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Comparative Study on Static Term Structure of Interest Rates

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Abstract: The term structure of interest rates has been a hot topic in the financial sector. With the accelerating process of interest rate liberalization, to seek a representative benchmark interest rate of the market is basis for the fixed income products pricing. This paper using Nelson-Siegel-Svensson model and polynomial spline model fitting analysis is carried out on bond transaction data of Shanghai stock exchange in China, through analysis and comparison of the two models, to choose the appropriate method to fit the term structure of interest rates**.**

Keywords: the term structure of interest rates, spot rate, polynomial spline, Svensson model

1. INTRODUCTION

The term structure of interest rate is at the same level of risk, the relationship between different term interest rate and maturity, or the zero coupon bond yield curve. The term structure of interest rate is a very important basis research in the field of financial economics. At the macro level, the term structure of interest rates can provide information support to establishment and implement of monetary policy for the central bank, it can not only provide information on the economic cycle and inflation, but also to provide the market expectations to microeconomic entity. At the micro level, the term structure of interest rates is a basis for all fixed income securities pricing, financial derivatives pricing, asset pricing, and revealing the overall level and direction of change in the market of interest rates, so it is the essential analysis tool for the investors. In addition, the term structure of interest rates is an important indicator in risk management for the financial market participants. Because most are coupon bond in the bond market, the variety of zero coupon bonds is not complete, so can not be through the zero coupon bond yields to determine the term structure of interest rates directly. Therefore, we need to use economic theory and econometric methods to deduce the zero coupon bonds yield curves by coupon bonds. But at present, our country is still in the development of the financial markets, the interest rate is also gradually to market rates. Interest rate liberalization make all fixed income products focus on the benchmark interest rate, and thus the term structure of interest rates reflecting the information for investors and market governor is very important.

The research for the term structure of interest rates method mainly includes dynamic model and static model, and this paper mainly uses two relatively mature static models: Svensson model and polynomial spline model. Svensson model[1] was based on parametric model of Nelson-Siegel[2] adding two new parameters. Polynomial spline was firstly proposed by McCulloch[3][4],McCulloch used quadratic and cubic spine functions to estimate the discounted functions, showing that the estimated effect of the cubic spline method was better than the quadratic spline function method. Currently, domestic scholars focused on fitting comparison and analysis to the several models. Among them, Zhu Feng[5]used Svensson model and B-spline smoothing technique to construct the term structure of interest rates of Shanghai Stock Exchange; Zhu Shiwu and Chen Jianheng[6] using the Nelson-Siegel model, Svensson model and polynomial spline model fitted the term structure of interest rates on the bond of Shanghai Stock Exchange, finding that Svensson model has better stability so being more suitable for China's market conditions. Fu Manli[7] found that by comparing several models, Svensson model compared with the Nelson-Siegel model, having better flexibility and can be better reflecting the shape of the yield curve, and the polynomial spline method has the obvious advantage over fitting the bond's price.

This paper compares with fitting effect about Svensson model and polynomial spline model, and generalizes advantages of the two models.

2. MODEL INTRODUCTION

2.1 Svensson model

Svensson model introduced new parameters basing on Nelson-Siegel model, and the functional form of the instantaneous forward rate is as follows:

$$f(0,\mathrm{t}) = \beta_0 + \beta_1 \exp(\frac{-t}{\tau_1}) + \beta_2 \exp(\frac{t}{\tau_1}) \exp(\frac{-t}{\tau_1}) + \beta_3 (\frac{t}{\tau_2}) \exp(\frac{-t}{\tau_2})$$

The advantage of this model is that each parameter has a specific economic implications, β_0 is representative of long-term interest rates, it represents asymptote curve of the instantaneous forward rate $f(0,\mathrm{t})$, with the increasing maturity of t, $f(0,\mathrm{t})$ curve trends to be β_0; β_1 is representative of short-term interest rates, it is to measure factors of instantaneous forward rate curve approaching to asymptote curve's speed. If β_1 is positive, then the instantaneous forward rate curve rises with increasing remaining term. Conversely, if β_1 is negative, then the instantaneous forward rate curve declines with increasing remaining term. β_2 and β_3 represents the mid-portion of the interest rate, which determines the nature and the curvature of the instantaneous forward rate. The parameters τ_1、τ_2 control the attenuation rate of the index, which determines decay rate of β_1, β_2 and β_3. That is, when β_0 is fixed, it may characterize various shapes of interest rates curves by different combinations of β_1, β_2 and β_3, including monotonic, hump and S-shaped curves.

Then, based on the spot rate is an average forward rate, so the spot formula is as follows:

$$R(0,\mathrm{t}) = \beta_0 + \beta_1 [\frac{1-\exp(\frac{-t}{\tau_1})}{\frac{t}{\tau_1}}] + \beta_2 [\frac{1-\exp(\frac{-t}{\tau_1})}{\frac{t}{\tau_1}} - \exp(\frac{-t}{\tau_1})] + \beta_3 [\frac{1-\exp(\frac{-t}{\tau_2})}{\frac{t}{\tau_2}} - \exp(\frac{-t}{\tau_2})]$$

After getting the spot rate, the discount factor is $D(0,\mathrm{t}) = \exp(-\mathrm{t}\cdot \mathrm{R}(0,\mathrm{t}))$.

Svensson model comparing with the Nelson-Siegel model is more flexible and can portray the multimodal state of the interest rate curves.

2.2 Polynomial spine model

Polynomial spline model commonly uses spline functions to fit discount factor, and the parameters is estimated to minimize the difference between theoretical price and market price of the sample bonds, and then get discount function ,and finally get the spot rate and the forward rate. Polynomial spline method using spline function to approximate discount function. After polynomial spline was applied to the term structure of interest rates by McCulloch, then Shea[8] disussed the demarcation point of polynomial spline. Otherwise, when using polynomial spline method fitted the rate curves, it is critical in selecting the order of the polynomial and the number of splines. Currently, there is no uniform criteria to choose, and the only criterion is to minimize the difference between the theoretical price and the actual price. According to the status quo of China's bond market and Li Mingyu & Guo Wei [9], Zhu Rongxi & Qiu Wanhua [10] suggested order of the polynomial spline should be defined as 3, and when the order is 2, its second derivative is discrete, but the order is greater than 3, it is difficult to verify the continuity of the derivative. Only that, this will not only ensure adequate goodness of fitting, but also reduce the number of estimated parameters. This paper selected bond transactions in 2013 April 23 in the Shanghai Stock Exchange, according to the reality of trading bonds on that day, and to ensure that the transaction number of species in each time period is rather, so the discounting function is expressed as:

$$D(t)=\begin{cases} D_1(t)=1+b_1t+c_1t^2+d_1t^3 & t\in[0,1] \\ D_2(t)=1+b_1t+c_1t^2+d_1[t^3-(t-1)^3]+d_2(t-1)^3 & t\in[1,7] \\ D_3(t)=1+b_1t+b_2t+c_1t^2+d_1[t^3-(t-1)^3]+d_2[(t-1)^3-(t-7)^3]+d_3(t-7)^3 & t\in[7,15] \end{cases}$$

3. EMPIRICAL ANALYSIS

3.1 Data selection

The data of this paper selects bonds spot transactions with fixed rate on 23 April 2013 in Shanghai Stock Exchange (Table 1), a total of 14, of which less than one year is 6, remaining period from 1 year to 7 years is 3, form 7 years to 15 years is 5.

Table 1. Transaction data

Bond Code	Frequency	Maturity	Years to Maturity	Coupon Rate	Price	Yield to Maturity
019211	1	1	0.142466	0.0215	101.65	0.0349638
019025	1	3	0.30411	0.023	101.11	0.0390121
010308	1	10	0.40274	0.0302	102.16	0.021056
019219	0	1	0.487671	0.0294	101.51	0.0289637
019302	0	1	0.736986	0.0281	100.69	0.0287191
010701	1	7	0.791781	0.0293	100.62	0.0291223
010501	2	10	1.852055	0.0444	104.64	0.0223985
010213	2	15	4.410959	0.026	98.28	0.0308057
019303	2	7	6.755992	0.0342	100.84	0.0341822
019031	2	10	7.400801	0.0329	100.04	0.0333687
010512	2	15	7.564735	0.0365	102.05	0.0358327
010107	2	20	8.271233	0.0426	106.49	0.0348667
010303	2	20	9.983562	0.034	98.64	0.0357147
010504	2	20	12.060274	0.0411	106.3	0.0364683

3.2 Parameter estimation

In front of the model, having introduced discount function has been given in Svensson model, thus, theoretical pricing formula of the sample bond can be given as follows:

$$P_i^* = \sum_t CF_t^i \cdot D(\mathrm{t}, \Phi)$$

P_i^* is the theoretical price of the bond i, CF_t^i is cash flows of the bond i at future time t, $\mathrm{D}(\mathrm{t}, \mathrm{t}+\theta)$ represents the discounted values at time t, Φ is a parameter vector of discounting function (or matrix).

The standard to estimate parameter is to minimize the error of bond pricing (the difference between the actual price and the theoretical price). If using the ordinary least squares estimation method, there is no doubt that gives the equal weight to different terms, and this is not right in practice. Because the term of the bond in sample is not the same, in the terms of the bond, its price fluctuations is not only affected by the changes of interest rates, but also affected by duration and convexity. Clearly, long-term bonds fluctuates more than short-term bonds. Therefore, in order to solve this problem, when the objective function should set duration as weighting coefficient, and gives short-term bonds a higher weight, long-term bonds given less weight. So the objective function becomes:

$$\min \sum_{i=1}^{n} \omega^2_i \frac{(p'_i - p_i)^2}{n}$$

And $\omega_1 = \dfrac{\frac{1}{Dur_i}}{\Sigma \frac{1}{Dur_i}}$, this can solve the problem of heteroscedasticity.

3.3 Fitting results

For Svensson model, it needs to uses the nonlinear optimization algorithm to estimate the parameters, and the estimated parameters were as follows:

$$\beta_0 = 0.0522234508; \beta_1 = -0.03796324; \beta_2 = 0.0047669649; \beta_3 = 0.0494497335; \tau_1 = 25; \tau_2 = 4.3630808256$$

Based on these parameters, the image of the function on spot rate R and time t is shown in figure 1 under conditions of continuous compounding.

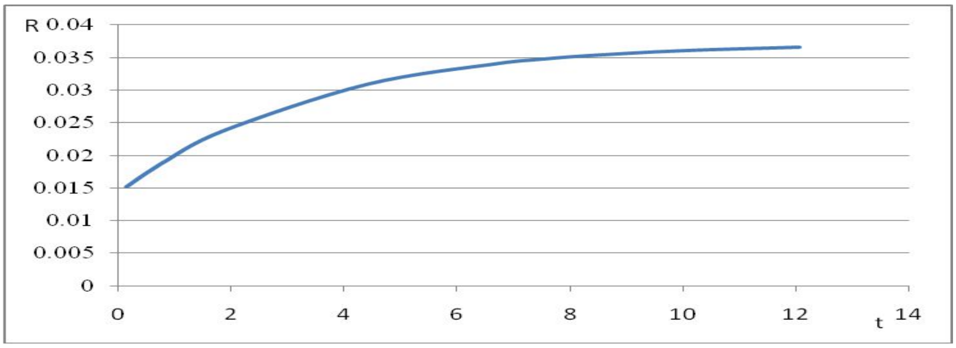

Figure 1. Svensson model

For polynomial spline model, the parameters' estimation uses regression analysis, and parameter estimation results are as follows:

$$b_1 = -0.014070299; c_1 = -0.009314312; d_1 = 0.0023187759; d_2 = 0.0002107801; d_3 = -0.00017213$$

After obtaining a function of time discount factor, and the discount factor will be converted into spot rate under continuously compounding.

According continuous compounding formula $R(t) = \frac{-\ln D(t)}{t}$, the spot curve is shown in Figure2:

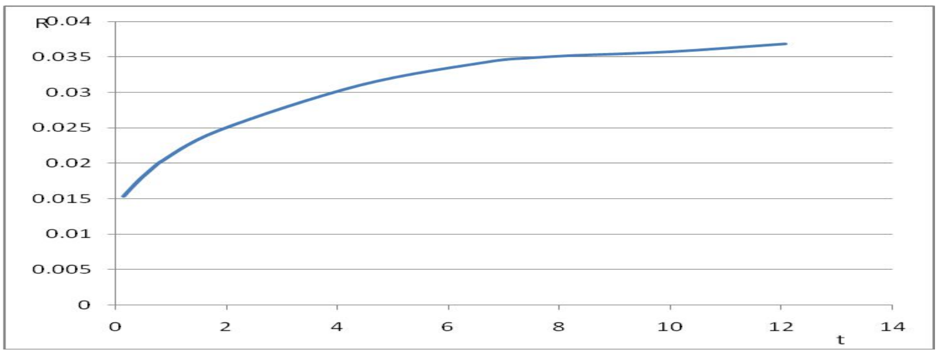

Figure 2. Polynomial spline model

And the two models to fit the spot rate is shown in Table2:

Table 2. Comparision of the two models

Years to Maturity	Spot Rate (Svensson)	Spot Rate (Polynomial spline)	Yield to Maturity
0.142466	0.015171677	0.015367022	0.034964
0.30441	0.016166057	0.016733353	0.039012
0.40274	0.01674891	0.017507027	0.021056
0.487671	0.017239987	0.018141186	0.028964
0.736986	0.018617792	0.019819445	0.038719
0.791781	0.018908278	0.02015145	0.029122
1.852055	0.023749321	0.024624409	0.022398
4.410959	0.030917191	0.031036239	0.030806
6.755992	0.034130801	0.034360959	0.034182
7.400801	0.03468338	0.034824488	0.033369
7.564735	0.034807256	0.034914011	0.035833
8.271233	0.035274675	0.03521221	0.034867
9.983562	0.036058951	0.035722928	0.035715
12.06027	0.036590803	0.036832294	0.036468

From the two graphs, both fitting effect is similar, interest rate curves are all upward sloping, this suggests that the longer the investment term, the higher the yield, and which is consistent with the theory of risk premium, and also reflects the time value of money.

Both Svensson model and polynomial spline model are all fitting better for medium-term, and both interest rates for short-term bonds in particular within 1 year bonds are underestimated phenomenon, this may be due to this several bonds whose remaining term is very closely within 1 year, and its pricing itself exists some errors. That leads to some difference between fitting result and the actual level. But for the remaining term of the bond in terms of more than 15 years, with the maturity's increasing, polynomial spline method presented in growth of power series for the remote, and this is clearly inconsistent with the actual, and relatively speaking, Svensson model fitting interest rate curves is relatively stable at the far end. Therefore, Svensson model fitting the curves is more realistic, but also is more reasonable.

4. CONCLUSION

This paper compares fitting effects between the polynomial spline method and Svensson model , and the two methods are both adapt to fit bonds of the medium –term, and the polynomial spline is closer to the actual price. But the research on the term structure of interest rates should not only stay in the fitting effect of historical data, because there is no sense, and forecasting the interest rates in future should is more meaningful. Svensson model itself is relatively stable, less affected by extreme values, form this perspective, it is more suitable for the development in the bond market.

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