Research Progress of Tie-Generative Mechanism in Network Based on ERGM

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Short Research Paper

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Abstract: The research on the tie-generative mechanism in network is conductive to explore the factors affecting network evolution and provide theoretical supports and decision-making suggestions for promoting (restraining) the formation (disappearance) of network edges. Based on the analysis of the current situation and development trend of ERGM from 2015 to 2020, this paper presents the research progress and limitations of ERGM. It has made good progress in the fields of network dynamic evolution mechanism, capturing network node heterogeneity, multi-layer network formation mechanism and Research on large (small) scale network formation mechanism. However, it must be admitted that the problems such as extending ERGM to obtain connection weighted information, modeling the heterogeneity of nodes in the network, detecting the multicollinearity that may exist in ERGM, studying the dynamic evolution mechanism of multi-layer network and using ERGM to deal with network missing data have not been well solved, waiting for further exploration.

Keywords: ERGM, tie-generative mechanism, social network, research progress

1. INTRODUCTION

Social network analysis radically changes quantitative analysis by shifting the focus from individuals to their relationships and interactions [1]. The traditional regression model is often used for social network analysis, which is based on the assumption of independence and unable to consider endogenous structural effect of the network [2]. However, this effect, that is, the relationship between nodes can be formed by the self-organization of the network, will affect the probability of edge generation [3]. Taiye Luo and Cuichang Ma also pointed out the formation of social network is the result of the joint influence of endogenous structure and exogenous variables. [4] Therefore, the relevant conclusions obtained from the traditional regression model which can't take the endogenous structure effect into account will be biased to some extent. Compared with the limitations of the independence assumption set by the traditional regression model for the observation object, the advantage of ERGM is that it can simultaneously consider endogenous structure and exogenous variables of the network to more comprehensively study tie-generative mechanism and evolution process of the network [5], which makes the analysis based on ERGM more rigorous and reliable. Guancan Yang, Tong Liu et al. [6] pointed out that ERGM can be well applied to analyze the influencing factors of citation relationship formation. Ghosh A , Ranganathan R also recognized ERGM as a network analysis tool in their research and stated ERGM should be an important part of the standard toolkit for future network tie-generative mechanism research. [7]

Understanding the formation mechanism of network could provide theoretical supports and decision-making suggestions for promoting (restraining) the generation of edges. Due to the advantages of ERGM, many scholars have applied ERGM to the research of network tie-generative mechanism in recent years. Helian Xu, Tianyang Sun et al. [8] studied the influence of endogenous structural variables, actor attribute variables and network covariables on the formation of high-end manufacturing trade network along the "One Belt and One Road" through ERGM. Song H discussed the factors affecting the generation of personal political

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discussion network's edges by using ERGM. [9] Anna Llupià, Puig J et al. used ERGM to evaluate the role of homophily and individual characteristics in social network. [10] Dang-Pham D, Pittayachawan S et al. also explored the formation mechanism of security consulting network by using ERGM. [11] Wenlong Yang and Debin Du studied the factors that affected the formation of investment network by comprehensively considering endogenous structural effect, actor-relationship effect and binary covariates. [12] Moreover, Guancan Yang, Zhanlin Liu et al. also analysed the tie-generative mechanism of the Nelarabine Drug patent citation network by using ERGM. [2]

However, although ERGM provides a series of flexible and highly scalable methods for network analysis, there is still a lack of an ERGM method that can take into account unobserve heterogeneity. [13] In addition, ERGM based on binary networks can't be used to simulate networks with weighted edges, which leads to the model unable to capture the network information of link strength between different nodes. [14] ERGM can only analyze static or cross-sectional data. The MCMCMLE algorithm for ERGM estimation and verification is very complex, which leads to the limited network scale that the model can simulate. This is consistent with the difficulty of directly estimating the ERGM parameters of very large scale networks proposed by stivala A and robins G.[15] These are new challenges that ERGM faces.From the perspective of literature, this paper will summarize the relevant concepts and development history of ERGM, as well as the present situation and future prospect of the research on the mechanism of network formation carried out by ERGM, so as to provide reference for future studies.

2. THE DEVELOPMENT OF ERGM AND RELATED CONCEPTS

The development of ERGM (Exponential Random Graph Model) can be traced back to the simple random graph model proposed by Erdos P and Renyi A [16] in 1959. The model contains the assumption that the relationships among network members are generated independently of the relationships among other members. Although the simple random graph model can not grasp the structural characteristics of the observed network well, it can provide a baseline for the comparison of other more complex models. In 1981, Holland P W and Leinhardt S developed a binary independent model to estimate the differences caused by reciprocation and differential attractiveness. [17] After that, Frank O , Strauss D introduced Markov dependence assumption into the model in 1986, and put forward a binary dependency model which assumes edges containing the same node are not independent of each other. [18] In 2006, Davidrh and Marksh extended the Markov model into p* model, which has a broader conditional dependence relationship, that is, the probability of the simultaneous existence of any two lines in the network is not equal to the probability combination of the respective existence of two lines. [19] Most ERGMs based on dependency assumptions can be regarded as p* models. However, p* model still has a series of problems such as the problem of degeneration. In order to solve these problems, Hunter D R put forward a method including GWD (Geometrically Weighted Degree Distribution), GWESP (Geometrically Weighted Edge Shared Partners), GWDSP (Geometrically Weighted Dyadwise Shared Partners) and other statistical items replace the complex structure and dependency conditions in observation network. [20] The current p* model, known as ERGM, has been able to incorporate high order dependence conditions into the analysis framework. In 2013, Wang P，Robins G extended ERGM to multilevel networks and demonstrated it with the collaboration network of the French Institute for Cancer Research. [21] Since then, multilevel networks have attracted the attention of many scholars. Brennecke J , Rank O studied the influence of enterprise knowledge network on inventor interaction network through the multi-level network that based on ERGM. [22]Smith M , Gorgoni S pointed out that a complete multi-level network includes networks at the micro, meso and macro levels, and applied multi-level ERGM to the study of complex interactions between activities at the enterprise level and international trade patterns. [23] At the same time, with the deepening of network research, academics are no longer satisfied with only statically studying the formation mechanism of networks. As proposed by Linqing Liu and Ziruo Chen, existing studies are mainly completed by analyzing static or cross-sectional network data, which makes it difficult to effectively reveal the dynamic evolution mechanism of the network. [24] People hope to analyze the evolution process of network dynamically so as to understand the formation mechanism of network more deeply. In 2014, Krivitsky P N , Handcock M S extended ERGM to STERGM (Separable Temporal Exponential Random Graph Mode). [25] STERGM is able to distinguish between newly created and previously existing relationships in the network, and longitudinally study the evolution process of the network. [26] Bjorklund P, Daly A J used ERGM and STERGM to study the influence of homogeneity and proximity dimension on the generation of social connections in pre-service teacher identification network. [27] At present, ERGM is still in the period of rapid development, and its flexible expansibility endows it with strong vitality. As Peng, Tai-Quan said, ERGM has a bright future. [28]

ERGM is a dynamic network model based on network binary relations. Different from traditional measurement models, ERGM emphasizes more on the dependence of relationships in the network [14]. According to the observation network, ERGM generates a random network graph, and through the steps of estimation, diagnosis, simulation, comparison and improvement, the generated network is more and more close to the observation network, so as to check which factors significantly affect the generation of the network [8].

As an innovative statistical inference method, ERGM allows a variety of deformation and expansion, and can include multiple factors that may affect network formation, including exogenous node attributes, endogenous structural effects and binary covariates, into the model [2],[3]. King S, Lusher D et al. used ERGM to study the influence of endogenous network features, attribute based exogenous features and geographical proximity dimensions on network formation. [29] Among them, exogenous node attribute refers to that the probability of linking between two nodes is affected by the node attributes itself. When the nodes have a certain attribute (refers to binary variables) or the larger the value of a node's attribute (refers to continuous variables), the higher the probability of link between nodes. The sender effect, receiver effect, homogeneous effect and heterogeneous effect all belong to node attribute effect. The sender effect refers to possess some properties of nodes are more likely to send a link to other nodes, correspondingly, the receiver effect refers to a certain attribute nodes are more likely to accept links from other nodes. Homophily (heterogeneity) means that the probability of forming a connection between nodes with the same (different) property is higher than the probability of forming between two nodes selected at random. In addition, as a complex system, the emergence of some relationship come from the internal process of the network relational system. Such self-organizing effects are usually called "endogenous structural effects". Cao Q, Liao L et al. pointed out that the social network among the residents of the treatment community of women was mainly generated from the endogenous structural factors of the treatment community itself. [30] The transitive effect, connectivity effect, preferential attachment effect and sparsity effect all belong to the self-organizing effect of network. The transitive effect refers to the influence of transitive closure structure on the formation of relationships in a network and the connectivity effect refers to the influence of 2-path structure. Preferential attachment can be divided into two types: convergence and expansion which refers to the influence of the in-degree (out-degree) distribution of network nodes on the relationship formation. The research on the effects of endogenous structure is indispensable, because it’s influence is stable and strong, and far beyond expectations. [31] Network synergistic effect refers to that in addition to generative node variables and endogenous structure, duality attributes can also affect the generation of the network. For example, the similarity of patent content will affect the generation of patent reference relationship.

3. RESEARCH STATUS AND LIMITATION OF TIE-GENERATIVE MECHANISM IN NETWORK USING ERGM

3.1 Research on the formation mechanism of different topic networks.

From 2016 to 2020, there are a lot of researches on the formation mechanism of different topic networks including transnational student exchange network [33], terrorist organization alliance network [34], strategic literature alliance network [6], healthy community network based on users' replies [35], patent technology diffusion network [31], patent collaborative innovation network [4], academic social network [36], patent citation network [37], scientific cooperation network [38], pyramid selling organization network [39] and the urban network based on the perspective of China's top 100 electronic information companies [14].

In addition to discussing tie-generative mechanism in network from common perspectives such as reciprocation, preferential attachment and preferential selection, some scholars have also studied from the angles of similarity and social factors. Qingfeng Duan and Xiaohuan Pan discussed the influence of similarity in social attributes of literature on citation preference. [40] In addition, Lee S K , Kim H [41]， Anna Llupià, Puig J [10] also pointed out that not only the purely academic factors, but also many complex and diverse factors such as political background, economic level, cultural atmosphere, individual psychology and intelligence may affect the formation of the network, which should be paid attention to in future research.

3.2 Studying the network tie-generative mechanism longitudinally.

In order to explore the evolution mechanism of the complex network, Peng, Tai-Quan [28] divided the journal citation network into four time series and skillfully studied the evolution of the network. Xiaoyan Liu, Jinpeng Li et al. also used a similar method to explore the evolution mechanism of technology trading network in the integrated circuit industry. [42] In 2014, Krivitsky P N , Handcock M S extended ERGM to STERGM. [25] The emergence of STERGM makes it more convenient to analyze the mechanism of network formation longitudinally. Linqing Liu and Ziruo Chen studied dynamic evolution mechanism of Chinese dominant industrial combination by TERGM. [43] Xiaoyan Liu and Jing Wang also discussed the different influences of exogenous node attributes and endogenous structure on the formation and dissolution of cooperation in different growth stages of OLED technology innovation network based on TERGM. [44]

3.3 Expolring network formation mechanism through multi-level ERGM.

The connections of different organizational levels in the network are not completely independent. They are interdependent at all levels and affect each other in a complex way. [45] The networks of different organizational levels and the connections between them constitute a multi-level network. The complex causal relationship in the multi-level network is not simply one-way. For example, the long-term transaction network between companies affects the relationship between companies, which in turn will bring new business opportunities and constraints to their companies. [46] Liu Xuan, Wang Linwei and others also pointed out in their research that there may be a great correlation among friend network, user access network and reply network in online health community, and it is of great significance to pay attention to the relationship and interaction between these networks. [35] ERGM was originally developed for single-level network. In 2013, Wang P , Robins G and others extended ERGM to multi-level network, and used the cooperative network of French cancer research elites and their affiliated institutions as an example to prove that a full understanding of the network requires cross level parameters. [21] This study was later seen as the beginning of multilevel network analysis using ERGM. Since multi-layer ERGM can capture the relationship that cannot be explored by a single network [47], and more fully reproduce the observation network, it has been widely concerned by the academic community since then. Brennecke J , Rank O used multi-level ERGM to research the influence of enterprises' knowledge network on inventors' interactive network [22], and Hollway J , Koskinen J also discussed how to embed bi-edge cluster into multi-level interdependent network system based on this model.[48] Smith M , Gorgoni S applied multi-level

ERGM to explore the complex interaction between enterprise-level activities and international trade. [23] In addition, Wang P, robins G et al. also proposed SSM (social selection model) as an extension of multi-level ERGM in 2016 [49], this model integrates the information about nodes into the modeling framework, which may help to determine the degree of homogeneity and other attributes that may affect the affiliation within and between levels and the structure of the whole multilayer network, so that people can have a more detailed and complete understanding of the network structure and basic network process.

However, as a developing model, multi-level ERGM has many limitations. Multi-level ERGM can only deal with binary data, which means that the observation network fitted by multi-level ERGM loses the important information of edge weight. [23] This model can not simulate the dynamic evolution mechanism of observation network vertically, which is also one of the limitations of the model. As Brennecke J and Rank O mentioned, the future vertical research should investigate the co-evolution of social network and knowledge network within the enterprise. [22] In addition, Wang P., Robins G. also pointed out that they often find that the homogeneity assumption under ERGM may be too strong, especially for large empirical networks. [49]

3.4 Extending ERGM to capture the heterogeneity of network nodes.

To understand the "unobserved heterogeneity" and what impact this heterogeneity will have on the research, we use a passage from Box-Steffensmeier J M , Christenson D P [13] to explain:

"However, we may suspect that there are other, intangible factors specific to each individual that are difficult if not impossible to measure, such as "friendliness" or "charisma," that are also related to network structure (people that are friendlier are likely to have more friends, increasing the centrality of friendly individuals above what we would expect given their other, known attributes). In other words, the observed and measured characteristics are not sufficient for explaining the network we observe. Further, because these unobserved characteristics may be correlated with both the outcome (network structure) and the other explanatory variables, there is the potential for mistaken inferences when such heterogeneity is not accounted for."

In order to fit the observation network more accurately, researchers begin to study how to extend ERGM to capture the heterogeneity existing in the network. Thiemichen S , Friel N [50] ameliorated ERGM to capture the heterogeneity of network nodes. Henry T R , Gates K M [51] also modeled the unobserved heterogeneity. Box-Steffensmeier J M , Christenson D P [13]and Koskinen J , Wang P [52] all discussed how to extend ERGM to alleviate the dilemma caused by failure to model heterogeneity. In 2020, Henry T R , Gates K M [51] developed SRFM-ERGM (Sender/Receiver Finite Mixed Exponential Random Graph Model) that can capture heterogeneity.

3.5 Extending ERGM to accommodate networks of different sizes.

Monte carlo estimation of ERGM parameters is a computationally intensive process, which results in strict limits on the size of the network that ERGM can fit [53]. Due to the difficulty of parameter estimation, the practical application of this kind of model is limited to relatively small networks, up to several thousand nodes, usually only a few hundred nodes (such as online social networks) or fewer nodes (such as church social networks). [15] It is rarely applied to the research of small networks (6 or less nodes in directional networks) from team or home [54] and very large-scale networks.

Most of the researches on small networks are based on descriptive statistics. The main limitation of this work is the availability of network models. ERGM, which is often used to fit large and medium-sized networks, can not be used because the problem of maximum likelihood estimation is more obvious in small networks. [54] In addition, there are a series of problems in it's application to the research of ultra large scale network. From a conceptual point of view, with the expansion of the network scale, the MCMLE (Monte Carlo maximum likelihood estimation) commonly used by ERGM requires that the nodes need to fully understand the assumption that all other nodes are connected when looking for the nodes that establish the connection, which makes it impractical. Stivala A and Robinson G also pointed out that it is very difficult to directly estimate the parameters of ERGM for a very large network. [15] From a technical point of view, a series of operations such as loading large-scale network data, estimation and testing have high requirements for computer memory, network analysis software and other related technologies. All of these restrict the application of ERGM in very large scale network.

Aiming at the problem, people have done some research. Yon G G V，Slaughter A et al. proposed the “ergmito” method based on ERGM extension to realize the fitting of small networks. [54] Weihua also pointed out that when modeling large-scale network, the key is to achieve a good balance between accuracy / consistency and speed / stability. [1] He elaborated two broad methods and seven large-scale network fitting methods based on ERGM, explained the advantages and disadvantages of each method, and provided suggestions for researchers to choose methods. AADS，BJHK demonstrated how to use snowball sampling and conditional estimation method to estimate ERGM parameters of large undirected networks, and verified the feasibility of this method. [53] In 2019, Stivala A , Robins G et al. demonstrated the EE (equilibrium expectation) algorithm, which can estimate the ERGM parameters of a social directed network model with more than one million nodes, and applied it to an online social network with more than 1.6 million nodes. [15] At present, the research on the application of ERGM in different scale networks is still in the development stage, and we believe that more scholars will join the research in the future.

4. FUTURE PROSPECTS OF TIE-GENERATIVE MECHANISM IN NETWORK BASED ON ERGM

Compared with the traditional regression analysis method, the advantage of ERGM is that it gets rid of the constraint of independent hypothesis and comprehensively considers the influence of network structure, node attributes and covariables on the network formation. At present, this research area has made good progress in the aspects of model extension and influence factors exploration, but there are still many troubles waiting for further research.

4.1 Extending ERGM to capture information on connection strength.

The weakness of ERGM is that it can only process binary network while can’t capture the information on connection strength, which makes scholars have to convert the weighted network into the binary network for research, this operation leads to the loss of rich information in the weighted network. [28] Extending ERGM to capture connection strength information in weighted networks is a topic worthy of further discussion.

4.2 Research on the dynamic mechanism of network formation.

Although there are TERGM and STERGM which can consider time dependence, there are still many gaps in this filed. BraillyJ, Favreg et al. pointed out that the influences among different levels of networks are not unidirectional, in fact, they influence and depend on each other. [46] However, current researches based on cross-sectional data can’t enable them to explore complex causal relationships among multi-level networks, which limits people's understanding of the evolution process of networks. Shaohua Shi and Yehong Sun also proposed the interaction between networks of different levels would be the future research direction. [55] In addition, the differences in the effects of influencing factors at different stages of network development also need to be further explored. [44]

4.3 Extending ERGM to capture heterogeneity.

Heterogeneity refers to the intangible factors that are not observed or difficult to measure in the research but have an impact on the formation of the network, such as personal charm and corporate culture. Some scholars have shown that this unobserved heterogeneity may be the main reason for the inappropriate fitting of the network model. [50] At present, although some academics have proposed SRFE-ERFM to alleviate the dilemma caused by heterogeneity, this model still has some limitations. The research object of this model must be a directional network and the receiver and sender can't be modeled simultaneously. Furthermore, this model is only applicable to cross-sectional data, and can't be userd for longitudinal research on the network. All of these limit the further application of SRFM-ERGM. How to extend the model so that it can broadly model various influencing factors while capturing the heterogeneity of nodes will be the focus of future research.

4.4 Developing methods to detect multicollinearity in ERGM.

Multicollinearity is when there is at least one independent variable in the model that changes as a function of the other independent variables. The undetected collinearity in ERGM would cause problems in the inference of model parameters, resulting in model degradation or non-convergence. In addition, the increasing number of structural items in the network also increases the possibility of collinearity [56] and eventually leads to the model being unusable. Unfortunately, multicollinearity has not received enough attention and is often not examined effectively. Although Duxbury S W proposed a method proved to be valid in detecting multicollinearity in 2017 [56], in general, few studies have been conducted on how to detect and eliminate multicollinearity in ERGM, and further studies are needed in the future.

In addition to the above outlook, such as exploring how to estimate and impute the missing data of the network based on ERGM [57], comparing ERGM with other models to discuss their respective advantages, disadvantages and applicable network types [58], extending the concept of network formed from the formation of the edge to the formation and disappearance of the edge [41], [44], and so on are all research points that should be paid attention to in the future.

5. CONCLUSIONS

This paper takes the literature that discussing network formation mechanism based on the ERGM as the research object, mainly analyzes the research progress and limitations in this field from the perspective of the ERGM algorithm expansion (modeling node's heterogeneous, proposing longitudinally ERGM model, appling ERGM in different scale network) and the complex factors that affect the formation of the network (the research on multistage network, the influence of multiple social factors on the network) during the five years from 2015 to 2020, and puts forward the future prospects in this field, which provides reference for future research.

ACKNOWLEDGEMENT

This work is partly supported by the National Natural Science Foundation of PRC (No. 72025101, 71729001, 71601100), and the Fundamental Research Funds for the Central Universities (No. FRF-TP-20-045A2).

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