The Realization of a Stochastic Optimization Model for the Empty Container Inventory Based on EDI Information

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Abstract

This paper explores the feasibility of realizing the optimization of the empty container inventory based on EDI information. Firstly the basic structure of the stochastic optimization model for the empty container inventory has been described and two models including the infinite and finite horizon model were constructed. Then several statistical test have been taken and the distribution of the inland turnover time of the empty container, the empty containers' supply and demand identified. Finally we adopt PB to develop the DSS for the stochastic optimization of container inventory based on the shipping EDI information.

Key words: EDI, empty container, DSS

1. Identification of the problem

With the rapid development of the container transportation and the fierce competition on the market, how to reasonably determine the empty container inventory so as to reduce the running cost has become one important issue that all the container transportation enterprises are facing to.

Essentially the reasonably determination of the level of the empty container inventory falls into the scope of the optimization of the empty transport vehicle's inventory. In general, the present research is focusing on the following two aspects: to estimate the imbalance coefficient of empty container flow by experience or statistic samples, to realize the optimization of the empty containers inventory by means of the optimization models mainly the programming model. Because of the quite great uncertainty of the container flow, it is unavoidable to collect large amount of data to support the realizing of the quantified model to optimize the empty container inventory. So this paper is ready to take a systemic study on the information transmitted through the shipping EDI platform and the construction of the stochastic optimization model, then to explore the feasibility of realizing the optimization of the empty container inventory based on EDI information.

2. The analysis on the function and data structure of shipping EDI platform

Transmitting the information is the traditional core function of the information service provided by the traditional EDI platform, but the content of the information service has begun to change under the present network environment, and to emphasize particularly on providing the service of the value-added information step by step. What is the value-added information? Comparing with the original information, it has an obvious increase in the content of the information obtained by reasoning, analyzing and processing a group of correlative information. For the shipping EDI platform, the developing tendency in the future is how to further dig and process the information transmitted through the shipping EDI platform rather than to transmit the information simply, and provides more value-added information to the EDI users in their daily operation. Among which, it has been one of the attentions how to provide the information support for the reasonable determination on the empty container inventory.

Now the types of information transmitted through the shipping EDI information platform are IFCSUM, BAPLIE, COSTCO, EXPUSL, CODECO, COEDOR, COARIS, ACKIAG, COARIO, COARID, and so on. From these information, we can get many valued results. For example, IFCSUM contains much information regarding the quantities and directions of the empty container flow. We can identify the running rule of the empty container flow by analyzing IFCSUM. We will explore this issue later. Here we only summarize that it is possible to realize the stochastic optimization of the empty container inventory based on this EDI information.

3. The stochastic optimization model for the empty container inventory

3.1 General Description

The basic structure of the stochastic optimization model for empty container inventory based on the storage theory can be described as follows:

\[ F = (T, Q, X, D, S, L) \]  (1)

In the formula above:
T stands for the length of the planning time, we classify it into the type of the finite and infinite;

Q stands for the fact whether the empty containers’ shortage in a short time is allowed or not;

X stands for the fact whether the model considers the empty container inventory of one point or multiple-points;

D/S stands for the fact whether the demand/supply of the empty containers is stable or random;

L stands for the fact whether the allocation and transportation of the empty containers need to take time or not;

We can construct several models based on the above basic structure. Because of the limitations of pages, we are only giving the infinite and finite horizon models which consider only one point.

3.2 The infinite horizon model

3.2.1 The basic model

Suppose the length of the planning time is infinite. The point for considering is lack of empty container. The shortage of supply of the empty container is not allowed. Let the demanding distribution of the empty containers follows the distribution of \( G(x) \), and the replenishing time of the empty containers is zero. The strategy \((s,S)\) is adopted.

According to the above conditions, we argue that with the infinite planning time horizon, the attention can be focused on the period of one cycle. Suppose the inventory level at the beginning of the period is \( I \), then the maximum level can be reached to \( R = I + Q \). We also suppose the demanding amount of the empty container during this period is \( x \). Hence,

If \( R \geq x \), then the expected cost is equal to:

\[
E(C_1(R)) = \int_{R+1}^{\infty} b(x-R)dG(x) + \delta \times k(R-I)
\]  

(2)

In the above formula, \( h \) is the storing cost of the empty container for the single period, and it is suppose that no storing costs has occurred for the empty container distributed in this period. \( k \) is the distributing cost.

If the distribution of empty containers happened, \( \delta = 1 \); otherwise \( \delta = 0 \)

If \( R < x \), then the expected cost is equal to:

\[
E(C_2(R)) = \int_{R+1}^{\infty} b(x-R)dG(x) + \delta \times k(R-I)
\]  

Here, \( b \) means the temporary leasing cost of the empty container.

