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Supply Chain Network Equilibrium Model for Perishable Products Based on Retailers' Utility

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Abstract: We study a supply chain network equilibrium model for perishable products under the theory of discrete choice models, which comprises multiple perishable product manufacturers, distributors and retailers. Our goal is to present equilibrium outputs for different firms in this supply chain network model for perishable products based on retailers' utility maximization. The network model is built upon transportation time function and deterioration rate to express the characteristics of perishable products, with the inclusion of the discarding costs associated with temperature control in transportation. Discrete choice model was used to elaborate retailers' choice according to their utility maximization. Furthermore, the variational inequality formulation is used to express the equilibrium solution of supply chain network for perishable products, which determines their profit shares in competition. A numerical example is present to support this model is reliable and reasonable.

Keywords: Supply chain network; Variational Inequalities; Perishable product; Modified Projection Method

1. INTRODUCTION

The supply chain practices of perishable products are currently under public scrutiny. Customers with more health conscious require more and more high quality for perishable products at a fair price [1, 2]; The price of perishable products fluctuates with the changes in time and inventory sharply[3]; Competition among firms results in strict cost, quality and time control of perishable products in supply chain network [4-6]; The structure of supply chain network for perishable products is increasingly complex and related to the superposition of many different types of networks [4, 7]; The supply chain of perishable products differentiating from other supply chains is the importance played by factors such as food quality and human safety [8]. In order to meet these requirements, it is necessary to urge an extensive, effective and sound method to understand, analyze and manage the supply chain network of perishable products, and to serve customers effectively and safely.

The supply chain of perishable products, as any other supply chain, is a network of organizations working together in different processes and activities in order to bring products and services to the market, with the purpose of satisfying customers' demands [2].The term perishable product has been coined to describe the product that has a fixed lifetime during which it can be used and after which it must be discarded [9]. Common examples of perishable products are human blood, fashion products, agri-food and medical drugs. The supply chain of perishable products is a network organization associated with different processes and activities of production, transportation, storage, marketing and disposal of perishable products so on in order to bring perishable products to customers and satisfy customers' demands.

To date, there are several relevant literature reviews in this field. Ahumada and Villalobos [4] review the main contributions in the field of production and distribution planning for agri-foods based on agricultural crops. Nahmias [10] provides excellent review of the inventory management of perishable products. The review by Beliën and Forcé [11] focus on inventory and supply chain management of blood products. Yu and Nagurney [12] describe some of the contributions about food products in supply chain, these literatures aim to integrate and synthesize two or more processes associated with food supply chains.

A number of papers have proposed and studied supply chain optimal model for perishable products from system-optimization perspective recently. Rong and Akkerman et al. [13] integrate food quality in decision-making on production and distribution in a food supply chain and provide a methodology to model food quality degradation in such a way that it can be integrated in a mixed-integer linear programming model used for production and distribution planning. Wang and Li[14] aim to reduce food spoilage waste and maximize food retailer’s profit through a pricing approach based on dynamically identified food shelf life. Asgari and Farahani et al. [15] determine the optimal amounts of wheat to be transported from each producing province to each consuming province per month across the year by using linear integer programming (LIP) model. Dabbene and Gay et al. [16, 17] present an approach for the optimization of fresh-food supply chains that manages a trade-off between logistic costs and some indices measuring the quality of the food itself as perceived by the consumer. A combined Markov Dynamic Programming (MDP) and simulation approach are presented and applied to a real life case of a Dutch blood bank supply chain by Haijema, van der Wal and van Dijk et al. [18, 19].

On the supply chain equilibrium model side, Yu and Nagurney [12] develop a network based food supply chain model under oligopolistic competition and perishability, with a focus on fresh produce through the introduction of arc multipliers. Chatwin [20] formulates a model and develops optimality conditions for selecting the optimal price, considering a continuous-time inventory problem in which a retailer sets the price on a fixed number of a perishable asset that must be sold prior to the time at which it perishes. Nagurney and Yu [21] develop a new model of oligopolistic competition for fashion supply chains in the case of differentiated products with the inclusion of environmental concerns. Baghalian and Rezapour et al. [6]develop a stochastic mathematical formulation for designing a network of multi-product supply chains for rice comprising several capacitated production facilities, distribution centres and retailers in markets under uncertainty. Masoumi and Yu et al [22] construct a generalized network oligopoly model with for supply chains of pharmaceutical products using arc multipliers. Nagurney and Masoumi et al. [23] develop a generalized network optimization model for the complex supply chain of human blood, which is a life-saving, perishable product. The multicriteria system-optimization approach on generalized networks with arc multipliers captures many of the critical issues associated with blood supply chains.

