Contagion in a Financial System

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Abstract
Financial contagion is often observed in recent financial crisis, which illustrates a critical need for new and fundamental understanding of its dynamics. So in this paper we mainly focus on modeling and analysing the financial contagion in a system where a large number of financial institutions are randomly connected by the direct balance sheets linkages own to the lending or borrowing relationships. We propose a simple contagion algorithm to study the effect of several determinants, such as the topology of financial network, exposure ratio, leverage ratio, and the liquidation ratio. One of our finding is that the financial contagion is weaker as the growth of connectivity of network, so a financial system with a higher connectivity is more stability or robustness; we also find that the exposure ratio increases the risk of financial contagion, but both the leverage ratio and liquidation ratio has a negative relationship on financial contagion.


Introduction
A crucial characteristic for the recent financial crisis is the contagion (or avalanche effect) of distress/failure, which is the potential of shocks hitting particular financial institutions to quickly spread across the whole financial system. For example, the default of Lehman Brothers on 15 September 2008, triggers a series of bankruptcy of firms in the financial system of USA, even in other countries. Many economists are attracted by this contagion phenomenon and produce a wealth of studies. Particularly, the using of network theory is the prominent direction. Indeed, the financial system can be viewed as a network with highly connected structure by interdependencies because of financial innovation—those interdependencies can be in the form of obligation, exposure, ownership and correlation [1]. Building on these interdependencies, the intertwined financial network and the diversified
financial institution can not only offer an explanation for the spread of crisis throughout the network, but also offer an implication for policy actions such as government intervention and bailout. These interdependent relationships, initially build-up with the purpose of risk sharing, have also created a channel of spreading of financial distress. Like what [2] said, the financial network exhibits a knife-edge, or robust-yet-fragile property: in normal times the interdependencies between institutions enhance the liquidity allocation and increased risk sharing[3]; however, in financial distress time, the same interdependencies can amplify initial shocks lead to the insolvency of a large number of institutions or even the collapse of the whole network [4, 5].

However, the study of the nature and causes of financial contagion reflects the uncertainty and conflicting views from the academic literatures. For example, in the paper of [3] and [6], the authors argue that with the financial network becoming more dense, the impact of shocks of individual institutions to the rest system is becoming small, as the losses of an distressed individual bank are divided into more creditors, However, In contrast to this view, Blume, Easley et al.[7] and Vivier-Lirimont [8] argue that the frangibility of a financial network is increased when the number of the counterparties of a bank is growing. This situation illustrates a critical need for new and fundamental understanding and analysis of financial contagion. So in this paper we mainly focus on modeling and analysing the financial contagion in a financial system where a large number of financial institutions are connected by the direct balance sheets linkages own to the lending or borrowing relationships. In this financial network an institution interacts with several other institutions, and so the default of one institution as some idiosyncratic shocks will affect its creditors, the creditors which are insolvent will also suffer default and cause further failures in the financial system. A number of determinants influence this kind of financial contagion, such as the topology of financial network, the size of exposures, and the capital buffer. We model this kind of financial contagion and propose a simple contagion algorithm to study the role of these determinants. In detail, focusing on a financial system with n banks randomly connected, we take the initial idiosyncratic shock as exogenous, and investigate and analyse how it spreads through different financial networks; we also study how it is absorbed or amplified by different size of exposures and the capital buffer. Our contribution is to the ongoing debate on the role of financial integration and diversification in the spreading of financial contagion.

The remaining part of this paper is organized as follows: Section 2 provides a brief review of relevant literature on the application of network theory to the study of financial contagion; Section 3 introduces the contagion mechanism and the algorithm; Section 4 presents the results of simulation experiments; Section 5 concludes.

**Literature Review**

Even financial crises have been frequently witnessed throughout the twentieth century, it is in recent times, following the global financial collapse as the Subprime Crisis of 2008-2009, that the network theory have been extensively employed to study financial contagion by economists and financial regulators [9-11]. The seminal literature of [3] pioneer this strand of theoretical study by showing how the network structure affects the risk sharing, they point out that the complete network can absorb idiosyncratic shocks, while the complete network might allow negative spillovers to spread throughout the system (financial contagion). After this outstanding work, a large number of literatures on financial contagion employ network or graph model. Financial contagion mainly comes through three mechanisms: 1), correlation risk because of overlapping portfolios exposure [12-15]; 2) liquidity hoarding risk because of rumor or imperfect information [5, 16, 17]; and 3), counterparty risk because of the direct bilateral exposures[9, 18-21]. We mainly focus on the third mechanism in which the bilateral exposures are the direct balance sheets linkages in the form of lending or borrowing relationships. Indeed, these lending or borrowing relationships can be act as a channel of spreading of contagion.

We broadly categorize the study of financial contagion into two branches, the first branch is considering financial system as random network, which emphasize the importance of
network topology structure, such as network connectivity, average degree and density. Those kinds of literatures model financial contagion as a result from an initial idiosyncratic shock to one or few financial institutions and spreading through the entire network in a cascade manner. This group of literatures includes the work of [4, 9, 18, 19, 22]. The other branch is studying the financial contagion in a deterministic network, which considers the financial network as either exogenous or endogenous and examines the impact of initial defaults as predetermined by network externalities, such as the configuration model[23], the tiering banking network[24], nested split graph[25].

The above mentioned theoretical literatures investigate the mechanism and influence of financial contagion under a series of determinants by some stylized model and a series of assumptions. There is an obvious shortcoming of such brands of research, as what Upper said: “analytical results on the relationship between market structure and contagion have been obtained only for a limited number of highly stylized structures of interbank markets, which are of limited use when it comes to assessing the scope for contagion in real world banking systems”[26]. “Given the scarcity of theoretical results, researchers have increasingly turned to computer simulations to study contagion”, actually, Upper presents an comprehensive review on using numerical simulations to study the mechanics of financial contagion in the paper of[27]. Here we also list some paper on simulation in recent year, [5, 9, 12, 18, 19, 28-35].

Financial Network and Balance Sheet

Here we consider a financial system in which $n$ financial institutions (banks for short) are randomly connected together by their exposures on each other. These exposures which reflect the lending or borrowing relationships in this financial system can be represented by a weighted directed network, denoted by an exposure matrix $W \in \mathbb{R}^{n \times n}$. In this network, each node is a bank and each link represents a directional lending relationship between two banks, the weight reflects the size of exposure which comprises assets as well as liabilities on other side. We should highlight that the magnitude of these exposures is important for study financial contagion. The exposure matrix $W$ is defined as follow, where $w_{ij}$ denotes the size of lending by Bank $i$ to bank $j$. ($i, j \in N, N = \{1, 2, ..., n\}$), $w_{ij} \neq 0$ reflects the presence of a link, while $w_{ij} = 0$ reflects the absence of a link.

$$W = \begin{bmatrix} 0 & w_{12} & \cdots & w_{1n} \\ w_{21} & 0 & \cdots & w_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \cdots & 0 \end{bmatrix}$$

Now we turn to consider the structure of assets and liabilities for individual bank. Figure 1 shows a stylized balance sheet for a financial institution. On the assets side of figure 1, the bank lends to other banks in the financial system, which form the “Internal Assets”, the remainder of assets consists a range of “External Assets” which are the holdings of other real economy, such as government bonds, mortgages, corporate lending and commercial real estate lending. On the other side of the balance sheet, the liabilities consists of the “Deposits” and “Internal Liabilities”, the deposits is held to be external outside of the system, as such household, internal liabilities is the borrowing from other banks, the “Equity” is the capital buffer which denotes the excess of total assets over total liabilities.

Considering the exposure matrix $W$, we can calculate the total exposures of bank $i$ to the financial system. The “Internal Assets” held by $i$, which is denoted by $A_i^I$, can be got based on $A_i^I = \Sigma_j w_{ij}$; and The “Internal Liabilities” $L_i^I$ can be got based on $L_i^I = \Sigma_j w_{ji}$. 
Since an internal asset of one bank is an internal liability of another bank, so the internal liabilities are endogenously determined based on the topology of the financial network, besides, the equation of (1) could be obtained.

\[ \sum_i A_i^I = \sum_i L_i^I = S \] (1)

We define the total of internal assets as \( S \), which provide a measure of the total risk exposures of the financial system. Considering the structure of balance sheet, the following equations are found.

\[ A_i = A_i^E + A_i^I \] (2)

\[ L_i = L_i^I + D_i + E_i \] (3)

\[ A_i = L_i \] (4)

Where \( A_i, L_i, A_i^E, D_i \) and \( E_i \) denote bank \( i \)'s total assets, total liabilities, external assets, deposits and equity, respectively.

What’s more, we introduce two ratios. The exposure ratio, which denotes as \( \alpha_i \), is the rate of internal assets to the total assets \( \alpha_i = A_i^I / A_i \) The exposure ratio reflects the risk exposures of bank \( i \); The leverage ratio, which denotes as \( \beta_i \), is the rate of equity to total assets \( \beta_i = E_i / A_i \), this leverage ratio is also named as “capital ratio” or “the ratio of net worth”, which represents the capacity of absorbing losses while remaining solvent. As we mentioned, the “Equity” is the excess of total assets over total liabilities, so when the total liabilities exceed the total assets \( (E_i \leq 0 \text{ or } \beta_i \leq 0) \), the bank insolvent.

**The Contagion Mechanism**

**Initial Failures**

Here we assume that the initial failures are caused by idiosyncratic shock which happened due to some credit risks (e.g., frauds) or operation risks (e.g., wrong decision). The idiosyncratic shock has a bad effect on the external assets of a subset of banks in the financial system (maybe one or several banks), in the form of reducing the amount of external assets and hence causing the default of these banks. It is worth noting that the idiosyncratic shock is not the
aggregated or correlated shock which influence almost all banks simultaneously in the financial system. The bank has to liquidate if it is default, while its creditors will lose a fraction of claims because the liquidation value of a firm is always smaller than its book value. Formal speaking, a bank \( i \) is insolvent when \( E_i \leq 0 \) because of the reduction of external assets, which cause the bank to liquidate; the liquidation of bank \( i \) induces a loss equal to \( \gamma_i w_{ij} \) for its counterparty \( j \), where \( \gamma_i \) is the liquidation ratio of bank \( i \). So we define the set of initially insolvent banks is as follow:

\[
Z_0 = \{ i \in N | \beta_i \leq 0 \}
\]  

(5)

In this paper, we study the case that the number of set \( Z_0 \) equal one, which means there is just one default bank at initial time.

**The Contagion Process**

In this financial network, the default of one or several banks may lead to other banks being insolvent, which generating a cascade effect of default. Figure 2 illustrates the mechanism of the cascade effect. At some time of this contagion process, bank A and bank B are insolvent and have to be liquidated, which lead to repay their internal liabilities to bank 1, 2, 3, …, m. Each creditor bank only receive one proportion of its claims, this induce bank 1 suffering a loss which exceed its equity, so bank 1 become insolvent and is to be liquidated in the subsequent step; besides, bank 3 also become insolvent because that the cumulative losses, incurred from both bank A and bank B, exceed its equity. It must be worth to note that bank A and B are not necessary to be liquidated in the same step.

To model the dynamics of default contagion, we suppose that all banks in the network are initially solvent and that the network is perturbed at time \( T=0 \) by the initial failure of one single bank. Considering the set of initially insolvent banks \( Z_0 \), we calculate the set of banks, which become insolvent at time \( T=1 \) due to their claims to initial default bank, based on the following equations.
\[ z_1 = \{ i \in N | E_i \leq \sum_{j \in Z_0} (1 - \gamma_j)w_{ij} \} \]

(6)

\[ Z_1 = Z_0 \cup z_1 \]

(7)

Actually, when introduce the initial failure, for bank \( i \), which is not in the set of \( Z_0 \), the internal assets \( A_i = \sum_{j \in Z_0} \gamma_jw_{ij} + \sum_{j \notin Z_0} w_{ij} \). So the change of internal assets \( \Delta A_i = \sum_{j \in Z_0} (1 - \gamma_j)w_{ij} \). According to equation (2), (3), (4), we can obtain that bank \( i \) will be insolvent when \( E_i \leq \Delta A_i \).

Following this procedure, we can calculate the set of default banks at time \( T=t \) based on \( Z_{t-1} \).

\[ z_t = \{ i \in N | E_i \leq \sum_{j \in Z_{t-1}} (1 - \gamma_j)w_{ij} \} \]

(8)

\[ Z_t = Z_{t-1} \cup z_t \]

(9)

Iterating the equation of 8 and 9, we can trace the contagion process initiated by one single bank (\( \#Z_0 = 1 \)). The process will terminate when \( Z_t = Z_{t-1} \).

**A Simple Contagion Algorithm**

This contagion process can be study by the tool of branching process which is widely used in the field of epidemiology for study the epidemic spreading[36]. Indeed there are some scholars adopt the branching process to study the probability of financial contagion or the extent of contagion [4]. However, there are several challenges for the theoretical analysis of financial contagion. Firstly, the structures of balance sheet for banks are diversity. The size of total assets, the leverage ratio, the exposures ratio and the in-degree and out-degree for different bank may be different. Secondly, the branching process usually occurs on a tree, but the financial contagion not necessarily a tree, but is rather a more general graph. Take the default of bank 3 as we illustrate in figure 2 as example. The default of either bank A or bank B will not induce the default of bank 3, but the default of both bank A and bank B can induce its default. Considering these challenges, we turn to simulation study of financial contagion with the following contagion algorithm.

**Step 1:** Introducing the initial failures. Random selecting one bank for default, so the size of the set of initially insolvent banks equal one (\( \#Z_0 = 1 \));

**Step 2:** Liquidating the default bank. Only repaying one proportional of internal liabilities for the default bank (\( \gamma_iw_{ij} \));

**Step 3:** Revising banks’ balance sheets. Mainly focusing on the creditors for default banks and revising these creditors’ balance sheets based on equations (2), (3), (4);

**Step 4:** Updating the set of default banks. Calculating the set of default banks \( Z_t \) based on equations (8), (9).

**Step 5:** Terminating this algorithm if \( Z_t = Z_{t-1} \), otherwise returning to step 2.

**Simulation Experiments and Results**

**Parameters Setting**

The algorithm mentioned above makes it possible to study the contagion process in a financial system when it is in a particular state, corresponding to a particular configuration of the network topology and the balance sheets for each bank. Our main goals are to understand whether and how the financial contagion depends on the network properties and the structure
of balance sheets. At the first step, we should make some specific instruction for the network topology and balance sheets.

Table 1. Summary of the variation for parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network topology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Random Network</td>
<td>$N$</td>
<td>The number of nodes in a financial network</td>
</tr>
<tr>
<td>$P_{random}$</td>
<td>The probability of forming a link between two nodes</td>
<td>0.01 to 0.30</td>
</tr>
<tr>
<td>Balance Sheet Structure</td>
<td>$\alpha$</td>
<td>The exposure ratio/the internal-assets-to-assets ratio</td>
</tr>
<tr>
<td>$\beta$</td>
<td>The leverage ratio/ the equity-to assets ratio</td>
<td>0.01 to 0.05</td>
</tr>
<tr>
<td>The liquidation ratio</td>
<td>$\gamma$</td>
<td>The repaying ratio for each internal liabilities</td>
</tr>
</tbody>
</table>

The financial network be studied is Erdős and Rényi random graph[37]. The random graph model can be defined by two parameters: $N$ disconnected nodes and the probability $P$ for forming a link between each couple of nodes, the link formation process is i.i.d., and the degree distribution is binomial. Based on this definition, we can construct a series of financial systems which comprises 1000 banks, the lending or borrowing relationships in the financial system is represented by the weight in the network, the weight is assigned according to the discovery of the paper of [38], in which the weight follows a Log-Normal distribution with mean 15.2 and standard deviation of 0.8.

Now turn to the structure of balance sheet, we assume that all banks have the same exposures and leverage in a financial network, but difference in different networks. So in a financial network, we set the exposure ratio and leverage ratio for all banks are the same $\alpha$ and $\beta$, respectively. We can determine the detail information for each bank’s balance sheet, such as, total assets, equity and deposits, based on the confirming of internal assets, internal liabilities $\alpha$ and $\beta$.

In a nutshell, a financial system is determined by the matrix $W$, exposure ratio $\alpha$ and leverage ratio $\beta$. So the diversification of the financial system is reflected by the variation of these parameters. Table 1 summary these variation which are considered in our simulation study. It is worth noting that we also assume that all banks in the same financial system have the same liquidation ratio.

Finally, in order to evaluate the magnitude of financial contagion, we introduce two measure indicators. At first, we define financial contagion as an event that at least one bank falls into default as a response to the initial failure. Following this definition, two measure indicators are derived: 1), contagion probability, defined as the probability of occurring of a contagion event (equation 10); 2), extent of contagion, defined as the average banks being defaulted induced by the initial failure if a contagion event occurs (equation 11). The contagion probability and the extent of contagion are suitable for measuring the magnitude of financial contagion, reflecting the stability or the robustness of a financial system. Particularly, contagion probability reflects the sensibility of a financial system for suffering financial contagion, while the extent of contagion reflects the fragility of a financial system. In the following subsections, we present the computational results which are performed 1000 simulations based on these two measure indicators.

\[
\text{Contagion Probability} = \frac{\text{Number of contagion events observed}}{\text{Number of total experiments}} \quad (15)
\]

\[
\text{Extent of Contagion} = \frac{\text{Number of total defaulted banks induced by contagion}}{\text{Number of contagion events observed}} \quad (16)
\]
The Probability $P$ and Contagion

We first investigate the effect of probability $P$, which denotes the probability of forming link between each couple of nodes when constructing a financial network. Figure 3 shows the changing of the contagion probability and extent of contagion under the varying of the probability $P$, here we also vary the leverage ratio $\beta$ from 0.01 to 0.05. Our first finding is that both the contagion probability and the extent of contagion decrease as the increasing of probability $P$, regardless of the varying of $\beta$. Especially, there is a sharp drop when probability $P$ varying approximately from 0.05 to 0.2. Moreover, considering the same probability $P$, we observe that a higher value of leverage ratio $\beta$, the lower value for the contagion probability as well as the extent of contagion. These observations show the negative influence of the probability $P$ and leverage ratio $\beta$ on the financial contagion. 

![Fig. 3. The influence of probability P on financial contagion](image)

This negative relationship between probability $P$ and financial contagion can be understand as follow: for a random network, the average degree is approximately $(N - 1)P$. So the average degree is increase as the growing of the probability $P$, which reflects a higher level of connectivity of the network; the high level of connectivity denotes the shock of defaulted banks can be shared or absorbed by more banks, so the contagion probability and the extent of contagion are small. We conclude that a higher value of probability $P$, which denotes the financial system is more stability, the lower probability of contagion and the lower of the extent of contagion. Turn to the negative relationship between leverage ratio $\beta$ and financial contagion, the intuition is simple: higher value of leverage ratio $\beta$ reflects higher capital buffer which act as a cushion, this situation denotes that banks can absorb more risk induced by other banks. This also leads us to conclude that the financial system with high leverage ratio is more robustness, because of the negative influence on financial contagion.

Exposure Ratio and Contagion

Figure 4 reports the effect of exposure ratio on financial contagion. We find that both the contagion probability and the extent of contagion increase as the growing of exposure ratio, for example, when leverage ratio $\beta$ equals 0.03, the contagion probability is changing approximately from 0.18 to 0.67, and the extent of contagion is changing from 0 to 400. The probable reason is following: the increasing of exposure ratio reflects the growing of risk for banks, because high exposure ratio denotes more assets are hold by other banks. From another perspective, the exposure ratio measures the concentration of bank’s asset, higher exposure ratio reflects lower concentration, so induce higher influence on it when failure hits the banks’ counterparty.
However, there are two special cases. The first case is the situation that the leverage ratio $\beta$ equals 0.01, the contagion probability is always 1 and the extent of contagion is almost 100, although the exposure ratio is varying from 0.2 to 0.4, the reason is that the financial network is so fragile that can’t bear any shocks because of low leverage ratio. The other case is that the leverage ratio $\beta$ equals 0.04 or 0.05, although there is a distinct changing for the contagion probability, the extent of contagion has almost no changings, this situation induces that the financial contagion can occur but the extent is very small. The underlying reason is obvious, the high leverage ratio denotes the high level of stability of the financial network, which reflects the initial idiosyncratic shock can be absorbed during the first few contagion process.