So, the total expected cost is:

\[
E(C(R)) = \int_{0}^{R} h(R-x)dG(x) + \int_{R+1}^{\infty} b(x-R)dG(x) + \delta \times k(R-I)
\]

(4)

Let the derivative of the \( E(c(R)) \) is zero, then we can get \( R^* \) (namely \( S^* \)). The next step is to obtain the value of the replenishing point (\( s^* \)) by taking marginal analysis in the following way:

If we replenish the empty container at the beginning of the period, then the expected cost is equal to:

\[
E(c(I)) = \int_{0}^{I} h(I-x)dG(x) + \int_{I+1}^{\infty} b(x-I)dG(x)
\]  

(5)

If we don’t replenish the empty container at the beginning of the period, then the expected cost is equal to:

\[
E(c(R)) = \int_{0}^{R} h(R-x)dG(x) + \int_{R+1}^{\infty} b(x-R)dG(x) + k(R-I)
\]

(6)

If \( E(c(I)) \geq E(c(R)) \), then it is not necessary to replenish the empty container. Namely:

\[
\int_{0}^{I} h(I-x)dG(x) + \int_{I+1}^{\infty} b(x-I)dG(x) \geq \int_{0}^{R} h(R-x)dG(x) + \int_{R+1}^{\infty} b(x-R)dG(x) + k(R-I)
\]  

(7)

When \( I=R \), the above inequality is obviously held, therefore, it is affirmative to have the solution.

When \( I<R \), it is also possible to held the above inequality, then the minimum of the \( I \) is \( s \).

3.2.2 The modification to the basic model
In the above model, we suppose that there is no delay in the replenishing of the empty containers, which can greatly decrease the complexity of the model. In practice, it is obvious that there is the delay of empty containers replenishing. So we must consider this issue and modify the basic model in the following.

The consideration of the delay of empty containers replenishing has two affects on the model:

One affect is the amount of empty containers stored at the beginning of this period which not only includes the amount of empty containers at the end of the last period, but also includes the empty containers that should have been distributed in the previous periods but arrive in this period. i.e. The empty container inventory of this period equals to the ones at the beginning of the period plus the ones which are in the procedure of transport.

The other affect is that the time length of decision-making should extend from this period to the delaying period. i.e. The demand of empty container considered not only includes the one of this period but also includes the empty container demand which happens in the delaying time.

Bases on the above two effects, we can make the following modification to the basic model:

$$ I' = I + \tau \int_0^T xdG(x) $$

In the above formula:

- $T'$ is the delaying time,
- $T_0$ is the span of this period which has been considered.

$$ E(c(R)) = \int_0^\infty h(R - (1 - \frac{\tau}{T_0}))xdG(x) + \int_{T_0}^\infty b((1 + \frac{\tau}{T_0})x - R)dG(x) + k(R - I') $$

In the above, we discussed the infinite model and take one period to consider. However, it is not suitable in the finite horizon model. We will construct the model in the following way.

Suppose there is $n$ periods. The correlative assumption is the same as the infinite model.

In the $jth$ period, the whole expected cost is equal to:

$$ E(C_j(R_j)) = \int_0^R h(R_j - x)dG(x) + \int_{R_j}^\infty b(x - R_j)dG(x) + k(R_j - I_j) $$

$$ \delta = \begin{cases} 1 & R_j > I_j \\ 0 & R_j \leq I_j \end{cases} $$

Then we can define the whole of the optimization object as:

$$ \text{Min} \quad C = \sum_{j=1}^n E(c_j(R_j)) $$

To the above model, we can adopt the dynamic programming theory to solve.

Assuming $I_j$ is the status variable, $R_j$ is the decision variable, the equation of status transferring is:

$$ I_j = R_j - D_j $$

Suppose $E(c_j(R_j))$ is index function, $E|g_j(I_j)|$ is the optimization function, then the reversing model can be constructed as follows:

$$ E(g_j(I_j)) = \min \{E[C_j(R_j)] + E|g_j(I_j)| \} $$

$$ E(g_{n+1}(I_{n+1})) = 0 $$

From the above models, We can get $R_j*$ of every period. The $R_j*$ is the $S_j$ of the strategy of $(s_j*, S_j*)$. The obtaining of $s_j*$ is the same as the infinite horizon model.

3.3 The finite horizon model with one point

3.3.1 Basic model

The above model is only suitable for this condition that delaying time of empty container replenishing is zero.
If the delay were considered, the following adjustment should be given:

\[ I_{j+i} = I_j + Q_{j+i-1} - D_j \]

4. The information support for the stochastic optimization of the empty container inventory provided by the shipping EDI platform

According to the above stochastic optimization model for empty container inventory, we can find the fact below. In order to realize numeric simulation and concrete application, it is essential to acquire some statistic distribution of random phenomena and then to obtain the result of optimization by running the models. The random phenomena mostly include the distribution of empty containers' demand and supply and the turnover time of empty containers and so on. The information can be acquired completely through the information that is transmitted by the shipping EDI information platform. In order to explain the feasibility further, we take advantage of IFCSUM, CODECO and COEDOR to determine the statistic distribution of all kinds of random phenomena.

4.1 The distribution of the inland cycling time of empty containers

According the information of EDI, We dig one company’s statistical data of the inland cycling time whose container ship anchored at Shanghai port.

