Fall 12-1-2023

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Brusov, P.N., Filatova, T.V., Kulik, V.L., Chang, S.I., Lin, Y.C.G. & Chang, L.M. (2023). Precision finance: Capital structure theories approach reality. In Li, E.Y. *et al.* (Eds.) *Proceedings of The International Conference on Electronic Business, Volume 23* (pp. 466-480). ICEB'23, Chiayi, Taiwan, October 19-23, 2023

Precision Finance: Capital Structure Theories Approach Reality

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ABSTRACT

Bringing existing financial models closer to real practice is the most important challenge in precision finance. Over the past couple of years, the two main theories of the capital structure (Brusov–Filatova–Orekhova (BFO) and Modigliani–Miller (MM)) have been adapted to the established financial practice of the functioning of companies, taking into account the real conditions of their work. They are generalized to the case of variable income, to paying income tax with arbitrary frequency, to the advance payments of income tax etc. Taking these effects into account significantly changed the results of both theories and brought both theories closer to reality.

Keywords: Precision finance, capital structure, Generalized Modigliani–Miller (MM) theory, Generalized Brusov–Filatova–Orekhova (BFO) theory, actual operating conditions of the company

INTRODUCTION

One of the most important challenges in precision finance is to bring existing financial models closer to real–world practice. The modern theory of the cost of capital and capital structure – the Brusov–Filatova–Orekhova (BFO) theory (Brusov et al. 2018, 2021, 2022, 2023) and its perpetual limit – the Modigliani–Miller theory (Modigliani and Miller 1958, 1963, 1966) describe the case of constant income and payment of income tax at the end of the year. But in practice, companies may make these payments upfront, and the income may be variable. Over the past couple of years, the two main theories of the capital structure (Brusov–Filatova–Orekhova (BFO) and Modigliani–Miller (MM)) have been adapted by the authors to the established financial practice of the functioning of companies, taking into account the real conditions of their work. They are generalized to the case of variable income (this is extremely important), as well as to the case of paying income tax with arbitrary frequency, to the case of advance payments of income tax and for the combinations of these effects. Account of these effects has changed the results of both theories significantly and made the Modigliani–Miller theory (which is perpetual limit of Brusov–Filatova–Orekhova) closer to Brusov–Filatova–Orekhova one, although they will never intersect, since the MM theory does not have a time factor, and the BFO describes companies of arbitrary age.

In case of variable income the role of discount rate for leverage company passes from the weighted average cost of capital, WACC, to WACC − g (where g is growing rate), for a financially independent company from k0 to k0 − g. The real discount rates WACC − g and k0 − g decrease with g, while WACC grows with g. This decrease leads to an increase in company value with g.

In case of payments of tax on profit with arbitrary frequency we derived modified BFO formulas and show that: (1) All BFO formulas change; (2) all main financial parameters of the company, such as company value, V, equity cost, ke, and the weighted average cost of capital, WACC, depend on the tax on profit payments frequency. The increase of the frequency of payments of income tax leads to a decrease in the cost of attracting capital, WACC, and to increase in the capitalization of the company, V. At a certain age n of the company and at certain frequency of tax on profit payments p, a qualitatively new anomalous effect takes place: the equity cost, ke(L), decreases with an increase in the level of leverage L. This radically changes the company′s dividend policy, since the economically justified amount of the dividends is equal to the cost of equity. For both parties–for the company and for the tax regulator more frequent payments of tax on profit are beneficial: for the company, because this increases the company capitalization, and for the tax regulator, because earlier payments are beneficial for it due to the time value of money.

The influence of variable incomes, the frequency of income tax payment, advance income tax payments on all major financial indicators (WACC, company value, V, equity cost, ke etc) and their dependence on debt financing was studied and it was shown that this influence is significant and quite important.

BASIC THEORIES OF CAPITAL STRUCTURE: A HISTORICAL POINT OF VIEW

From a historical point of view, **five stages** in the development of the capital structure theory can be distinguished: **First** (before 1958) – the traditional approach, based on practical experience and existed before the appearance of the first quantitative theory by Modigliani and Miller (**the second stage)** (Modigliani and Miller 1958, 1963, 1966). **The third stage** (1964– 2008) is the numerous attempts of scientists to modify the Modigliani – Miller theory. **The fourth stage** (2008–2019) is the appearance of Brusov–Filatova–Orekhova **(**BFO) theory, which removed the main restriction of the Modigliani – Miller (MM) theory associated with the infinite life–time of the company (Brusov, Filatova 2023). And, finally, **the fifth stage** (2019 up to now), which began a couple years ago and is associated with the adaptation of the two main theories of the capital structure (Brusov–Filatova–Orekhova and Modigliani–Miller) to the established financial practice of the company's functioning by taking into account the real conditions of their work (Brusov et al. 2021, 2022,2023).

One of the most important assumptions of the Modigliani – Miller theory is that all financial flows and all companies are perpetuity. This limitation was lift out by Brusov–Filatova–Orekhova in 2008 (Filatova et al 2008), who have created BFO (Brusov–Filatova–Orekhova) theory – modern theory of capital cost and capital structure for companies of arbitrary age (BFO–1 theory) and for companies of arbitrary life time (BFO–2 theory) (Brusov et al 2018). **Fig.1** shows the historical development of the theory of capital structure from the empirical traditional approach to the general theory of capital structure, BFO, through the perpetuity Modigliani-Miller theory.

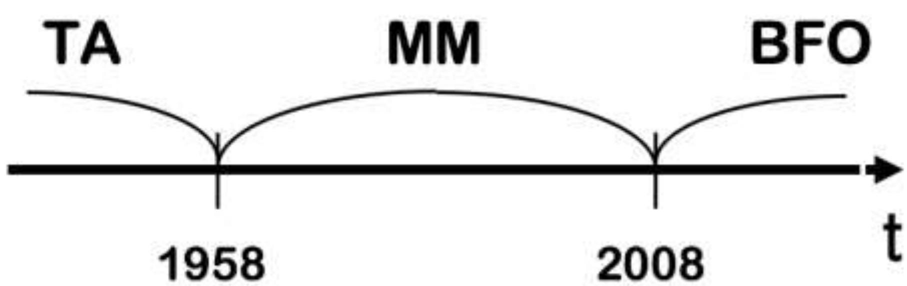

Figure. 1: The historical development of the theory of capital structure from the empirical traditional approach (TA) to the general theory of capital structure, BFO (Brusov–Filatova–Orekhova), through the perpetuity Modigliani-Miller theory.

One year companies were studied by Steve Myers in 2001, who showed that the weighted average cost of capital WACC is greater in this case than in the perpetual Modigliani-Miller case, and the value of company V is therefore less. Only two results for capital structure of the company were known up to 2008, when BFO theory has appeared: Modigliani–Miller for perpetuity companies and Myers for one–year company (see **Fig. 2**). The created BFO theory filled the entire interval between t=1 and $t=\infty$. This expands capital structure theory for the companies of arbitrary ages and/or arbitrary life–time. Many new meaningful effects have been discovered in the BFO theory.

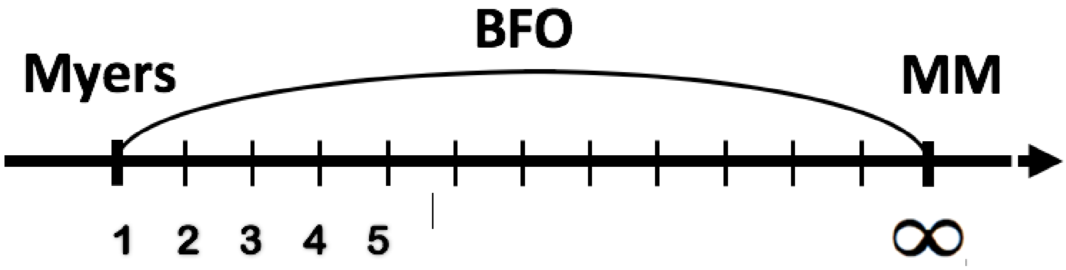

Figure 2: Only two results for capital structure of the company were known before BFO theory: Modigliani–Miller (MM) for perpetuity companies and Myers for one–year company, while the created BFO theory filled the entire interval between n=1 and $n = \infty$.

The Empirical (Traditional) Approach

In the traditional approach, based on practical experience and existing before the advent of the first quantitative theory of Modigliani and Miller, the weighted average cost of capital WACC and the associated capitalization of the company, $V = \mathrm{CF}/\mathrm{WACC}$, depend on the capital structure, the level of debt load, L. The cost of debt is always lower than the cost of equity, because the former has less risk due to the fact that the claims of creditors are satisfied before the claims of shareholders in the event of bankruptcy. As a result, an increase in the share of cheaper borrowed capital in the total capital structure to the limit that does not cause a violation of financial stability and an increase in the risk of bankruptcy leads to a decrease in the weighted average cost of capital WACC. The return required by investors (equal to the cost of equity) is growing; however, its growth did not offset the benefits of using cheaper borrowed capital. Therefore, the traditional approach welcomes the increase in leverage $L = D/S$ and the associated increase in the value of the company $V = \mathrm{CF}/\mathrm{WACC}$. The empirical traditional approach existed until the appearance of the first quantitative theory by Modigliani and Miller (1958).

Within the based on existing practical experience traditional approach the competition between the advantages of debt financing at a low leverage level and its disadvantages at a high leverage level forms the optimal capital structure, defined as the leverage level, at which WACC is minimal and company value, V, is maximum.

BENEFITS OF ADVANCE PAYMENTS OF TAX ON PROFIT: CONSIDERATION WITHIN BRUSOV–FILATOVA–OREKHOVA (BFO) THEORY

The modern theory of the cost of capital and capital structure – the Brusov–Filatova–Orekhova (BFO) theory and its perpetual limit – the Modigliani–Miller theory describe the case of constant income and payment of income tax at the end of the year. But in practice, companies may make these payments upfront, and the income may be variable. Recently we have modified the Modigliani–Miller theory for the case of advanced payments of income tax and have shown that obtained results are quite different from ones in "classical" Modigliani–Miller theory. In current paper for the first time we modify the Brusov–Filatova–Orekhova (BFO) theory for the case of advanced payments of income tax and show that the impact of the transition to advance payments is much more significant than in the case of an perpetuity limit (the MM theory) and even leads to a qualitatively new effect in the dependence of equity cost on leverage. An important conclusion made in this paper is that the tax shield is very important and the way it is formed (payments at the end of the year or in advance) leads to very important consequences, changing, in particular, the company's dividend policy. The final Brusov–Filatova–Orekhova equation for WACC for the case of advanced payments of income tax

$$\frac{\left(1-(1+WACC)^{-n}\right)}{WACC} = \frac{\left(1-(1+k_0)^{-n}\right)}{k_0\left(1-w_d\,t\cdot\left(1-(1+k_d)^{-n}\right)\right)\cdot(1+k_d)}\,. \tag{1}$$

This formula differs from the classical Brusov–Filatova–Orekhova equation by the factor $(1+k_d)$ in the left denominator

$$\frac{\left(1-(1+WACC)^{-n}\right)}{WACC} = \frac{\left(1-(1+k_0)^{-n}\right)}{k_0\left(1-w_d\,t\cdot\left(1-(1+k_d)^{-n}\right)\right)} \tag{2}$$

Results

Here the dependence of the weighted average cost of capital, WACC, capital value, V, equity cost, ke, on leverage level L for three–year and six–year companies, using Microsoft Excel is being studied. We consider two types of payments of income tax: (1) at the end of the year and (2) in advance. As we mentioned above, for WACC we use formulas (12) and (13), for capital value, V, we use formulas (6) and (7) and for equity cost, ke, we use formula (15).

We use the following parameters: k_0=0.2; k_d=0.18; t=0.2; n=3; 6; *CF*=100.

Dependence of the Weighted Average Cost of Capital, WACC, Capital Value, V, Equity Cost, ke, on Leverage Level L for Three–year Company

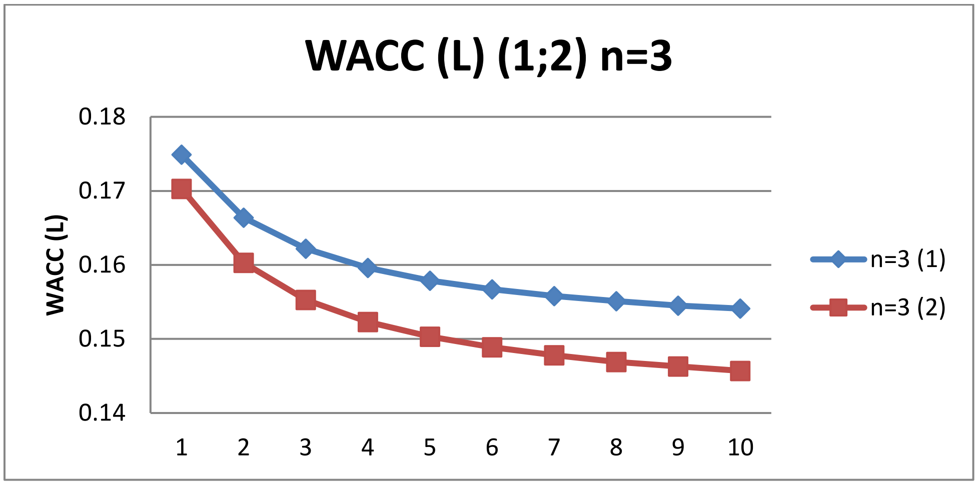

Figure 3: Dependence of the weighted average cost of capital, WACC, on leverage level L in the cases of payments of tax on profit at the end of the year (1) and in the beginning of the year (2) for three–year company

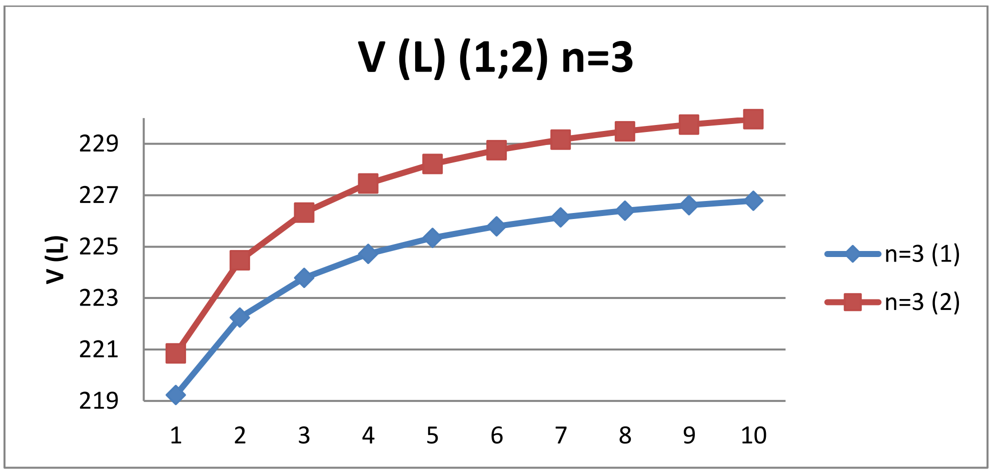

Figure 4: Dependence of the company value, V, on leverage level L in the cases of payments of tax on profit at the end of the year (1) and in the beginning of the year (2) for three–year company

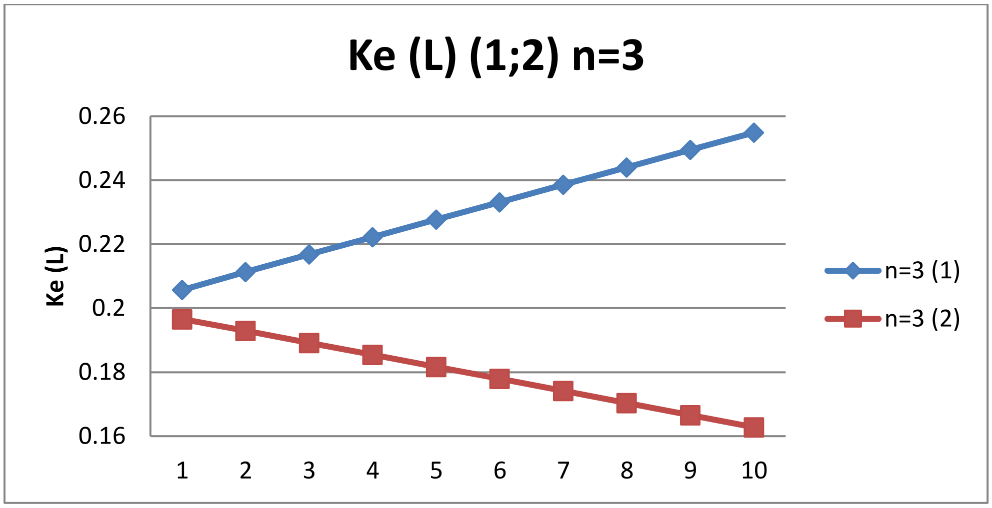

Figure 5: Dependence of the equity cost, ke on leverage level L in the cases of payments of tax on profit at the end of the year (1) and in the beginning of the year (2) for three–year company

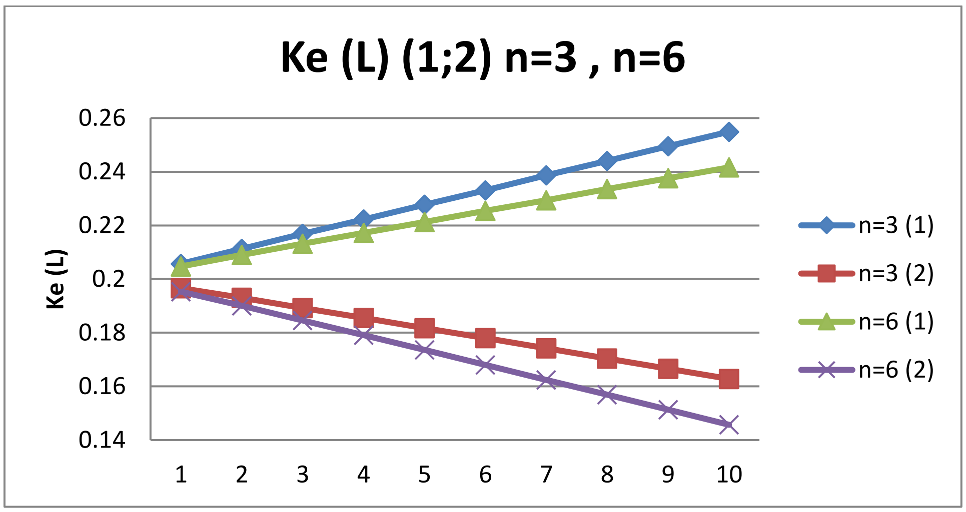

Figure 6: Dependence of the equity cost, ke, on leverage level L in the cases of payments of tax on profit at the end of the year (1) and in the beginning of the year (2) for three–year and six –year companies

Discussion

We derive BFO formulas for WACC, V, ke for the case of advanced payments of income tax. Making the calculations using these formulas within Microsoft Excel we get the following results (see Figures 3–6).

1. WACC (L) decreases with L in both cases. This means that debt financing is important and should be used by company – it leads to decrease of attracting capital cost with L.
2. WACC turns out to be lower in the case of advanced payments of income tax, this tells about importance of the use of advanced payments of income tax for companies.
3. WACC decreases with company age: this is one of the important results of classical BFO theory.
4. Company value, V, increases with L in both cases, this follows from the decrease of attracting capital cost with L.
5. Company value, V, turns out to be bigger in the case of advanced payments of income tax: this tells about importance of the use of advanced payments of income tax for companies.
6. Company value, V, increases with company age
7. Equity cost ke decreases with company age in both cases: this is one of the important results of classical BFO theory.
8. Equity cost ke increases with leverage level L in the case of payments of income tax at the end of the year.

9. Equity cost ke decreases with leverage level L in the case of advanced payments of income tax. This means the appearance of a qualitatively new effect that can greatly change the company's dividend policy, because the economically justified amount of dividends is equal to the equity cost.

Summary and Conclusions

Advance income tax payments are beneficial to both parties: to companies, because they lead to decrease of cost of attracting capital and increase of company values; to regulator, because earlier replenishment of the budget ensures an increase in the stability of budget revenues.

An important conclusion made in this paper is that the tax shield is very important and the way it is formed (payments at the end of the year or in advance) leads to very important consequences, changing, in particular, the company's dividend policy. If with payments at the end of the year, the amount of dividends should increase with the increase in the use of debt financing, with advance payments of income tax the amount of dividends should decrease with the increase in the use of debt financing: this is a pioneering result that radically changes the company's dividend policy.

THE GENERALIZATION OF THE BRUSOV–FILATOVA–OREKHOVA THEORY FOR THE CASE OF PAYMENTS OF TAX ON PROFIT WITH ARBITRARY FREQUENCY

Both main theories of capital cost and capital structure – the Brusov–Filatova–Orekhova (BFO) theory and its perpetuity limit – the Modigliani–Miller theory– consider the payments of tax on profit once per year, while in real economy these payments are made more frequently (semi–annual, quarterly, monthly etc.). Recently the Modigliani–Miller theory has been generalized by us for the case of tax on profit payments with an arbitrary frequency. Here for the first time we generalized the Brusov–Filatova–Orekhova (BFO) theory for this case. The main purpose of the paper is bring the BFO theory closer to economic practice, taking into account one of the features of the real functioning of companies – the frequent payments of tax on profit. We derive modified BFO formulas and show that: (1) all BFO formulas change; (2) all main financial parameters of the company, such as company value, V, the weighted average cost of capital, WACC, and equity cost, ke, depend on the frequency of tax on profit payments. It turns out that the increase of the number of payments of tax of profit per year leads to decrease of the cost of attracting capital, WACC and increase of the company value, V. At a certain age n of the company and at certain frequency of tax on profit payments p, **a qualitatively new anomalous effect** takes place: the equity cost, ke(L), decreases with an increase of the level of leverage L. This radically changes the company's dividend policy, since the economically justified amount of the dividends is equal to the cost of equity. More frequent payments of income tax are beneficial for both parties – for the company and for the tax regulator: for the company, this leads to an increase in the value of the company, and for the tax regulator, earlier payments are beneficial due to the time value of money.

We have derived the modified formula BFO for weighted average cost of capital (WACC) for company of age n years for the case of p payments of tax on profit per year (payments at the end of periods)

$$\frac{1-(1+WACC)^{-n}}{WACC}=\frac{\left(1-(1+k_0)^{-n}\right)}{k_0\left(1-\frac{k_d w_d t\left(1-(1+k_d)^{-n}\right)}{p\left((1+k_d)^{1/p}-1\right)}\right)} \tag{3}$$

For the the Modigliani–Miller limit one has

$$WACC=k_0\left(1-\frac{k_d w_d t}{p\left((1+k_d)^{1/p}-1\right)}\right) \tag{4}$$

At p=1 we get **classical** BFO formula

$$\frac{1-(1+WACC)^{-n}}{WACC}=\frac{1-(1+k_0)^{-n}}{k_0\cdot\left(1-w_d t\left[1-(1+k_d)^{-n}\right]\right)} \tag{5}$$

For the the Modigliani–Miller limit one has

$$WACC = k_0\left(1 - w_d\, t\right) \tag{6}$$

Formulas for Capital Value, V, and Equity Cost, Ke

Below we investigate the dependence of the weighted average cost of capital, WACC, capital value, V, equity cost, ke, on leverage level L at different frequency of payment of tax on profit p for three–year and six–year companies, using Microsoft Excel. For WACC we will use formula (3) and for capital value, V, and equity cost, ke, we will use formulas (7) and (8) respectively (see below).

Company of age n capitalization could be calculated by the following formula

$$V = \frac{CF \cdot \left(1 - \left(1 + WACC\right)^{-n}\right)}{WACC}. \tag{7}$$

k_e should be found from the equation

$$WACC = k_e w_e + k_d w_d \left(1 - t\right)$$

$$k_e = \frac{WACC}{w_e} - \frac{k_d\left(1 - t\right) w_d}{w_e} = WACC\left(1 + L\right) - Lk_d\left(1 - t\right) \tag{8}$$

where one should substitute WACC from the formula (3).

Results

In this section we study the dependence of the weighted average cost of capital, WACC, capital value, V, equity cost, ke, on leverage level L at different frequency of payment of tax on profit p for three–year and six–year companies, using Microsoft Excel. As we mentioned above, for WACC we use formula (3) and for capital value, V, and equity cost, ke, we use formulas (7) and (8) respectively.

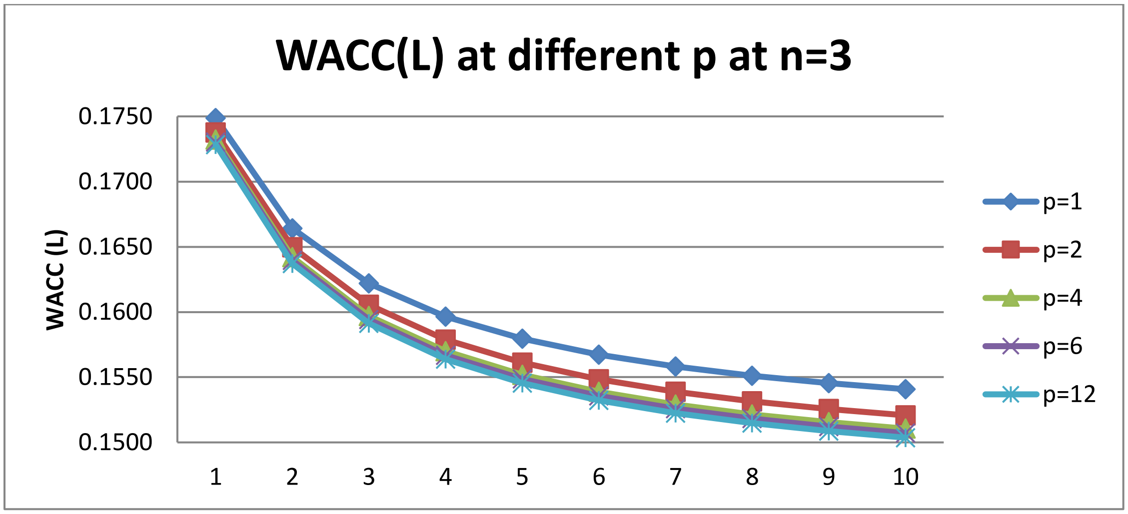

Figure 7: Dependence of the weighted average cost of capital, WACC, on leverage level L at different frequency of payments of tax on profit p for three–year company

The weighted average cost of capital, WACC, decreases with leverage level L at any frequency of payments of tax on profit p. The difference between the WACC (L) curves is maximum when moving from annual (p=1) to semi–annual (p=2) income tax payments and decreases when moving from semi–annual (p=2) to quarterly (p=4) payments and from quarterly (p=4) to monthly (p=12) payments.

Dependence of The Company Value, on Leverage Level L at Different Frequency of Payment of Tax on Profit p for Three–year Company

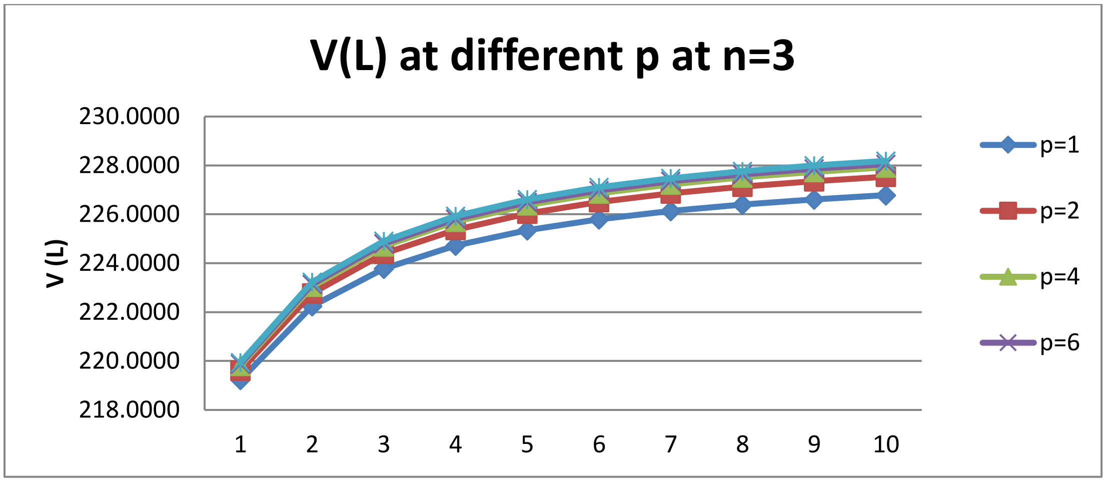

Figure 8: Dependence of the company value, V, on leverage level L at different frequency of payments of tax on profit p for three–year company

The value of company V increases with the frequency p. The largest increase occurs when moving from annual (p=1) to semi–annual (p=2) income tax payments and decreases when moving from semi–annual (p=2) to quarterly (p=4) payments and from quarterly (p=4) to monthly (p=12) payments.

Dependence of The Equity Cost, ke, on Leverage Level L at Different Frequency of Payment of Tax on Profit p for Three–year Company

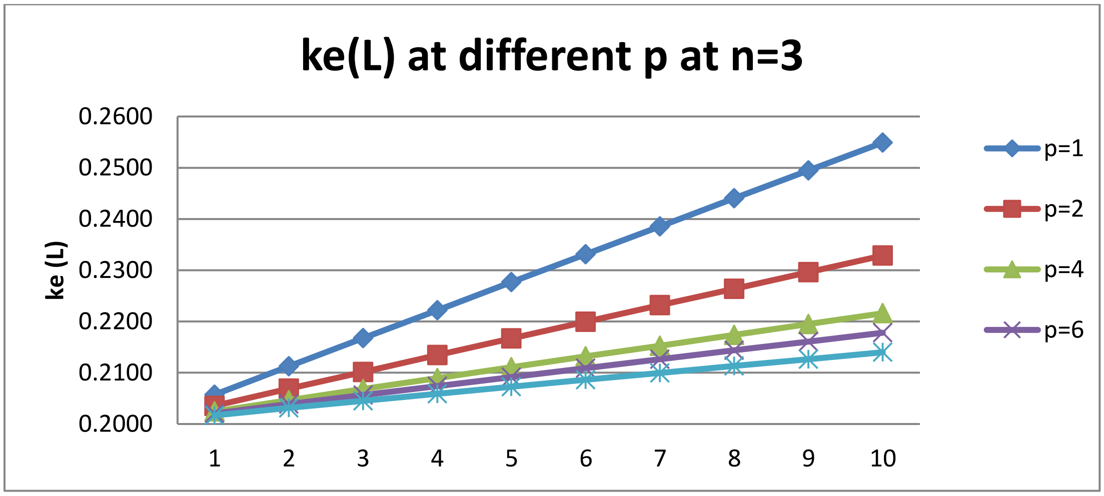

Figure 9: Dependence of the equity cost, ke, on leverage level L at different frequency of payments of tax on profit p for three–year company

The cost of equity ke increases linearly with the level of leverage L. The slope of the curve ke(L) depends on the frequency of paying income tax: it decreases with increasing p, most rapidly when moving from annual (p=1) to semi–annual (p=2) payments of income tax and slower in the transition from semi–annual (p=2) to quarterly (p=4) payments and from quarterly (p=4) to monthly (p=12) payments.

GENERALIZATION OF THE MODIGLIANI – MILLER AND BRUSOV–FILATOVA–OREKHOVA THEORY FOR THE CASE OF VARIABLE PROFIT

The main theory of capital cost and capital structure Brusov–Filatova–Orekhova (BFO) theory and its perpetuity limit – theory by Nobel Prize winners Modigliani and Miller consider the case of constant profit, while in practice profit of the company is, of cause, variable. Recently we have generalized the Modigliani – Miller theory for the case of variable profit, and here for the first time we have generalized the Brusov–Filatova–Orekhova theory for the case of variable profit.

This generalization significantly expands the applicability of this modern capital structure theory, which is valid for companies of arbitrary age, in practice, in particular, in corporate finance, in investments, in business valuation, in banking, in ratings, etc.

We derive the generalized Brusov–Filatova–Orekhova formula for WACC and, using this formula in MS Excel, we show that the role of the discount rate shifts from the weighted average cost of capital WACC to WACC–g (where g is the growth rate) for financially dependent companies and k0– g (for financially independent companies). While WACC increases with g, the real discount rates WACC–g and k0–g decrease with g and, accordingly, the value of the company increases with g. For the cost of equity ke, the slope of the curve ke (L) increases with g. Since the economically justified amount of dividends is equal to the cost of equity, this should change the company's dividend policy. It turns out that at the rate g < g* the slope of the curve ke(L) becomes negative, which can significantly change the principles of the company's dividend policy. This means the discovery of a qualitatively new effect in corporate finance.

We have derived the BFO equation for the case of variable profit of the company

$$\frac{1-\left(\frac{1+g}{1+WACC}\right)^n}{WACC-g}=\frac{1-\left(\frac{1+g}{1+k_0}\right)^n}{(k_0-g)\cdot\left(1-w_d t\left[1-(1+k_d)^{-n}\right]\right)}. \tag{9}$$

This is the main theoretical result of paper Brusov et al. 2022.

In perpetuity (Modigliani – Miller) limit ($n\to\infty$) we get the following equation for WACC in the Modigliani–Miller theory in the case of variable profit

$$WACC-g=(k_0-g)\cdot(1-w_d T) \tag{10}$$

$$WACC=(k_0-g)\cdot(1-w_d T)+g \tag{11}$$

Let us consider first the case of Generalized Modigliani–Miller theory (GMM theory).

The Case of Generalized Modigliani–Miller Theory (GMM theory).

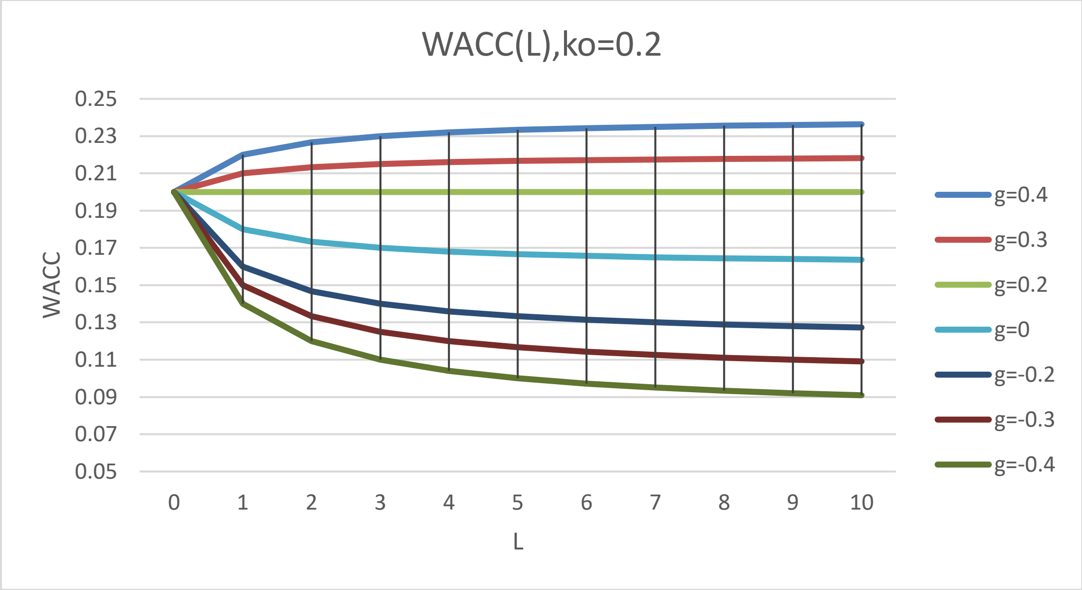

Figure 10: Dependence of WACC on leverage level L in Generalized Modigliani–Miller theory (GMM theory) at k_0=0.2 and g = 0;±0.2; ±0.3; ±0.4

From Fig.10. it is seen that all curves WACC(L) for different g start from one point k_0, in this case from point (0; 0.2). They decrease with leverage level L at g<0.2 (at g = 0;±0.2;−0.3;−*0.4*) and increase at g>0.2 (at g=0.3; 0.4). The curves WACC(L) increase with growth rate, g. Note, that cut–off value of g, which separate increasing curves WACC(L) from decreasing ones is equal to k_0=0.2, and WACC is constant at g=k_0 and equal to k_0. Below we check this observation at different value of k_0 (k_0=0.3).

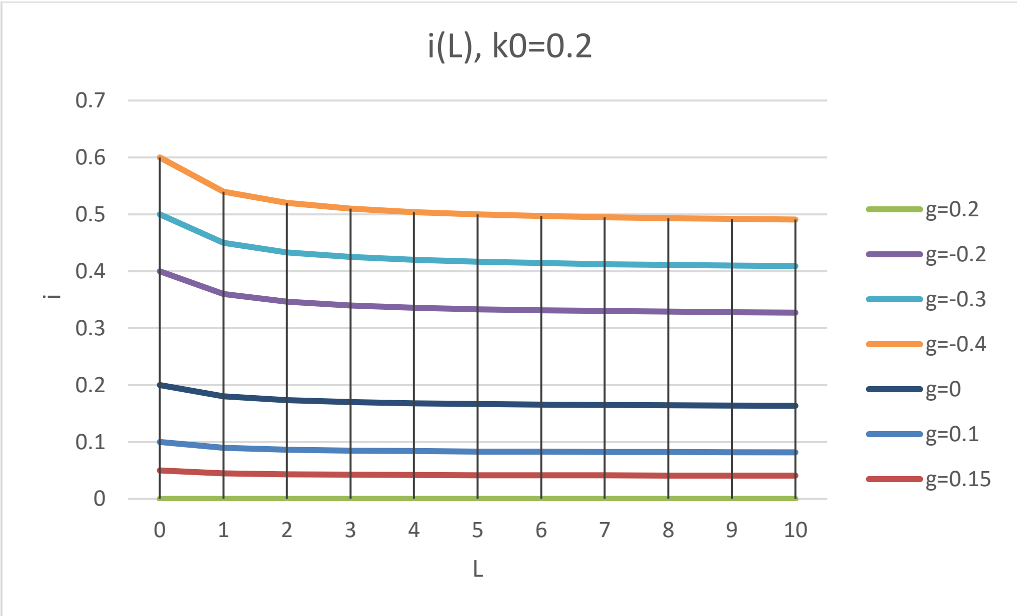

Figure 11: Dependence of discount rate i on leverage level L in Generalized Modigliani–Miller theory (GMM theory) at k_0=0.2 and g = 0; 0.1; 0.15;±0.2; –0.3;–0.4

From Fig. 11 it is seen that discount rate i decreases with leverage level L at growth values g<k_0 (at g = 0; 0.1; 0.15;±0.2; –0.3;–0.4). Discount rate i in opposite to WACC decreases with g: this provides the increase of company value V with g.

At g>k_0 discount rate i increases with L, being negative (it is not shown at Fig.11).

Dependence of discount rate i on leverage level L in Generalized Modigliani–Miller theory (GMM theory) at k_0=0.3 and different values of g

Let us study the dependence of discount rate i on leverage level L in Generalized Modigliani–Miller theory (GMM theory) at k_0=0.3 and different values of g.

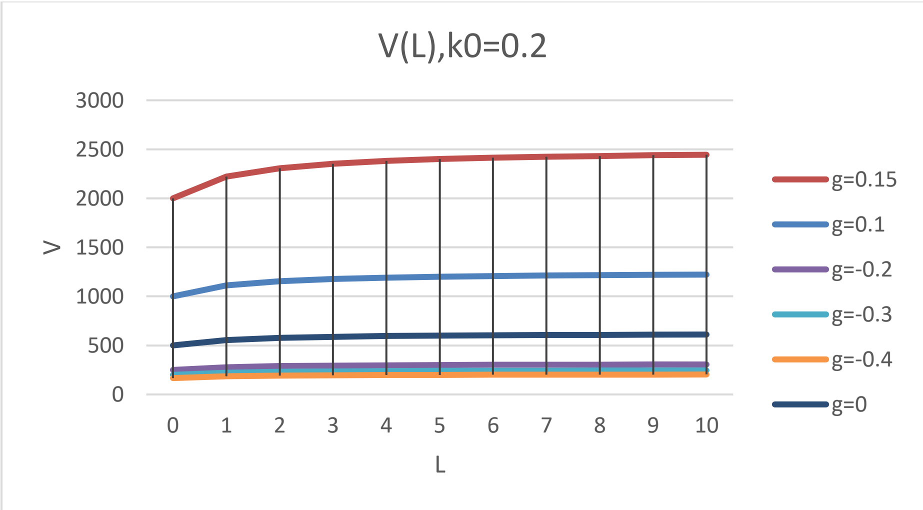

Figure 12: Dependence of company value V on leverage level L in Generalized Modigliani–Miller theory (GMM theory) at k_0=0.2 and g = 0; 0.1; 0.15;–0.2; –0.3;–0.4

From Fig.12, it is seen, that at k_0=0.2 and g = 0; 0.1; 0.15;–0.2; –0.3;–0.4 the company value V at fixed growth rate g increases with leverage level L in Generalized Modigliani–Miller theory (GMM theory). The company value V as well increases with growth rate g.

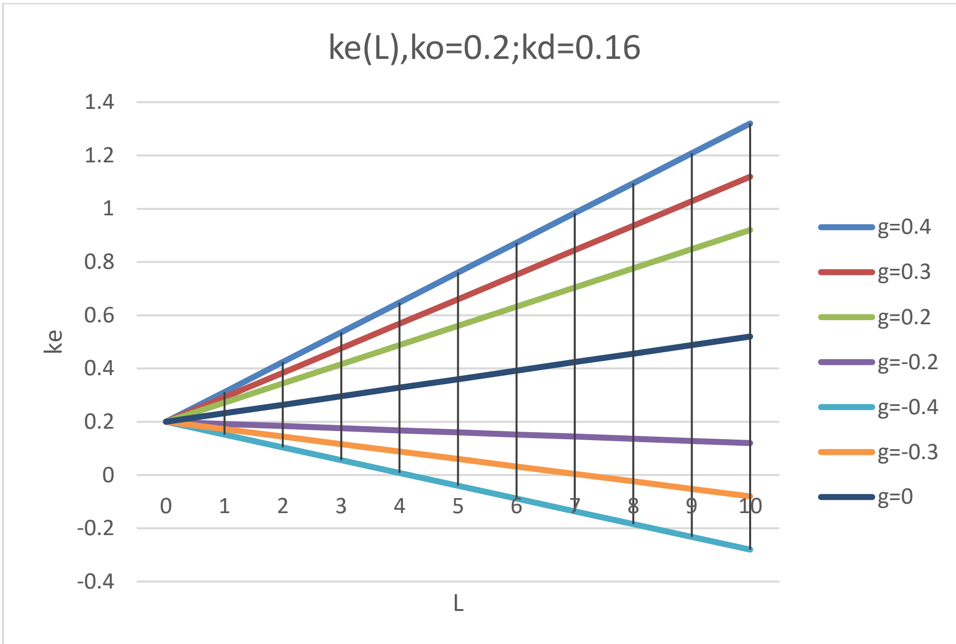

Figure 13: Dependence of equity cost k_e on leverage level L in Generalized Modigliani–Miller theory (GMM theory) at k_0=0.2; k_d=0.16 and g = 0;±0.2; ±0.3; ±0.4

We have investigated the dependence of equity cost k_e on leverage level L in Generalized Modigliani–Miller theory (GMM theory) at k_0 =0.2 and g = 0;±0.2; ±0.3; ±0.4. From Fig.13 it is seen that the equity cost, ke, which linearly grows with leverage level L increases with g: the tilt angle ke (L) grows with g. It is interesting, that at k_0=0.2; k_d=0.16 and at g* = –0.16 in accordance with formula

$$g^* = -\frac{(k_0 - k_d)\cdot(1-T)}{T}$$

the equity cost ke turns out to be equal to k_0 and does not change with leverage level L.

This should change the dividend policy of the company, because the economically justified value of dividends is equal to equity cost.

The Case of Brusov–Filatova–Orekhova Theory

Calculations of the discount rate, WACC–g

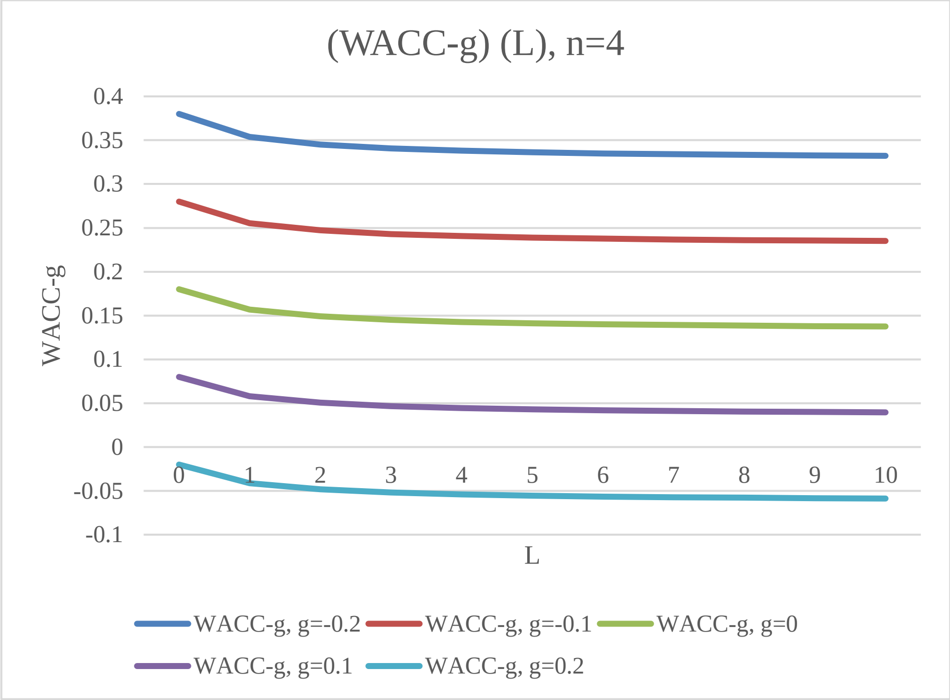

Figure 14: The discount rate, WACC–g, depending on the level of leverage L at different growth rates g=0.2; 0.1; 0.0; –0.1; –0.2 in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) at k_0=0.18; k_d=0.16; t=0.2 for four –year company

As it could be seen from Figure 14 all the curves (WACC–g)(L) with leverage level L at all g values. The (WACC–g) values at fixed leverage level L decrease with growth rate, g. This means that WACC–g is a suitable candidate for the discount rate.