**Liquidation Ratio and Contagion**

We finally investigate the effect of liquidation ratio on financial contagion. The liquidation ratio reflects the repaying proportion for internal liabilities when bank needs to liquidate. The liquidation ratio also can be considered as a measure of the magnitude of the shocks: a higher liquidation ratio, the lower magnitude of the shock, because high liquidation ratio reflects more internal liabilities can be repaid.

Figure 5 shows the changing of the contagion probability and extent of contagion under the varying of the liquidation ratio. We can find that both the contagion probability and extent of contagion decrease due to the increasing of liquidation ratio. As discussed above, the reason is that high liquidation denotes the magnitude of shock is small, so the financial contagion can’t spread further. Of course, there are also two special cases, one is that the financial system is fragile when the leverage ratio $\beta$ equals 0.01, where the contagion probability and the extent of contagion are almost 1 and 1000, respectively; the other is the situation that financial system is stability when leverage ratio $\beta$ equals 0.04 or 0.05, where the extent of contagion is almost zero.
Discussion and Conclusion

Indeed, during the past few decades, global financial systems have seen considerable growth in size, complexity and diversification. However, it is our understanding of the mechanism of such systems that has not necessarily kept pace. On the other hand, the recent financial crisis has made a profound demonstration that modern financial systems can amplify and disseminate financial distress on a global scale. Motivated these situations, in this paper we analyse how the topology of financial network and the balance sheet structure affect financial contagion, which is evaluated by the contagion probability and the extent of contagion, by simulation study based on a simple contagion algorithm. We find that the financial contagion is weaker as the growth of connectivity of the network in the form of increasing probability $P$, a high level of connectivity denotes the shock of defaulted banks can be shared or absorbed by more counterparties, so a financial system with a higher probability $P$ is more stability or robustness. For the structure of balance sheet which is determined by exposure ratio $\alpha$ and leverage ratio $\beta$, we find that exposure ratio has a positive relationship with financial contagion, but a negative relationship for leverage ratio and financial contagion. The exposure ratio measures the concentration of bank’s asset, higher exposure ratio reflects lower concentration, this situation induces that bank exposes more risk to its counterparty. The leverage ratio determines the magnitude of bank’s capital buffer which reflects the capacity of absorbing shocks. Finally we investigates the role of liquidation ratio, which evaluates the magnitude of the shocks, on financial contagion, the results show that both the contagion probability and extent of contagion decrease due to the increasing of liquidation ratio.

Our study partly clarifies the interplay between the network topology and financial integration in the disseminating financial contagion. Besides, this study also provides implications for regulation of financial system. For example, the regulation of financial stability should not only seek to minimize the risk of failure of individual institutions, but also should focus on the whole financial system. In detail, the strategy of diversification indeed looks like sensible for sharing risk from the perspective of individual institutions—eggs are placed in more baskets; however the diversification may not optimal being viewed from systemic perspective, even can generate a bad result.

However, some factors are not taken into account in this study. For a financial network, we assume that banks are randomly connected, but this may be not true in reality, because banks lend or borrow money depend on many factors, such as bank’s credit. So the network topology may be not determined by random connection, for example, Chinazzi, Fagiolo et al find a core-periphery structure of the International Financial Network (IFN) architecture [39]. Of course, the exposure ratio and leverage ratio also may be not the same for all banks. Those
factors should be investigated in our future study, what’s more, other interesting problems are also worth to study, such as how the role of governmental intervention and bailout under financial crisis, how to response for individual institution to mitigate contagion and so on.

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