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Statistical Modelling for Simulating and Interpreting an Egg Packaging Process for Giveaway Mitigation

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Abstract

Giveaway, the excess product being packed into orders, contributes to revenue loss that pre-packaged food manufacturers care about the most. In collaboration with an egg packaging company, this study aims to discover operation rules to mitigate the giveaway in egg orders. For that, two variables have been raised as potential controllable factors of the giveaway. One statistical model has been developed to better interpret the experimental results by understanding the underlying rules of the egg grading machine. The experiments have been accurately reproduced by a simulation using the estimated model parameters, indicating the model's success. Based on the experiment results, we claim that the number of accepted egg grades significantly influences the final giveaway ratio. Limitations and further potentials of the statistical model have also been discussed.

Keywords Data Analytics, Statistical Modelling, Giveaway Control, Egg Packaging

1 Introduction

Many efforts have been made to ensure conformity of pre-packaged goods. In Australia, trade measurement laws regulate the measurement of pre-packaged goods, including eggs, and restrict each package's deficiency (Egg Standards Australia 2010). Eggs sold in Australia are graded by individual weight, and eggs in the same grade are packed together in fixed quantities. For example, medium, large, extra-large, jumbo and super-jumbo belong to a standard grading scheme. A dozen (12) or a tray (30) of eggs are commonly packed together as a carton. By regulation, each egg in a carton must be equal to or heavier than the minimum weight. For example, an egg in an extra-large carton must weigh at least 58.3 grams (Australian Eggs 2020).

Egg producers commonly pack excessive-weight eggs into a carton because the extra weight can buffer water evaporation during eggs' shelf life. However, too much giveaway weight will increase production costs because heavier eggs consume more feed (Hy-Line 2020). So, we work closely with a local egg packaging company to investigate this issue. The company's grading floor has a proprietary grading machine (grader) that performs: weighing incoming eggs, assigning eggs to egg grades (weight categories), and dispatching eggs into orders. The egg giveaway occurs when eggs from a heavier grade are packed into cartons of a lower grade order (Figure 1 in Appendix 1). The grading floor operator inputs egg orders into the grading machine and allocate packaging lanes to each order. There are 16 packaging lanes, and each lane can only be assigned to one egg order at a time. The egg scheduling and dispatching algorithm used in the grading machine is unknown to the operator. So, we can only focus on the controllable machine configurations to mitigate the giveaway weight. One example is assigning which grade of eggs can be downgraded into an order demanding a lower egg grade. In this paper, we report a controlled experiment to analyze the impact of different machine configurations.

The commercial egg grading machine used in our factory trials measures each incoming egg's weight and assigns it to a weight category. But it only retains the final aggregated statistics like the total egg number and weight in each category. These summary statistics are sufficient to calculate the overall level of the production giveaway. But we can neither assign confidence intervals to the giveaway estimations nor deduce the operational rules of the machine. The former issue prohibits us from applying rigorous statistical comparisons between different configurations. The latter drawback forces the end users to rely solely on the machine manufacturers to optimize their production. Therefore, we also report a parametric model to interpret the experiment and understand the underlying grading machine rules. The fitted model is also proven successful in reproducing the experiment in simulation. So, we can adapt it to a broader supply chain modelling in the future to further manage production costs.

The egg packaging process is an information system as a whole. It collects egg order and weight data to fulfil orders efficiently. This study builds on top of the system with the goal of giveaway mitigation. Achieving this requires collecting data in the egg packaging process and fitting a model to describe the grading machine's behaviour. Based on the acquired insights, we may predict and manage egg giveaways by optimizing the operational rules in the future.

2 Literature Review

Sarkar et al. (2022) discuss various optimization goals and algorithms used in food processing projects. These projects focus on high-level algorithms that coordinate multiple actuators in the processes to improve efficiency, which require full access to all the equipment. Multiple projects employ Neural Networks (NN) as one of the optimization tools. On the other hand, egg grading involves many discrete value variables, different from other food processes that receive and manipulate continuous variables such as temperature and processing time. Dewil et al. (2019) introduce fundamental knowledge of how egg scheduling and dispatching work inside the grading machine. Greedy Randomized Adaptive Search Procedure (GRASP) suggests that the machine actively searches for "optimum" solutions based on available eggs in production. Thus, access to the algorithm is required to predict the exact outcome. Alternatively, we may be able to walk around this with statistical modelling, which may provide a good enough prediction of the grader's behaviour and allow us to tweak the process. Eiselt et al. (2022) provide in-depth information about modelling methods in Operations Research. It is handy to describe a black-box process mathematically and statistically without accessing its algorithms. While not much research has been done specifically for the egg packaging process, We find literature in other food fields to provide more perspectives and information. Defraeye et al. (2019) digitally reproduce how temperature changes mango quality with mathematical models and physical simulations. It is used in the digital twin of mango transportation. Koulouris et al. (2021) demonstrate a detailed brewery plant model consisting of chemical, scheduling, and labour factors. Both projects work with processes that have higher transparency and more control in actuators which is slightly different from our case.