Firstly, we show the statistical graph of the samples as follows:

Figure 1 The statistical analysis of cycle time

From the above graph, we found: the possible density of the inland cycling time is resembled as the distribution of negative exponent. So we take the statistical test as follows:

The density function of the distribution of the negative exponent is:

\[ f(t) = F'(t) = \frac{1}{\theta} e^{-\frac{t-\eta}{\theta}} \]  

As

\[ E(t) = \int_{-\infty}^{+\infty} t \cdot \frac{1}{\theta} e^{-\frac{t-\eta}{\theta}} dt = \eta + \theta \]

So we can get:

\[ \bar{u} = \eta + \theta \Rightarrow \bar{u} - \eta \]

From the sample data, we work out:

\[ \bar{u} = 3.70, \eta = 2, \theta = 1.70, F(t) = 1 - e^{-\frac{t-2}{1.70}} \]

Then we can get the following table:

Table1: The Result of Statistical Test

<table>
<thead>
<tr>
<th>( i )</th>
<th>( t_i )</th>
<th>( p_i )</th>
<th>( n \hat{p}_i )</th>
<th>( n \hat{p}_i - f_i )</th>
<th>( \frac{(n \hat{p}_i - f_i)^2}{n \hat{p}_i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_1 ): ( 0 \leq t &lt; 6.5 )</td>
<td>23</td>
<td>0.4872</td>
<td>18.0277</td>
<td>-4.9723</td>
<td>1.3714</td>
</tr>
<tr>
<td>( \lambda_2 ): ( 6.5 \leq t &lt; 12.5 )</td>
<td>10</td>
<td>0.3496</td>
<td>12.9351</td>
<td>2.9351</td>
<td>0.6660</td>
</tr>
<tr>
<td>( \lambda_3 ): ( 12.5 \leq t &lt; 18.5 )</td>
<td>3</td>
<td>0.1112</td>
<td>4.1161</td>
<td>1.1161</td>
<td>0.3026</td>
</tr>
<tr>
<td>( \lambda_4 ): ( 18.5 \leq t &lt; 18.5 )</td>
<td>1</td>
<td>0.0519</td>
<td>1.9211</td>
<td>0.9211</td>
<td>0.4416</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.7817</td>
</tr>
</tbody>
</table>

With the above table, we can get:

\[ x^2_{0.05} (k - r - 1) = x^2_{0.05} (3 - 1 - 1) = x^2_{0.05} (1) \]

\[ = 3.841 > 1.3402 \]

Based on the above analysis, we conclude that the turnover time of empty container in Shanghai port follows the distribution of the negative exponent.

By using the statistical information of other ports, we can also get the above conclusion. Then we can conclude that the arriving time of the empty container follows the Poisson distribution.
It should be noted that in different port, because of the difference of the delivering distance of the inland, the difference of the storing and loading time in the port and the difference of the velocity of collecting and delivering by highway, water and railway, although the inland cycling time of the empty container still follows the distribution of negative exponent, the parameters of $\theta$, $\eta$ are different.

4.2 The statistical distribution rule of the empty container’s supply and demand

In the above, we have identified the statistical rule of container of the inland cycling time, then, we will use sample date which was dogged from the database of EDI platform to analyze the distribution rule of empty container’s supply and demand.

Firstly we show the statistical graph of the fact sample as follows:

![Figure 2 The statistical analysis of importing container](image)

From the above graph, we found: the possible distribution of importing empty container is resemble to negative exponent and the possible distribution of importing empty container follows normal distribution. So we take the similar statistical test shown in the above and found the assumption is correct.

4.3 Conclusion

From the analysis, we can find the following result: generally speaking, the distribution of the empty containers' demand follows the distribution of negative exponent, the distribution of the empty containers' supply follows normal distribution and that of the turnover time of the empty container follows the distribution of negative exponent. It is worthy of pointing out that it has different patterns of the distribution both at different ports and at different time. Therefore, when making use of the stochastic optimization model, we must acquire correlative EDI information actually and promptly to choose the appropriate distributive pattern.

5. The developing of the decision support system (DSS) for stochastic optimization of the empty container inventory based on the shipping EDI information.

Based on the model above, we adopt PB to design the DSS for stochastic optimization of container inventory based on the ports and shipping EDI information.

The system structure can be described as follows:

(see next page)

In the above system:

The functions of the application level include the analysis on the container market; the optimization of container distribution; the analysis on the container index.

The function of the data warehouse is to store the original data dug from the source of the EDI platform.

The function of database is to store some parameters used by the application level.

The function of knowledge base is to store the knowledge used by the application level.

The function of model base is to store the models used by the application level.
Because of the limitation of time, we only design the structure and function of the whole of the system and start to develop the original system by means of PB. The task of the first phase is to realize stochastic optimization model for the empty container inventory based on EDI information. It includes three functions: digging correlative data from the EDI platform, making statistic analysis to get the distribution pattern and some parameters, and realizing the numeric simulation of the optimization models.

Bibliography: