisfying all constrains takes the follow form [1].

$$E=\frac{W_1}{2}\left\{\sum_{i=1}^{n}\left[\sum_{j=1}^{n}x_{ij}-1\right]^2+\sum_{j=1}^{n}\left[\sum_{i=1}^{n}x_{ij}-1\right]^2\right\}+\frac{W_2}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}(x_{k,j+1}+x_{k,j-1})x_{ij}d_{ik} \tag{10}$$

where $x_{i0}=x_{in}$ and $x_{i,n+1}=x_{i1}$. W_1 and W_2 are the coupling parameters corresponding to the constrains and the cost function of the tour length, respectively. d_{xy} is the distance between city x and city y.

Therefore, the equation (2) can be described as follows.

$$y_i(t+1)=ky_i(t)+\alpha\left\{W_1\left[\sum_{\substack{l=1\\l\neq j}}^{n}x_{il}(t)+\sum_{\substack{k=1\\k\neq i}}^{n}x_{kj}(t)\right]-W_2\left[\sum_{\substack{k=1\\k\neq i}}^{n}d_{ik}x_{k,j+1}(t)+\sum_{\substack{k=1\\k\neq i}}^{n}d_{ik}x_{k,j-1}(t)\right]+W_1\right\}-z_i(t)(x_i(t)-I_0) \tag{11}$$

This paper adopts the unitary coordinates of 10 cities same with the conference [3]. The shortest distance of the 10 cities is 2.6776.

In the numerical simulations, we remain the parameters $W_1= W_2=1$, $\varepsilon_1=20$, $\varepsilon_2=0.8$, $k=1$, $\alpha=1$ unchanged and change the simulated annealing parameter β and the temperature $z_i(1)$. For $g_1(x)$, $z_i(1)=0.4$, $I_0=0.65$; For $g_2(x)$, $z_i(1)=0.4$, $I_0=0.47$; For $g_3(x)$, $z_i(1)=0.1$, $I_0=0.48$. 500 different initial conditions of y_{ij} are generated randomly in the region [-1, 1] for different β. The results of the numerical simulations are shown in the Table 1 (RGM= Rate of global minima; RLM= Rate of local minima; RIS=Rate of infeasible solutions; AIC=Average iterations for convergence.).

As is the Table 1 shown, the performance of the network with $g_1(x)$ is superior to that of the one with $g_2(x)$ and the performance of the network with $|g(x)|<|x|$ such as $g_1(x)$ and $g_2(x)$ is superior to that of the one with $|g(x)|>|x|$such as $g_3(x)$. The explanations are given as follows.

Table 1 The results of the numerical simulations for the proposed network with nonlinear self-feedback

$g(x)$	β	RGM	RLM	RIS	AIC
	0.004	499(99.8%)	1(0.2%)	0(0.0%)	124
$g_1(x)$	0.003	500(100%)	0(0.0%)	0(0.0%)	135
	0.002	500(100%)	0(0.0%)	0(0.0%)	153
	0.004	497(99.4%)	3(0.6%)	0(0.0%)	126
$g_2(x)$	0.003	500(100%)	0(0.0%)	0(0.0%)	131
	0.002	500(100%)	0(0.0%)	0(0.0%)	164
	0.004	496(99.2%)	4(0.8%)	0(0.0%)	123
$g_3(x)$	0.003	498(99.6%)	2(0.4%)	0(0.0%)	142
	0.002	500(100%)	0(0.0%)	0(0.0%)	166

First, for $|g(x)|<|x|$, the too small value of $g(x)$ will make the states decreased so fast that it is difficult to suffer from the local minima. In contrast, the appropriate value of $g(x)$ can remain a certain searching ability of the states so that it can prevent the states trapping into the local minima and converge to the globally optimal minima with the help of an appropriate temperature. Seen from the Fig.1, the corresponding value of $g_2(x)$ are much smaller than those of $g_1(x)$ in the region $x \in [-1,0) \cup (0,1]$. So the performance of the network with $g_2(x)$ is inferior to that of the network with $g_1(x)$ and the value of $g_1(x)$ is fit for the network solving 10-city TSP.

Second, the value of nonlinear self-feedback has a strong impact on the state jump. The temperature of simulated annealing has an effect on controlling the possibility of random jump in state space [5]. For $|g(x)|>|x|$, it is easy to jump out the state space controlled by the temperature. That is to say, under the condition of the same temperature, the possibility is larger to jump out the controlled state space for $|g(x)|>|x|$ than that for $|g(x)|<|x|$. Therefore, in order to control the state jump the network with $|g(x)|>|x|$ needs a lower temperature.

In order to be convenient to compare the performance of the proposed network with that of the network with the linear self-feedback $g_4(x) = x$ (WCNN), the parameters of transiently WCNN are taken as β=0.003, $z_i(1)$=0.29~0.4, I_0=0.65, and the other parameters remain unchanged. The results of the numerical simulations with 500 iterations are summarized in Table 2.

Table 2 The results of the numerical simulations for the network with $g_4(x)$

$g(x)$	$z_i(1)$	RGM	RLM	RIS	AIC
	0.25	498(99.6%)	2(0.4%)	0(0.0%)	125
$g_4(x)$	0.30	499(99.8%)	1(0.3%)	0(0.0%)	127
	0.35	500(100%)	0(0.0%)	0(0.0%)	144
	0.40	500(100%)	0(0.0%)	0(0.0%)	204

By the comparison between Table 1 and Table 2, the network with $g_1(x)$ is not only more effective than the network with the linear self-feedback $g_4(x)=x$ in the same temperature $z_i(1)$=0.4 but also superior to the network with the smaller temperature $z_i(1)$. Moreover, the number of the average iterations for the network convergence to globally optimal minima at β=0.003 is also smaller than that of the network proposed by Chen and Aihara in reference [3], which costs 398 iterations at β=0.003 before reaching 100% globally minima. Such ability of the proposed network owes to the non-monotonous activation function and especially the nonlinear self-feedback. And by the above analysis one can believe that there exists a better nonlinear self-feedback with appropriate function values in the region $x \in [-1,1]$ which is more effective to solve the 10-city TSP.

By the above analysis, now we can make sure that $g_1(x)$ is the most effective nonlinear self-feedback which can improve the performance of the network with maximum limit among the above four self-feedback functions and believe that there is a better nonlinear self-feedback more effective to solve the 10-city TSP. Now we try to find such a function based on $g_1(x)$. As is known, $g_1(x)$ is the mother function of Gauss wavelet family and so Wavelet Analysis theory can be applied to solve 10-city TSP. For continuous wavelet $\Phi_{s,l},(x)=\Phi[s(x-l)]$ the scale index s and the location index l are real numbers that scale and dilate the mother function to generate wavelets. s indicates the wavelet's width, and l gives its position. What makes wavelet bases especially interesting is the self-similarity caused by the scales and dilations. $g_1(x)$ applied to solve 10-city TSP can be seen as a generated wavelet with 1.0 as the scale index, I_0 as the location index. The effect of the scale index on the $g_1(x)$ is shown in Fig.2.

Fig.1 shows that the generated wavelet accords with the analysis in Section 3. The numerical simulations are taken with different scale and location indexes, and the results are shown in Table 3. The parameters are set as $z_i(1)=0.4$, $\beta=0.003$.

As is seen from the Table 3, the performance is improved due to the decrease of the scale index. Therefore, the generated wavelet as nonlinear self-feedback is more effective to solve 10-city TSP.

Table 3 The results of the numerical simulations for the network with the generated wavelet

scale	location	RGM	RLM	RIS	AIC
0.9	0.60	500(100%)	0(0.0%)	0(0.0%)	123
0.9	0.65	500(100%)	0(0.0%)	0(0.0%)	127
0.85	0.70	500(100%)	0(0.0%)	0(0.0%)	129
0.85	0.75	500(100%)	0(0.0%)	0(0.0%)	121
0.75	0.80	500(100%)	0(0.0%)	0(0.0%)	119

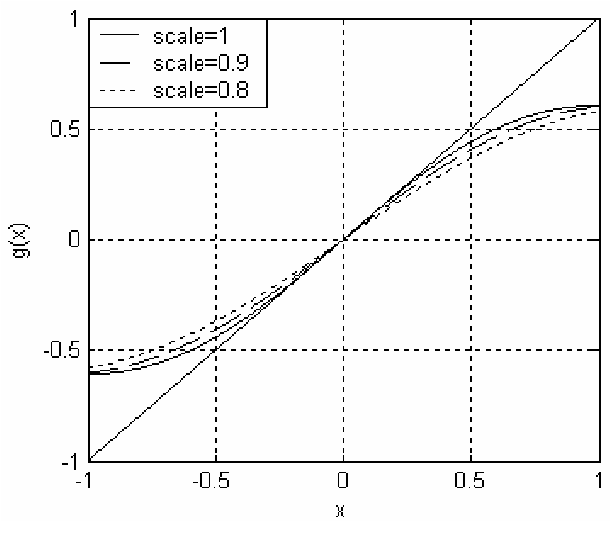


Fig.1 the generated wavelets with different scales

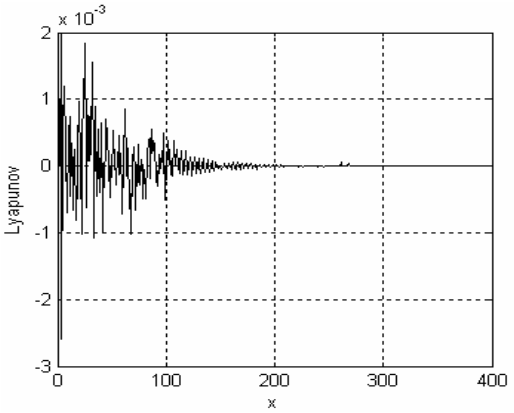
5 CHAOTIC DYNAMICS OF THE INTERNAL STATE

Fig.2 (*a*) indicates that the transiently chaotic searching of WCNN is distinct with that of the network proposed by Chen and Aihara in that the searching state can be negative. Therefore, the transiently chaotic researching area of the proposed model with nonlinear self-feedback is wider than that of the proposed model by Chen and Aihara. And Fig.2 (*b*) and the left three pictures of (*c*) indicate that transiently chaotic dynamics behavior exists during chaotic neural networks solving the 10-city TSP and the chaotic area distribution of the chaotic neural network with different nonlinear self-feedback is distinct with each other.

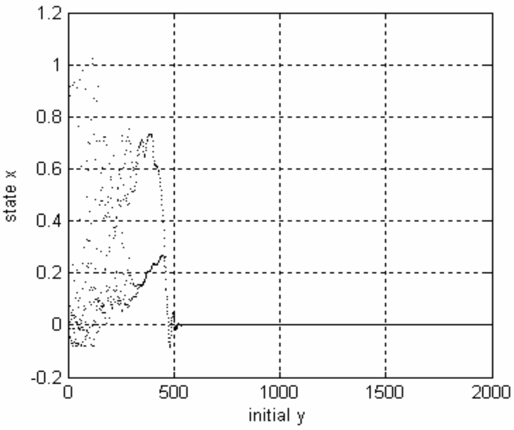
Chaotic area distribution can instruct people to choose the parameters in order to generate chaos. What's more, chaotic area distributions of networks in solving TSP can predict the performance of the networks. The right three pictures of Fig.2 (*c*) indicate that the evolutions of chaotic area distributions at k=1 with the decrement of the simulated annealing temperature generate transiently chaotic searching. The simulation results are summarized in Table 4. Table 4 indicates that the simulation results accords with the above analysis.

Table 4 The results of the numerical simulations for the network with $g_1(x)$, $y=x(g_4(x))$ and $g_3(x)$.

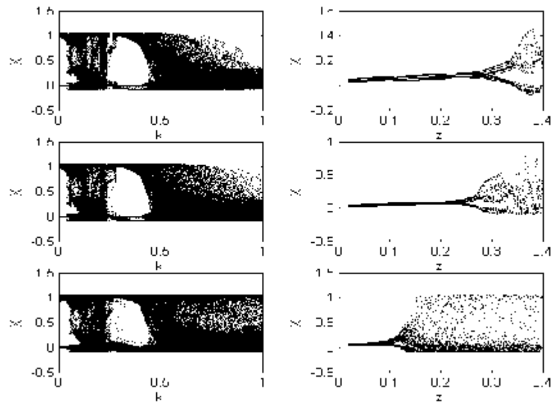
$g_i(x)$	RGM	RLM	RIS	AIC
$g_1(x)$	1000(100%)	0(0.0%)	0(0.0%)	131
$g_4(x)$	1000(100%)	0(0.0%)	0(0.0%)	213
$g_3(x)$	996(99.6%)	4(0.4%)	0(0.0%)	580



(*a*)



(*b*)



(*c*)

Fig.2 Chaotic dynamics of an internal state, the largest Lyapunov exponents and chaotic area distribution with the parameters $z_i(1)$=0.4, I_0=0.65. (*a*) The transiently chaotic searching of an internal state; (*b*) The largest Lyapunov exponents of the network for the 10-city TSP; (*c*) The chaotic area distribution of WCNN respectively with $g_1(x)$, linear self-feedback $y=x$ and $g_3(x)$ for the 10-city TSP; the left three pictures are chaotic area distribution on *k* and the right three pictures are chaotic area distribution on $z(t)$ at k=1.

6 CONCLUSION

The proposed nonlinear self-feedback of the model has a full impact on the performance of the model for 10-city TSP and has a significant meaning for chaotic neural network. The numerical simulations indicate that WCNN with wavelet self-feedback is more effective to solve combinational optimization problem than the others.

7 ACKNOWLEDGMENTS

This work is supported by the Nature Science Foundation of Heilongjiang Province (F2010-35).

REFERENCES

[1] J. -J. Hopfield, D. -W. Tank, (1985),Neural Computation of Decision in Optimization Problems, Biol. Cybern. 52: 141-152.

[2] J. -J. Hopfield, (1982),Neural Networks and Physical Systems with Emergent Collective Computational Abilities," In Proc Natl Acad Sci, 79: 2554-255.

[3] L.-N. Chen, K. Aihara, (1995),Chaotic Simulated Annealing by a Neural Network Model with Transient Chaos, Neural Networks, 8(6): 915-930.

[4] C.-S Zhou, T.-L. Chen, W.-Q. Huang, (1997),Chaotic neural network with nonlinear self-feedback and its application in optimization," Neurocomputing, 14: 209~222.

[5] D. Zhen, H. Leung, Z. –W. Zhu, (2003),Critical temperature of the transiently chaotic neural network, Mathematical and computer modelling, 37: 779~783.

[6] J.-S. Lin, (2001),Annealed chaotic neural network with nonlinear self-feedback and its application to clustering problem, Pattern recognition, 34: 1093~1104.

[7] Y.-Y. He, L.-P. Wang, (2000),Chaotic neural networks and their application to optimization problems, Control theory and application, 6(7): 847~852.

[8] L. Wang and K. Smith, (1998),On chaotic simulated annealing, IEEE Trans on NN, 9(4): 716~718.

[9] Q. Zhang, A. Benveniste,(1992)Wavelet networks, IEEE Trans on NN, 3(11):889~898.

[10] J. Zhang, et al, (1995),Wavelet neural networks for function learning, IEEE Trans on SP, 43(6): 1485~1497.

[11] Y.-Q. Xu, M. Sun, G.-R. Duan, (2006),Wavelet Chaotic Neural Networks and Their Application to Optimization Problems,Lecture Notes in Computer Science, vol. 3791, Springer press,:379-384.

[12] Y.-Q. Xu, M. Sun, (2006),Shannon Wavelet Chaotic Neural Network and Its Application to TSP, In Proceedings of the 25th Chinese control congress,: 1172~1176.

[13] L.-N. Chen and K. Aihara, (1997),Chaos and asymptotical stability in discrete-time neural networks, Physica D, 104: 286~325.

[14] A. Potapove, M. Kali, (2000),Robust chaos in neural networks, Physics Letters A, 277(6): 310-322.

[15] Shuai J W, Chen Z X, Liu R T, et al. (1996))Self-evolution Neural Model. Physics Letters-A, 221(5): 311-316

[16] S. Geman, D. Geman, (1984))Stochastic relaxation, Gibbs distributions and the Baysian restoration of image, IEEE Trans. Pattern Anal. Mach. Intell, vol.6,:721-732

[17] J-S Lin, (1999),An annealed Hopfield neural network to vector quantization for image compression, Opt. Engng. vol,38,:599-603