Our key in this paper is to address time sensitive problem in supply chain network for perishable products by using ideas of transportation time function and deterioration rate from user-optimization perspective. Anna Nagurney, Min Yu and Masoumi [12, 21, 24] adopted different methods to describe the characteristics of perishable products. This supply chain network equilibrium model that we focus on changing characteristics of perishable products with time is distinct from other literatures in several aspects.

(1) In order to investigate how the time-sensitive characteristics of perishable products affect supply chain firms’ profits, we capture the changing characteristics of perishable products with time by using transportation time function and deterioration rate of perishable products.

(2) In order to investigate how the different transportation methods affect supply chain firms’ profits, we consider the associated cost along with the selection of multiple transportation methods. Such as UPS, FedEx, and DHL provide shipment specific pricing contracts to different requirements. Furthermore, we consider discarding costs associated with temperature control in transportation.

(3) For the sake of explaining product choice process of retailers, we adopt Multinomial Logit model to express the product alternatives probability that retailers are willing to choice based on their utility.

(4) We study the free competition in supply chain network with complete information.

The rest of this paper is outlined as follows. In section 2, we develop a supply chain network equilibrium model for perishable products according to retailers’ utility maximization. It is important in this section that we describe the changing characteristics of perishable products with time. At the same time, Multinomial Logit model was used to explain the retailers' choice. The Algorithm method was given in section 3. We illustrate our results with a numerical example in section 4. Section 5 is the conclusion.

2. SUPPLY CHAIN NETWORK EQUILIBRIUM MODEL FOR PERISHABLE PRODUCTS

In this section, we develop a supply chain network model with perishable products manufacturers, perishable products distributors and perishable products retailers with the inclusion of the production, transportation, marketing and store of product associated with the different costs. The decision-makers on the supply chain network in the form of manufacturers and distributors are considered as profit maximization under time constraints. Retailers make a decision under their utility maximization.

We consider N manufacturers, M distributors, K retailers, a typical manufacturer by n, a typical distributor by m, and a typical retailer by k (Fig. 1). The figure 1 is abstracted from figure 2. The supply chain network for perishable products includes information network, logistic network and transportation network so on. The link between node n and node m can be production and transportation of perishable products. The link between node m and node k can be transportation of perishable products from distributor m to retailer k. Dotted lines denote other transportation methods with different cost functions.

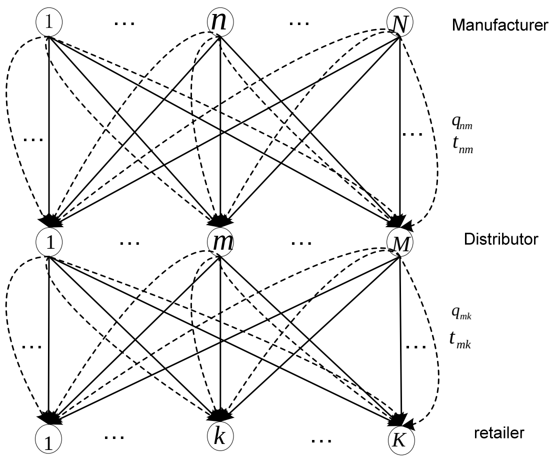

Fig. 1. The supply chain network topology for perishable products

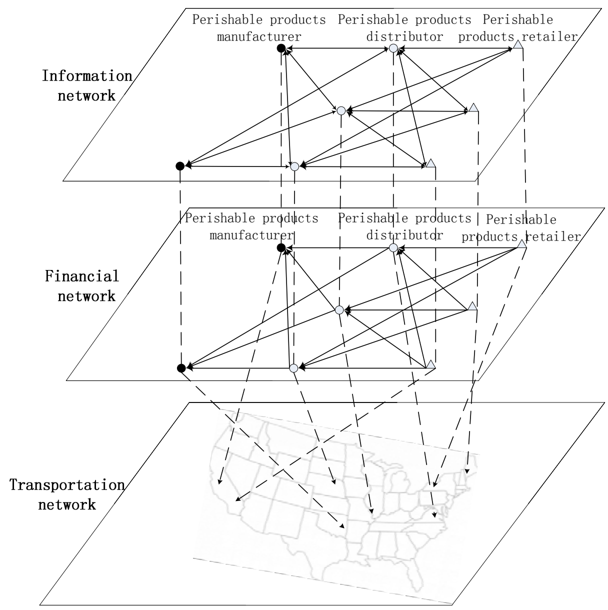

Fig. 2. The supply chain network three-dimensional structure

Consider a general network $H=[G,L]$, where G denotes the set of nodes in the network and L denotes the set of directed links. There are n manufacturers that produce a homogeneous perishable product. There are m distributors that ship this kind of perishable product to k retailers. Consider d_k is the demand for the perishable product at retailer k. The process of production, transportation and marketing was assumed to be acyclic. Horizontal firms will compete each other, vertical firms will cooperate each other. γ denotes the deterioration rate of perishable product in unit of time, which is given and fixed under the same or fixed temperature and other environmental conditions [10, 25]. So we assume γ is a function of temperature that can be controlled in transportation, $\gamma=\gamma(l)$, l denotes the optimal temperature that need be controlled in transportation. Different temperature control has a different cost function. t_{nm} denotes the transportation time on link nm.

Transportation time is the function of the flow on this link, $t_{nm} = t_{nm}(q_{nm})$.

2.1 The behavior of manufacturers and their optimality conditions

Let q_n denote the nonnegative production output of manufacturer n , $q_n \in R_+^N$. q_{nm} denotes the all products between manufacturer n and distributor m . q'_{nm} denotes the futility products between manufacturer n and distributor m . There are I kinds of transportation ways between manufacturer n and distributor m , a typical one is called i . The production output of manufacturer n satisfies this conservation of product flow equation (1). The production output of manufacturer n is equal to the sum of the quantities from manufacturer n to all distributors through all transportation ways. Here we assume that every manufacturer has a production cost function f_n which depends not only on his production output, but also on other manufacturer's output, because of raw materials competition.

$$q_n = \sum_{m=1}^{M}\sum_{i=1}^{I} q_{nm}^i \tag{1}$$

$$f_n = f_n(q_n) \ , \forall n. \tag{2}$$

The transaction cost between manufacturer n and distributor m includes transportation cost and discarding cost of perishable products. The discarding cost of perishable products is the cost that can be used to discard the futility product. Different transportation way has a different cost function. The transaction cost between manufacturer n and distributor m depend not only on the volume of product flow between these two firms, but also on the temperature, are given by:

$$c_{nm}^i = c_{nm}^i(q_{nm}^i, l_{nm}^i) \ , \quad \forall n, m, i. \tag{3}$$

The transportation time t_{nm}^i between manufacturer n and distributor m is related to the volume of product flow, so the transportation time functions are given by:

$$t_{nm}^i = t_{nm}^i(q_{nm}^i) \ , \quad \forall n, m, i. \tag{4}$$

The discarded volume of product between manufacturer n and distributor m can be given through time function:

$$q'^i_{nm} = \gamma(l_{nm}^i) \cdot t_{nm}^i \cdot q_{nm}^i \ , \tag{5}$$

Every manufacturer goes for profit maximization. So the profit of manufacturer n is equal to the price p_{nm}^1 that manufacturer n charges for the utility product times the utility volume of perishable products shipping to all distributors, then minus production cost and transaction cost. The function of profit maximization for manufacturer n takes discarded products into consideration, which can be expressed as:

$$\max \sum_{m=1}^{M}\sum_{i=1}^{I} p_{nm}^1 \cdot (1-\gamma(l_{nm}^i)\cdot t_{nm}^i)\cdot q_{nm}^i - f_n(q_n) - \sum_{m=1}^{M}\sum_{i=1}^{I} c_{nm}^i(q_{nm}^i, l_{nm}^i) \tag{6}$$

$$s.t. \quad q_{nm}^i \geq 0 \ ; \quad \forall n, m, i. \tag{7}$$

The manufacturers are assumed to compete in a noncooperative fashion with complete information. Assuming that the production cost functions and the transaction cost functions for each manufacturer are continuous and convex. The optimization function (6) for all manufacturers can be expressed as the following variational inequality [26-30]:

$$\sum_{n=1}^{N}\left[\frac{\partial f_n(q_n^*)}{\partial q_n}\right]\times\left[q_n - q_n^*\right] + \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{i=1}^{I}\left[\frac{\partial c_{nm}^i(q_{nm}^{i*}, l_{nm}^i)}{\partial q_{nm}^i} - \frac{\partial p_{nm}^1\cdot(1-\gamma(l_{nm}^i)\cdot t_{nm}^i)\cdot q_{nm}^{i*}}{\partial q_{nm}^i}\right]\times\left[q_{nm}^i - q_{nm}^{i*}\right] + \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{i=1}^{I}\left[\frac{\partial p_{nm}^1\cdot\gamma(l_{nm}^{i*})\cdot t_{nm}^i\cdot q_{nm}^i}{\partial l_{nm}^i} + \frac{\partial c_{nm}^i(q_{nm}^i, l_{nm}^{i*})}{\partial l_{nm}^i}\right]\times\left[l_{nm}^i - l_{nm}^{i*}\right] \geq 0$$

$$\forall q_{nm}^i, l_{nm}^i \in R_+^{NMI} . \tag{8}$$

The optimality conditions are expressed that a manufacturer will ship a positive amount of the utility product to a distributor if the price that the distributor is willing to pay for the utility product is equal to the manufacturer's marginal production and transaction costs by using optimal temperature control to reduce discarding cost and transportation cost.

2.2 The behavior of distributors and their optimality conditions

The transaction cost between distributor m and retailer k may include display and storage cost associated with this perishable product. $\hat{c}_m^j$ denotes transaction cost function using transportation way j. $\hat{c}_m^j$ is a function of $\sum_{m=1}^{M} q_{mk}$. There are J kinds of transportation ways between distributor m and retailer k, a typical one is called j. The transaction cost function may write:

$$\hat{c}_m^j = \hat{c}_m^j(q_{mk}^j, l_{mk}^j), \quad \forall m, k, j. \tag{9}$$

The transportation time t_{mk} between distributor m and retailer k is related to the volume of product flow, so the transportation time functions are given by:

$$t_{mk}^j = t_{mk}^j(q_{mk}^j), \quad \forall m, k, j. \tag{10}$$

The discarded volume of product between distributor m and retailer k can be given through time function:

$$q'^{j}_{mk} = \gamma(l_{mk}^j) \cdot t_{mk}^j \cdot q_{mk}^j, \tag{11}$$

Assuming that every distributor is profit-maximizer, so the profit of distributor m is equal to the price p_{mk}^2 that distributor m charges for the utility product times the utility volume of perishable products shipping to all retailers, then minus transaction cost. The function of profit maximization for distributor m takes discarded products into consideration, which can be expressed as:

$$\max \sum_{k=1}^{K}\sum_{j=1}^{J} p_{mk}^2 \cdot (1-\gamma(l_{mk}^j)\cdot t_{mk}^j)\cdot q_{mk}^j - \sum_{k=1}^{K}\sum_{j=1}^{J} \hat{c}_m^j(q_{mk}^j, l_{mk}^j) - \sum_{n=1}^{N}\sum_{i=1}^{I} p_{nm}^1 \cdot (1-\gamma(l_{nm}^i)\cdot t_{nm}^i) q_{nm}^i \tag{12}$$

$$s.t. \sum_{n=1}^{N}\sum_{i=1}^{I} q_{nm}^i \ge \sum_{k=1}^{K}\sum_{j=1}^{J} q_{mk}^j, \ \forall n, m, k, i, j. \tag{13}$$

The distributors are assumed to compete in a noncooperative fashion with complete information. Assuming that the transaction cost functions for each distributor is continuous and convex. The optimization function (12) subject to (13) for all distributors can be expressed as the following variational inequality [26-30]:

$$\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{i=1}^{I}\left[\frac{\partial p_{nm}^1(1-\gamma(l_{nm}^i)\cdot t_{nm}^i) q_{nm}^{i*}}{\partial q_{nm}^i} - \xi_m^1\right]\times\left[q_{nm}^i - q_{nm}^{i*}\right] + \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{j=1}^{J}\left[\frac{\partial \hat{c}_m^j(q_{mk}^{j*}, l_{mk}^j)}{\partial q_{mk}^j} + \xi_m^1 - \frac{\partial p_{mk}^2\cdot(1-\gamma(l_{mk}^j)\cdot t_{mk}^j) q_{mk}^{j*}}{\partial q_{mk}^j}\right]\times\left[q_{mk}^j - q_{mk}^{j*}\right] + \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{i=1}^{I}\left[-\frac{\partial p_{nm}^1\cdot\gamma(l_{nm}^{i*})\cdot t_{nm}^i \cdot q_{nm}^i}{\partial l_{nm}^i}\right]$$

$$\times\left[l_{nm}^i - l_{nm}^{i*}\right] + \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{j=1}^{J}\left[\frac{\partial \hat{c}_m^j(q_{mk}^{j*}, l_{mk}^j)}{\partial l_{mk}^j} + \frac{\partial p_{mk}^2\cdot\gamma(l_{mk}^{j*})\cdot t_{mk}^j\cdot q_{mk}^j}{\partial l_{mk}^j}\right]\times\left[l_{mk}^j - l_{mk}^{j*}\right] + \sum_{m=1}^{M}\left[\sum_{k=1}^{K}\sum_{j=1}^{J} q_{mk}^{j*} - \sum_{n=1}^{N}\sum_{i=1}^{I} q_{nm}^{i*}\right]\times\left[\xi_m^1 - \xi_m^{1*}\right] \ge 0$$

$$\forall q_{mk}^j, l_{mk}^j \in R_+^{MKJ}. \tag{14}$$

Where the term ξ_m^1 is the Lagrange multiplier associated with constraint (13) for distributors m. The economic interpretation of the distributors' optimality conditions is the same with the manufacturers'.

2.3 The behavior of retailers and their optimality conditions

The retailers, in turn, also have transaction cost to display and store perishable products before sell these products to customers. Denote the transaction cost associated with retailer k by $\bar{c}_k$, that is,

$$\bar{c}_k = \bar{c}_k(q_{mk}), \quad \forall m, k. \tag{15}$$

Let p_k^2 denote the price of the perishable product at retailer k. Further, denote the demand for the perishable product at retailer k by $d_k(p^2)$. The demand of retailers for the perishable product depends not only on the price at this retailer, but also on the price at other retailers, because distributors compete each other in different retailers.

The retailer takes the price charged by a distributor p_k^2, plus the transaction cost into consideration. The price plus the transaction cost does not exceed the price that the retailers are willing to pay for the perishable product. The equilibrium conditions for retailers take following forms:

$$\left.\begin{aligned} p_k^{2j*} + \frac{\partial \overline{c}_k(q_{mk}^{j*})}{\partial q_{mk}^j} = \xi_k^{2j*}, if \quad q_{mk}^{j*} > 0 \\ p_k^{2j*} + \frac{\partial \overline{c}_k(q_{mk}^{j*})}{\partial q_{mk}^j} \geq \xi_k^{2j*}, if \quad q_{mk}^{j*} = 0 \end{aligned}\right\} \tag{16}$$

Every distributor decides the proportion of product flow by using of different transportation ways according to retailers choices. For example, some retailers like to acquire and enjoy the products as soon as possible; the others hope to get these products as cheap as possible. They will to wait for a while to purchase these products. Let ω^j denote the probability of the retailer choice the transportation way j. $U^j = (U^1, \cdots, U^j, \cdots, U^J)$ denotes the vector of utilities associated with transportation way j. The utility U^j is expressed as a function of price, transportation time and an additive random.

$$U^j = \alpha \cdot P^j + \beta \cdot T^j + \theta^j \quad \forall j \in J. \tag{17}$$

The random component of the utility satisfies $\mathrm{E}(\theta^j) = 0$. One of the most widely used discrete choice models is the logit model [31]. The product choice probability is then given by:

$$\omega^j = \frac{e^{U^j}}{\sum_{j=1}^{J} e^{U^j}} \quad \forall j \in J. \tag{18}$$

Let now $d_k(p^{2j*})$ denote the demand function of retailer k by using of transportation way j. If the equilibrium price the retailers are willing to pay is positive, the utility shipments from distributors must be equal to the demand for the perishable product of retailers.

$$d_k(p^{j2*}) \begin{cases} = \omega^j \cdot \sum_{k=1}^{K} q_{mk}^{j*}, & if \quad \xi_k^{2j*} > 0 \\ \leq \omega^j \cdot \sum_{k=1}^{K} q_{mk}^{j*}, & if \quad \xi_k^{2j*} = 0 \end{cases} \tag{19}$$

Where the term ξ_k^2 is the Lagrange multiplier for retailers k.The retailers are assumed to compete in a noncooperative fashion with complete information. The function (16) and (17) for all retailers can be expressed as the following variational inequality [26-30]:

$$\sum_{m=1}^{M}\sum_{k=1}^{K}\left[p_k^{2*} + \frac{\partial \overline{c}_k(q_{mk}^*)}{\partial q_{mk}} - \xi_k^{2*}\right] \times \left[q_{mk} - q_{mk}^*\right] + \sum_{k=1}^{K}\sum_{j=1}^{J}\left[\sum_{m=1}^{M} q_{mk}^{j*} - \omega^j \cdot d_k(p_k^{2j*})\right] \times \left[\xi_k^{2j} - \xi_k^{2j*}\right] \geq 0 \tag{20}$$

2.4 The equilibrium conditions of the supply chain

The equilibrium conditions in the entire supply chain network must satisfy the sum of the variational inequalities (8), (14) and (20). The utility production quantity and shipments that the manufacturers ship to the distributors must be equal to the production quantity and shipments that the distributors accept from the manufacturers. The amounts of the product purchased by the retailers must be equal to the utility shipments that the retailers accept from the distributors. We state the equilibrium conditions of the supply chain in the following:

$$\begin{aligned} &\sum_{n=1}^{N}\left[\frac{\partial f_n(q_n^*)}{\partial q_n}\right] \times \left[q_n - q_n^*\right] + \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{i=1}^{I}\left[\frac{\partial c_{nm}^i(q_{nm}^{i*}, l_{nm}^i)}{\partial q_{nm}^i} - \xi_m^1\right] \times \left[q_{nm}^i - q_{nm}^{i*}\right] + \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{i=1}^{I}\left[\frac{\partial c_{nm}^i(q_{nm}^i, l_{nm}^{i*})}{\partial l_{nm}^i}\right] \times \left[l_{nm}^i - l_{nm}^{i*}\right] + \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{j=1}^{J}\left[\frac{\partial \widehat{c}_m^{\,j}(q_{mk}^j, l_{mk}^j)}{\partial q_{mk}^j} + \xi_m^1 - \frac{\partial p_{mk}^2 \cdot (1 - \gamma(l_{mk}^j) \cdot t_{mk}^j) q_{mk}^{j*}}{\partial q_{mk}^j} + p_k^{2*} + \frac{\partial \overline{c}_{mk}(q_{mk}^*, l_{mk}^j)}{\partial q_{mk}^j} - \xi_k^{2*}\right] \\ &\times \left[q_{mk}^j - q_{mk}^{j*}\right] + \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{j=1}^{J}\left[\frac{\partial \widehat{c}_m^{\,j}(q_{mk}^{j*}, l_{mk}^j)}{\partial l_{mk}^j} + \frac{\partial p_{mk}^2 \cdot \gamma(l_{mk}^{j*}) \cdot t_{mk}^j \cdot q_{mk}^j}{\partial l_{mk}^j}\right] \times \left[l_{mk}^j - l_{mk}^{j*}\right] + \sum_{m=1}^{M}\left[\sum_{k=1}^{K}\sum_{j=1}^{J} q_{mk}^j - \sum_{n=1}^{N}\sum_{i=1}^{I} q_{nm}^i\right] \times \left[\xi_m^1 - \xi_m^{1*}\right] + \sum_{k=1}^{K}\sum_{j=1}^{J}\left[\sum_{m=1}^{M} q_{mk}^{j*} - \omega^j \cdot d_k(p_k^{2j*})\right] \times \left[\xi_k^{2j} - \xi_k^{2j*}\right] \geq 0 \end{aligned}$$

$$\forall (q, \xi^1, \xi^2, l) \in R_+^{NMI+MKJ}. \tag{21}$$

Definition Ⅰ: Variational inequality problem

The finite-dimensional variational inequality problem, $VIP(F, K)$, is to determine a vector $x^* \in K \subset R^n$, such that

$$\left\langle F^{'}(X^*), (X - X^*) \right\rangle \geq 0, \quad \forall X \in K \tag{22}$$

Where F is a given continuous function from K to R^n, K is a given closed convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in n dimensional Euclidean space.

Definition Ⅱ: Supply chain network Cournot-Nash equilibrium for perishable product

The Cournot-Nash equilibrium state of the perishable product supply chain network is one where product shipments and temperature control pattern $(Q^*, l^*) \in K$ constitute an equilibrium if for each firm $i; i = 1, \cdots, n$,

$$U_i(Q_i^*, l_i^*, \hat{Q}_i^*, \hat{l}_i^*) \geq U_i(Q_i, l_i, \hat{Q}_i^*, \hat{l}_i^*), \forall (Q_i, l_i) \in K^i, \tag{23}$$

Where $\hat{Q}_i^* \equiv (Q_1^*, \cdots, Q_{i-1}^*, Q_{i+1}^*, \cdots, Q_n^*)$;

$$\hat{l}_i^* \equiv (l_1^*, \cdots, l_{i-1}^*, l_{i+1}^*, \cdots, l_n^*) . \tag{24}$$

According to (23), equilibrium is established if no firm can unilaterally improve its profits by changing its supply chain network product shipments and temperature in transportation, given the decisions of the other firms. Some literatures derived the variational inequality formulations of the Cournot-Nash equilibrium for supply chain network under free competition with complete information satisfying Definition Ⅱ [12, 21, 22]. Next, we derive the variational inequality formulations of the Cournot-Nash equilibrium for the perishable product supply chain network under free competition satisfying Definition Ⅱ, in terms of both product shipments and temperature.

Theorem 1: *Assume that, for each firm* $i; i = 1, \cdots, I$ *, the profit function* $\hat{U}_i(X)$ *is concave with respect to the variables in* x_i *, and is continuously differentiable. Then* $X^* \in K$ *is a supply chain generalized network Cournot-Nash equilibrium according to Definition* I *if and only if it satisfies the variational inequality:*

$$\sum_{i=1}^{I} \left\langle \nabla x_i \hat{U}_i(X^*)^T, X_i - X_i^* \right\rangle \geq 0, \forall X \in K. \tag{25}$$

Where $\langle \cdot,\cdot \rangle$ *denotes the inner product in the corresponding Euclidean space and* $\nabla x_i \hat{U}_i(X^*)$ *denotes the gradient of* $\hat{U}_i(X)$ *with respect to* X_i *.*

Variational inequality (25), in turn, for our model, is equivalent to the variational inequality (21): determine the vector of equilibrium shipments and temperature $q^*, l^* \in K$ *.*

Proof: See this proof in Masoumi and Yu et al, Nagurney and Yu, Yu and Nagurney. [12, 21, 22].

Theorem 2: *(Existence [21, 29]).There exists at least one Nash Equilibrium, equivalently, at least one solution to variational inequality (21), since in the light of the demand price functions (2), there exists* $a, b > 0$ *, such that variational inequality (25) admits a solution in* K_b *with*

$$x_b \leq b \tag{26}$$

Theorem 3: *(Uniqueness [21, 29]). With Theorem 2, variational inequality (25) and, hence, variational inequality (21) admits at least one solution. Moreover, if the function* $F(X)$ *of variational inequality (21), is strictly monotone on* $\kappa \equiv K^2$ *, that is,*

$$\left\langle (F(X^1) - F(X^2))^T, X^1 - X^2 \right\rangle > 0, \forall X^1, X^2 \in \kappa, X^1 \neq X^2. \tag{27}$$

then the solution to variational inequality (21) is unique, that is, the equilibrium shipments, temperature and the equilibrium demand pattern are unique.

3. MODIFIED PROJECTION METHOD

In this section, modified projection method is presented to solve the variational inequality problem. Of course, Projection Method, Relaxation Algorithm, Method of Successive Average (MSA) and Projected Dynamical Systems can also be applied to solve this problem [30, 32]. Modified Projection Method has some advantages. Note that a necessary condition for convergence of the general iterative scheme is that $F(x)$ is strictly monotone. In the case that such a condition is not met by the application under consideration, a modified projection method may still be appropriate. This algorithm requires, instead, only monotonicity of $F(x)$, but with the Lipschitz continuity condition holding, with constant L. The algorithm is now stated in the following.

4. Step 0: Initialization

1. Start with an $x^0 \in K$. Set $k := 1$ then select ρ , such that $0 < \rho \le \frac{1}{L}$, where L is the Lipschitz constant for function $F(x)$ in the variational inequality problem.
2. **Step 1: Construction and Computation**
3. Compute $\bar{x}^{k-1}$ by solving the variational inequality subproblem:
4. $[\bar{x}^{k-1} + (\rho F(x^{k-1}) - x^{k-1})]^T \cdot [x' - \bar{x}^{k-1}] \ge 0, \quad \forall x' \in K.$
5. **Step 2: Adaptation**
6. Compute x^k by solving the variational inequality subproblem:
7. $[x^k + (\rho F(\bar{x}^{k-1}) - x^{k-1})]^T \cdot [x' - x^k] \ge 0, \quad \forall x' \in K.$
8. **Step 3: Convergence Verication**
9. If $|x^k - x^{k-1}| \le \varepsilon,$ for $\varepsilon > 0,$ a prespecied tolerance, then, stop; otherwise, set $k := k+1$ and go to Step 1.
10. **Numerical Examples**

In this section, we present an example of strawberry supply chain network in a city to illustrate this model. There are two retailers to sell strawberry that shipped from three distributors. Two manufacturers ship the strawberry to these three distributors. The Modified Projection Method is applied to solve this numerical example. The preference that the retailer choice the products by using a same transportation way is same.

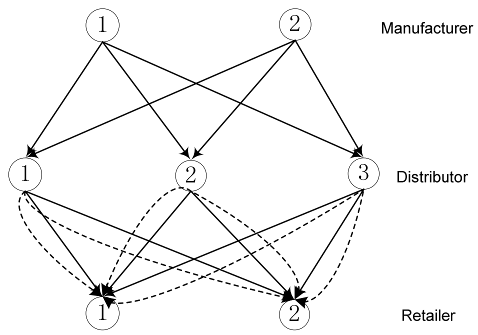

Fig. 3. The strawberry supply chain network topology

The production cost functions for the manufacturer 1 and manufacturer 2 were given by:

$f_1 = 3q_1^2 + 4q_1 + q_1 q_2, \quad f_2 = 3q_2^2 + 4q_2 + q_1 q_2$

There is only one transportation method between manufacturers and distributors. The transaction cost functions faced by manufacturers and the transportation time functions were given by:

$c_{nm} = q^2 + 2q + 5 + 3(l_0 - l_1), \; t_{nm} = 0.01 \cdot q + 2$

There are two transportation methods between distributors and retailers. The transaction cost functions faced by distributors using general cargo transportation and the transportation time functions were given by:

$\hat{c}_{mk}^1 = q^2 + 8q + 6 + 3(l_0 - l_1), \; t_{mk}^1 = 0.01 \cdot q + 2, \; \alpha = 0.75, \beta = 0.25$

The transaction cost functions faced by distributors using express transportation and the transportation time functions were given by:

$\hat{c}_{mk}^2 = 2q^2 + 16q + 12 + 3(l_0 - l_1), \; t_{mk}^2 = 0.002 \cdot q + 1, \; \alpha = 0.25, \beta = 0.75$

The transaction cost functions and demand functions faced by retailers given by:

$\bar{c}_k = 9q - 4$, $d_1 = -2(\omega_1^1 p_1^1 + \omega_1^2 p_1^2) + 1.5(\omega_2^1 p_2^1 + \omega_2^2 p_2^2) + 400, \; d_2 = -2(\omega_2^1 p_2^1 + \omega_2^2 p_2^2) + 1.5(\omega_1^1 p_1^1 + \omega_1^2 p_1^2) + 400$

The strawberry deterioration rate in unit of time under the same or fixed temperature was assumed as $\gamma(l_1) = 0.01 - 0.0001(l_0 - l_1), \; l_0$ =70 . The algorithm was implemented in matlab 7.0 (step length ρ =0.01, convergence precision ε =0.002), the modified projection method yielded the following solution through 16 iterations.

The product shipments between two manufacturers and three distributors were:

$q_{11} = q_{12} = q_{13} = q_{21} = q_{22} = q_{23}$ =132.5499

The optimal temperature between two manufacturers and three distributors were:

$l_{11} = l_{12} = l_{13} = l_{21} = l_{22} = l_{23} =$ 33.8059℉

The discarding product shipments between two manufacturers and three distributors were:

$q'_{11} = q'_{12} = q'_{13} = q'_{21} = q'_{22} = q'_{23}$ =2.8993

The product shipments between two manufacturers and three retailers by using general cargo transportation were:

$q^1_{11} = q^1_{12} = q^1_{21} = q^1_{22} = q^1_{31} = q^1_{32}$ =125.6283

The optimal temperature between three distributors and two distributors by using general cargo transportation were:

$l^1_{11} = l^1_{12} = l^1_{21} = l^1_{22} = l^1_{31} = l^1_{32}$ =33.773℉

The discarding product shipments between two manufacturers and three retailers by using general cargo transportation were:

$q'^1_{11} = q'^1_{12} = q'^1_{21} = q'^1_{22} = q'^1_{31} = q'^1_{32}$ =2.6866

The product shipments between two manufacturers and three retailers by using express transportation were:

$q^2_{11} = q^2_{12} = q^2_{21} = q^2_{22} = q^2_{31} = q^2_{32}$ =4.2076

The optimal temperature between three distributors and two distributors by using express transportation were:

$l^2_{11} = l^2_{12} = l^2_{21} = l^2_{22} = l^2_{31} = l^2_{32}$ =33.8054℉

The discarding product shipments between two manufacturers and three retailers by using express transportation were:

$q'^2_{11} = q'^2_{12} = q'^2_{21} = q'^2_{22} = q'^2_{31} = q'^2_{32}$ =0.0273

According to elastic demand function, the demand of two retailers respectively was: $d^1_1 = d^1_2$ =376.8849, $d^2_1 = d^2_2$ =12.6233. $p^1_1 = p^1_2$ =1.0970, $p^2_1 = p^2_2$ =4.8194.

5. CONCLUSIONS

This paper developed a supply chain network equilibrium model for perishable products based on retailers' utility maximization, which comprises multiple perishable products manufacturers, distributors and multiple retailers. The model expresses the characteristics of perishable products by using transportation time function and deterioration rate. And discarding cost associated with temperature control in transportation was considered. Furthermore, Finite-dimensional variational inequality was adopted to formulate the supply chain equilibrium conditions for perishable product. Finally, a numerical example is provided to illustrate the model and computational procedure. The main contributions of this paper are the following:

1. We develop a supply chain network equilibrium model for perishable products to capture the changing characteristics of perishable products with time by using transportation time function and deterioration rate of perishable products. The framework of variational inequalities is a main tool in our analysis.

2. To the best of our knowledge, this is the first paper that uses ideas of transportation time function and deterioration rate to describe characteristics of perishable products.

3. We introduce and study the Modified Projection Method for computing equilibrium shipments and temperature. In addition, the proof of existence and uniqueness of solution was introduced.

4. Multinomial Logit model was used to express the product alternatives probability that retailer are willing to choice based on their utility for price and time.

5. We illustrate our results through an example of strawberry supply chain network.

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