However, they inspire us and allow us to foresee what may be required in the upcoming stages, and therefore being able to plan the model accordingly.

Another aspect of solving the cost issue related to giveaway could be pricing the eggs by weight instead of by grade. Alexey Kavtarashvili (2021) provides a glimpse of the benefits of adopting this pricing strategy. This methodology removes giveaway from the manufacturer's side, as all weights of eggs are covered in the price. Potentially this could benefit all egg packaging companies in the market. It is an interesting point of view related to the giveaway problem we are attempting to solve. However, changing the market requires a lot of stakeholders, and in the scope of our study, we aim to mitigate the giveaway problem solely on the manufacturer's side.

3 Research Method

3.1 Experiment design

We aim to study the feasibility of giveaway reduction of two controllable operations — the number of accepted downgrade grades and packaging lanes. Three accepted egg-grade settings and two lane number configurations are designed. For the first operation, we configure the grading machine to control which egg grades to accept into an order. As presented in Table 1, an order can accept the order's egg grade or take one or two heavier egg grades. Because extra-large and large eggs are most in demand, we can change the ratio of their allocated lane numbers to achieve the second operation. As seen in Table 3, we allocate six lanes to extra-large orders. But we can assign three (2:1 lane ratio) or four lanes (3:2 lane ratio) to large orders.

This experiment controls other variables, such as the supply egg distribution and the egg order profile. All six trials (Table 2) consist of large, extra-large, and jumbo orders. Because we only compare trials with the same egg supplier ID, we exclude trial 3 from the analysis.

Order Type		Large Order (L)	Extra-Large Order (X)	Jumbo Order (J)
Accepted Egg Grades	One heavier egg grade	L & X eggs	X & J eggs	J & S eggs
	Two heavier egg grades	L, X, & J eggs	X, J, & S eggs	J & S eggs
	Two heavier egg grades except X	L, X, & J eggs	X eggs	J & S eggs

Trial	**Lane Ratio**	**Accepted Egg Grade**	**ID**
1	3:2	One heavier egg grade	1
2	3:2	Two heavier egg grades	1
3	3:2	Two heavier egg grades except for Order X	1 & 2
4	2:1	Two heavier egg grades except for Order X	3
5	2:1	Two heavier egg grades	3
6	3:2	Two heavier egg grades	3

Table 1 (left): Details of accepted egg grade settings. S, J, X, & L represent super-jumbo, jumbo, extra-large & large, respectively. Table 2 (right): Number of accepted downgrade grades and packaging lane ratios between X and L orders.

Grader Outlet	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	**Lane Id**
	S		J		X						N/A		L			(L)	**Order Type**

Table 3: Lane allocation for orders. Eggs are transported from left to right. Orders with heavier egg grades are closer to the outlet of the grading machine. Due to the production schedule, lanes 1 and 2 are reserved for a super jumbo order that is not part of the experiment. Lane 16 is on standby.

3.2 Supply Modelling

We approximate the egg weight of a single batch supply as a normal distribution $f(w;\mu,\sigma^2) = N(\mu,\sigma^2)$ (Dewil et al., 2019), where w is the weight of a single egg, μ is the mean and σ^2 is the variance. We model the egg grades on the demand side with a discrete distribution: 1. Medium eggs: $p_M(w), w \in [41.7\text{g}, 50.0\text{g})$. 2. Large eggs: $p_L(w), w \in [50.0\text{g}, 58.3\text{g})$. 3. Extra-large eggs: $p_X(w), w \in [58.3\text{g}, 66.7\text{g})$. 4. Jumbo eggs: $p_J(w), w \in [66.7\text{g}, 74.0\text{g})$. 5. Super-jumbo eggs: $p_S(w), w \in [74.0\text{g}, 79.0\text{g})$. The total probability of these demands is $\sum_G p_G \approx 1$, where $G \in \{M, L, X, J, S\}$ denotes an egg grade.

Minimizing the average egg weight supplied to orders reduces production costs. The optimal case is to fill orders with the minimum egg weight, and we treat all the extra weight as giveaway. For a bundle of orders demanding egg grades G, we scale the weight and define the total giveaway ratio as:

$$giveaway\ ratio = R_G = \frac{\sum_G \sum_{i=0}^{\widetilde{N}_G} (\widehat{w}_G^{actual} - w_G^0)}{\sum_G \sum_{i=0}^{\widetilde{N}_G} w_G^0} \qquad Equation\ 1$$

where $\widetilde{N}_G$ is the egg number of an order demanding egg grade G (we denote it as order G for simplicity), w_G^0 is the lower bound of egg grade G, and $\widehat{w}_G^{actual}$ is the average weight packed into order G. Because eggs from heavier egg grades can also downgrade to an order G, we define the total downgrade ratio as:

$$downgrade\ ratio = R_D = \frac{\sum_G \sum_{i=0}^{\widetilde{N}_G} (\widehat{w}_G^{actual} - \widehat{w}_G)}{\sum_G \sum_{i=0}^{\widetilde{N}_G} w_G^0} \qquad Equation\ 2$$

where $\widehat{w}_G$ is the mean of the supplied eggs in grade G. However, suppose we avoid egg downgrade, the continuous supply distribution still mismatches with the egg grades' lower bounds. We define the ratio of this contributor to egg giveaway as $ingrade\ variation\ ratio = giveaway\ ratio - downgrade\ ratio$.

3.3 Trial Modelling

Because there is a single egg transportation frame, only the first egg packing lane has access to the initial supply distribution. To model this scenario, we define four truncated normal distributions $f_G(w; \mu, \sigma^2, a_G, b_G)$ for four involved egg grades $G \in \{L, X, J, S\}$, where $a_G \leq w < b_G$. The initial mixture probabilities P_G^0 equal to the proportions under the initial distribution. After we pack some supplied eggs in the earlier lanes, the supply distribution will differ for the later lanes. So, we implicitly update the four mixture probabilities, P_G^k, after passing k previous egg packing lanes. For simplicity, we define $k \in \{0,1,2\}$ to match the three orders (J, X, and L) in the trials. The mixture distribution is $f_{mix}^k(w) = \sum_G P_G^k f_G$.

The grading machine records the total egg number $\widetilde{N}_G$ and the total weight $\widetilde{W}_G$ packed into an order G. We can calculate the average weight $\widehat{w}_G^{actual} = \widetilde{W}_G / \widetilde{N}_G$ of the packed eggs and claim that egg downgrade only exists if $\widehat{w}_G^{actual} > \widehat{w}_G$. Since one order accepts eggs from up to two heavier egg grades, the egg number allocation to an order G_1 can be solved by the following linear equations:

$$\begin{cases} n_{G1} + n_{G2}^- + n_{G3}^- = \widetilde{N}_{G1} \\ n_{G1}\widehat{w}_{G1} + n_{G2}^-\widehat{w}_{G2} + n_{G3}^-\widehat{w}_{G3} = \widetilde{W}_{G1} \\ n_{G2}^-/n_{G3}^- = r_{23} \end{cases} \qquad Equation\ 3$$

where n_{G1} is the number of eggs in the grade G_1 that are packed in order G_1, n_{G2}^- and n_{G3}^- are the downgradable egg number from the two heavier egg grades, and r_{23} is an unknown ratio. We define $n_{G3}^- = r_{23} = 0$ if only one heavier grade is accepted and the linear equations have a simple analytical solution. However, if both heavier grades are accepted, we fit the ratio $\hat{r}_{23}$ by defining two more parameters:

$$P_{G2\ or\ G3 \to G1} = \frac{n_{G2\ or\ G3}^-}{N_{G2\ or\ G3}^k};\ P_{G1 \to G1} = \frac{n_{G1}}{N_{G1}^k} \qquad Equation\ 4\ and\ 5$$

where N_G^k is the remaining egg number of egg grade G after passing the first k packing lanes, $P_{G2\ or\ G3 \to G1}$ measures the probability of downgrading an egg from a heavier egg grade, and $P_{G1 \to G1}$ measures the probability of an egg getting packed into an order of the same egg grade. We iteratively update N_G^k by subtracting n_G or n_G^- for all $k \in \{0,1,2\}$.

To estimate the best $\hat{r}_{23}$, we reproduce the factory trial using a grid of $r_{23}^{(i)}$ values. The simulation generates random egg weights from a fitted normal distribution. Each "egg" will pass three packing lanes by turns for jumbo, extra-large and large orders. The simulated probabilities ($\hat{P}_{G1 \to G1}^{(i)}$, $\hat{P}_{G2 \to G1}^{(i)}$ and $\hat{P}_{G3 \to G1}^{(i)}$) randomly decide whether to pack an egg into an order. Once all three orders are fulfilled, we calculate the simulated $R_G^{(i)}$ using allocated egg weights and Equation 1. Comparing it with the measured giveaway ratio R_G^*, we fit $\hat{r}_{23}$ using $argmin_{r_{23}^{(i)}}\left\{\left|R_G\left(r_{23}^{(i)}\right) - R_G^*\right|\right\}$. We also acquire the final simulated probabilities ($\hat{P}_{G1 \to G1}$, $\hat{P}_{G2 \to G1}$ and $\hat{P}_{G3 \to G1}$) for all orders, as well as the total simulated ratios ($\hat{R}_D$ and $\hat{R}_G$) for the trial.

4 Current Results and Discussion

4.1 Experiment Results

We use Monta Carlo method to estimate the sampling distributions (expected values and errors) of all $\hat{w}_G$'s and N_G^0's from the fitted supply normal distribution. Then, we chain multiple error propagations to estimate the errors of other subsequent results and model parameters. We first present the measured giveaway and downgrade ratios in Table 4. For each trial, we calculate the total ratios (R_G^* and R_D^*) using Equation 1 and 2. Because each trial consists of three individual orders, we also calculate the ratios of the large (L), extra-large (X), and jumbo (J) orders by fixing a single egg grade G in the two equations.

We then present the estimated downgrade probabilities ($\hat{P}_{G2\ or\ G3\to G1}$) and the same-grade-packing probabilities ($\hat{P}_{G1\to G1}$) in Table 5. The subscript "S" in the table represents the super-jumbo egg grade. In some trials, it can be downgraded to orders demanding jumbo and extra-large grades (Order J and Order X). Afterwards, we use the estimated probabilities to control the destination of eggs generated from the fitted supply normal distribution (trial simulation mentioned in Section 3.3). The results are then used to calculate the total simulated giveaway and downgrade ratios $\hat{R}_D$ and $\hat{R}_G$.

We also find some interesting observations related to the grading machine configuration. For example, trial 2 and 4 are configured to accept two heavier grades. But we observe no downgrade in two extra-large orders and the large order of trial 4. Hence, we conclude that accepting two downgradable grades does not necessarily mean two grades being downgraded in the actual production (it can be zero or one grade). This fact might hint at why specific configurations may cause counterintuitive ramifications. In trial 1 and 2, increasing from one to two accepted downgrade grades will implicitly prevent any downgrade to extra-Large order. In trial 4, explicitly avoiding a downgrade to extra-Large order will implicitly prevent any downgrade to large order. Therefore, it is clearer to interpret the packing procedures using the model parameters.

Orders	**Measurements**	**Trial 1**	**Trial 2**	**Trial 4**	**Trial 5**	**Trial 6**
Order J	$R_{G,J}^*$	3.65 ± 0.02 %	3.61 ± 0.02 %	3.5 ± 0.02 %	3.59 ± 0.02 %	3.54 ± 0.02 %
	$R_{D,J}^*$	0.19 ± 0.02 %	0.15 ± 0.02 %	0.05 ± 0.01 %	0.15 ± 0.01 %	0.09 ± 0.01 %
Order X	$R_{G,X}^*$	7.5 ± 0.02 %	7.31 ± 0.01 %	7.11 ± 0.01 %	7.42 ± 0.02 %	7.21 ± 0.02 %
	$R_{D,X}^*$	0.19 ± 0.01 %	0 ± 0 %	0 ± 0 %	0.31 ± 0.01 %	0.1 ± 0.01 %
Order L	$R_{G,L}^*$	18.9 ± 0.02 %	15.48 ± 0.02 %	11.32 ± 0.01 %	14.84 ± 0.02 %	15.62 ± 0.02 %
	$R_{D,L}^*$	7.28 ± 0.02 %	3.85 ± 0.02 %	0 ± 0 %	3.52 ± 0.02 %	4.3 ± 0.02 %
Total	R_G^*	9.35 ± 0.01 %	8.44 ± 0.01 %	7.41 ± 0.01 %	8.39 ± 0.01 %	8.42 ± 0.01 %
	R_D^*	1.76 ± 0.01 %	0.86 ± 0.01 %	0.01 ± 0.01 %	0.99 ± 0.01 %	1.02 ± 0.01 %

Table 4: Measured giveaway and downgrade ratios for each trial and order. "Total" means the overall ratios. J, X, & L represents jumbo, extra-large, and large orders.

Orders	**Model Parameters**	**Trial 1**	**Trial 2**	**Trial 4**	**Trial 5**	**Trial 6**
Order J	$\hat{P}_{S\to J}$	40 ± 4%	34 ± 4%	12 ± 3%	37 ± 4%	21 ± 3%
	$\hat{P}_{J\to J}$	73.5 ± 0.7 %	78 ± 0.7 %	79.8 ± 0.7 %	89.9 ± 0.9 %	86.3 ± 0.8 %
Order X	$\hat{P}_{S\to X}$ (*top*)				0.0 ± 10%	0.0 ± 10%
	$\hat{P}_{J\to X}$ (*single or bottom*)	19 ± 1%	0.0%	0 %	0.96 ± 0.09	0.23 ± 0.02
	$\hat{P}_{X\to X}$	80.8 ± 0.3 %	89.4 ± 0.3 %	80.8 ± 0.2 %	90.1 ± 0.3 %	88.1 ± 0.3 %
Order L	$\hat{P}_{J\to L}$ (*top cell*)		0 ± 2 %		30 ± 90 %	1 ± 2 %
	$\hat{P}_{X\to L}$ (*single or bottom*)	100 ± 2 %	100 ± 3 %	0 %	101 ± 4 %	100 ± 3 %
	$\hat{P}_{L\to L}$	57.0 ± 0.5 %	94.7 ± 0.8 %	100.1 ± 0.7 %	85.2 ± 0.7 %	75 ± 0.5 %
Total	$\hat{R}_G$	9.36 ± 0.02 %	8.44 ± 0.02 %	7.41 ± 0.02 %	8.37 ± 0.02 %	8.42 ± 0.02 %
	$\hat{R}_D$	1.77 ± 0.03 %	0.85 ± 0.02 %	0.01 ± 0.02 %	0.98 ± 0.02 %	1.02 ± 0.02 %

Table 5: Model parameters for three orders and the simulated total giveaway and downgrade ratios

4.2 Model Validation

Orders in trials 1 and 4 accept none or a single downgradable grade. So, we set $n_{G3}^- = r_{23} = 0$ in Equation 3 to estimate egg numbers ($\hat{n}_{G1}$ and $\hat{n}^-{}_{G2}$) packed into each order. We can estimate packing probabilities ($\hat{P}_{G1\to G1}$ and $\hat{P}_{G2\to G1}$) using Equation 4 and 5 without fitting $\hat{r}_{23}$. Since the simulated total ratios $\hat{R}_D$ and $\hat{R}_G$ in Table 5 matches the measured R_D^* and R_G^* in Table 4, our model is successful in trial reproduction.

Orders in trials 2, 5 and 6 accept up to two downgradable grades. So, we need to fit $\hat{r}_{23}$ before estimating egg numbers ($\hat{n}_{G1}$, $\hat{n}^-{}_{G2}$ and $\hat{n}^-{}_{G3}$) packed into each order. Using trial 5 as an example, we test a grid of

21 different ratios ($r_{23.L}^{(i)}$ and $r_{23.X}^{(i)}$) for large and extra-large orders — 441 ratio pairs in total. The simulated $\hat{R}_D$ and $\hat{R}_G$ overlap with the measured ratios in all cases, further assuring the model quality.

4.3 Results Analysis

All orders in trial 1 accept egg downgrades from one heavier grade. In contrast, we configure the extra-large and large orders in trial 2 to accept eggs from two heavier grades. We observe in Table 4 that the measured R_G^* and R_D^* ratios are both around 1% smaller in trial 2. So, configuring more grades to downgrade can significantly reduce giveaways by depressing downgrades. This conclusion is counterintuitive. But the order-specific giveaway and downgrade ratios in Table 4 and the modelled egg packaging probabilities in Table 5 provide a possible interpretation. Because $R_{G,L}^*$ and $R_{D,L}^*$ are the only ratios much lower in trial 2, Order L contributes most of the giveaway and downgrade. We can explain this by a sharp decrease in the downgrade probability $\hat{P}_{J\to X}$ and a universal increase in the probabilities $\hat{P}_{J\to J}$, $\hat{P}_{X\to X}$, and $\hat{P}_{L\to L}$. Therefore, configuring more egg grades to downgrade relieves the constraint of packaging eggs into orders with the same grade. So, fewer eggs are subject to downgrade.

However, some conditions may overturn the effect of allowing more grades to downgrade. In trial 5, the egg-grade acceptance arrangement is the same as in trial 2. We compare it with trial 4, which has the same configuration except for forbidding downgrades into Order X. Although trial 5 configures more downgradable grades, the measured R_G^* and R_D^* ratios are both around 1% smaller in trial 4. Since we also observe a universal increase in the probabilities $\hat{P}_{J\to J}$ and $\hat{P}_{X\to X}$, our justifications in the previous paragraph are still valid. However, the differences here are the zero downgrade ratios $R_{D,L}^*$ and $R_{D,X}^*$ in trial 4. So, large and extra-large eggs are sufficient to fulfil Order X and L in this particular supply-demand match-up. This condition induces a sharp increase in the probability $\hat{P}_{L\to L}$. The increase is significant because large orders contribute the most to the giveaway and downgrade in all five trials. We can conclude that forbidding egg downgrades to a self-sufficient grade can lower production giveaway.

We also check if the number of packaging lanes influences production giveaways. Trial 5 has three lanes for the large order (2:1 lane ratio), while trial 6 has four lanes (3:2 lane ratio). The total giveaway and downgrade ratios do not have a significant difference. Because there is one less packaging lane for the large order in trial 5, the probabilities $\hat{P}_{S\to J}$, $\hat{P}_{J\to X}$ and $\hat{P}_{L\to L}$ are all higher. This result indicates that more large eggs are likely to pack in Order L, so causing a lower downgrade ratio $R_{D,L}^*$; Order X and J tend to accept more downgrades, so having higher downgrade ratios $R_{D,J}^*$ and $R_{D,X}^*$. The two opposite effects offset each other, which explains the same R_G^* and R_D^* ratios.

4.4 Limitations and Future Work

There are a few limitations of our study. First, we have neglected the variations in the supply distribution from a single farm. For example, eggs used in trial 4, 5 and 6 are supplied from the same farm but might have slightly different weight distributions. So, actively recording supply egg distribution is required to validate our study further.

Second, we have ignored production noises, such as lane interruptions and congestion. We only include their impacts in the average probabilities like $P_{G2\to G1}$and $P_{G1\to G1}$. Hence, the fitted model parameters are too simplistic for further generalization. We cannot directly use it to predict giveaways of new orders. Because recording real-time interruption data is challenging, we propose using machine learning to extract more information from our model. Our model generates eight additional parameters (Table 5). So, we can map production profiles like order numbers and accepted egg grade numbers into a higher dimension. This operation is likely to enhance data predictability. We need to record production configurations for numerous more orders to achieve this purpose. After that, we can build a historical database of giveaway ratios and modelled parameters by running our model fitting for all those orders.

Lastly, the giveaway mitigation is inefficient by simply adjusting the operation configurations. We have only observed around 1% of giveaway and downgrade reduction in this experiment. Hence, we might need to study and manage other variables like the egg supply distribution to achieve better results.

5 Conclusion

This study introduces a statistical model to investigate the egg production giveaway based on selected operation variables. Simulations of egg-packaging processes have been performed based on the estimated model parameters. Because the simulated giveaway and downgrade ratios have coincided with the observed ratios, we claim that the model can interpret the underlying rules of the egg-packaging procedures. Based on the experiment results, we conclude that with sufficient egg supply, only the "accepted egg grade number" setting can affect production giveaways. We have further discussed the limitations to address in the next step. The final model could later be part of the digital-twin tool used by the production operator to find the optimal configurations.

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Appendix 1 Egg Packaging Process

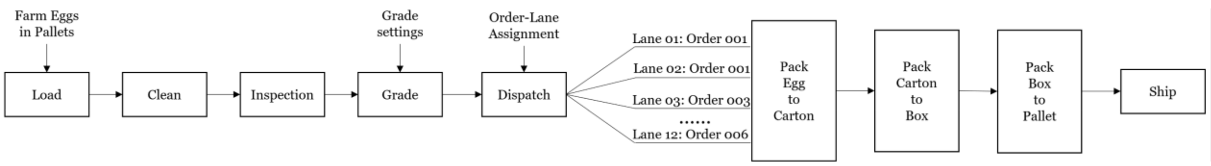

Figure 1 Process Flow Chart of the Egg Packaging Process on the grading floor (similar to Banjarat et al. 2019)