As in the case of the two–year company, explaining the behavior of (WACC–g)(L) with g growth can be as follows: all WACC(L) curves originate from the same point (L=0; WACC=0.18). The (WACC–g)(L) curves will be ordered as follows for L=0: the larger g, the lower the starting point and hence the entire graph lies, since the curves do not intersect. As we'll see below the decrease of (WACC–g)(L) with growth rate, g, will lead to increase of the company value, V, with g.

Calculations of the company value, V

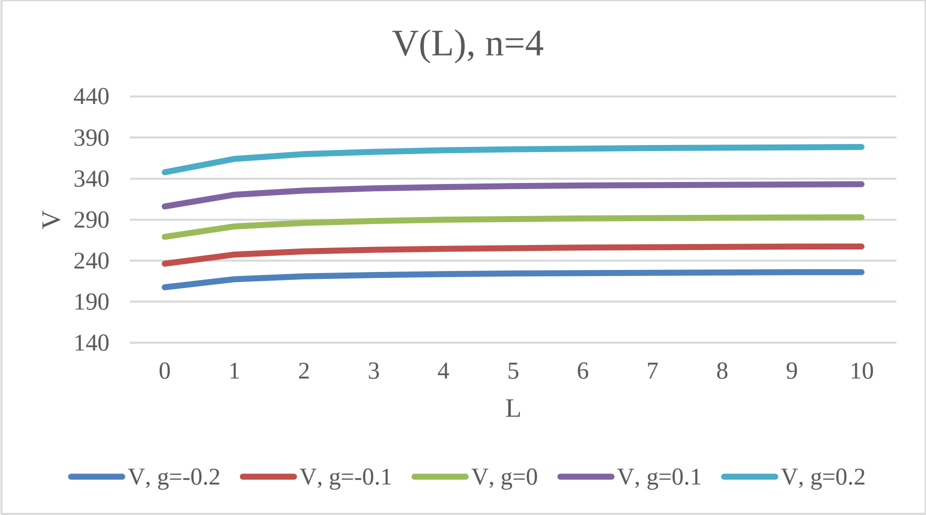

Figure 15: The company value, V, depending on the level of leverage L at different growth rates g=0.2; 0.1; 0.0; –0.1; –0.2 in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) at k_0=0.18; k_d=0.16; t=0.2 for four–year company

As it is seen from Figure 15, the company value V at fixed growth rate g increases with leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory). The company value V as well increases at fixed leverage level L with growth rate g at fixed. This is a consequence of a decrease in the discount rate (WACC–g)(L) with an increase in the growth rate g. Comparing with the results for the two year old company, we see that the value of the company V increases with the age of the company: we have a range of 149 to 157 with L=1 for g from –0.2 to 0.2 for the two year old company and a range of 217 to 364 with L=1 for g from –0.2 to 0.2 for a four year old company. And this is the obvious conclusion, because it is well known, that the value of any asset (company, stock, bond etc) is equal to the sum of the discounted returns generated by this asset. Since this value is proportional to the lifetime of this asset, the capitalization of the company will grow with its age.

The dependence of the cost of equity ke on the level of leverage L in the Generalized theory of Brusov–Filatova–Orekhova (GBFO theory) was studied below for a four–year company with growth rates g = 0;±0.1; ±0.2.

Calculations of the cost of equity ke

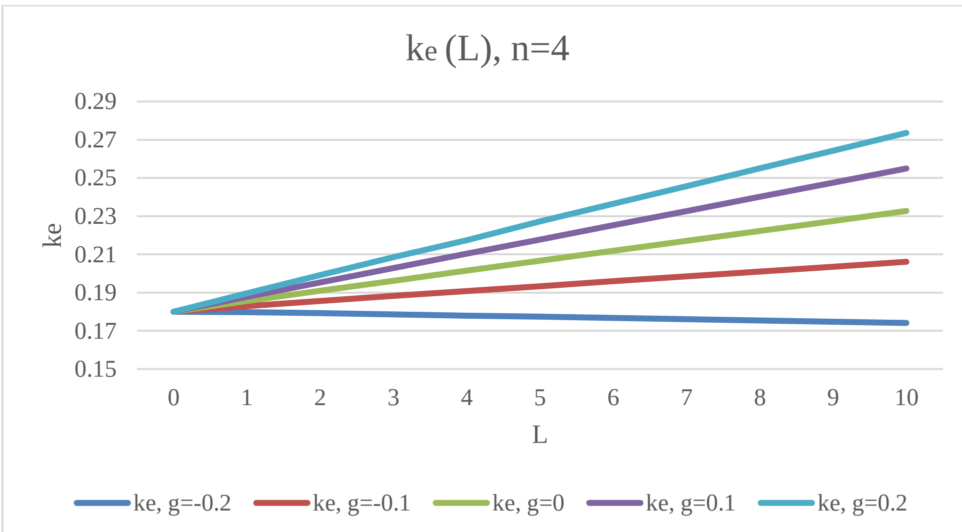

Figure 16: The equity cost, ke, depending on the level of leverage L at different growth rates g=0.2; 0.1; 0.0; –0.1; –0.2 in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) at k_0=0.18; k_d=0.16; t=0.2 for four–year company

From Figure 16 it can be seen that the cost of equity ke increases practically linearly with leverage level L at all growth rates g (except for g=–0.2 where we see a decrease in ke with leverage level L). The slope angle ke(L) increases with g. This should change the dividend policy of an enterprise with variable profit, since economically the reasonable amount of dividends is equal to the cost of equity.

But the biggest change in the company's dividend policy is related to the discovery of a qualitatively new effect in corporate finance: at a rate g < g*, the slope of the ke (L) curve turns out to be negative (one can observe this effect here for g=–0.2 where a decrease in ke with leverage level L takes place). This effect, which is absent in the classical Modigliani–Miller theory and the classical Brusov–Filatova–Orekhova theory with constant profit, exists in the Modigliani–Miller theory with variable income and in the Brusov–Filatova–Orekhova theory with variable income at a certain age of the company, n, which exceeds some cutoff age value n*. The latter effect is similar to a qualitatively new effect in corporate finance, discovered by Brusov–Filatova–Orekhova within the framework of the BFO theory: anomalous dependences of the cost of equity ke on the leverage level L when income tax T exceeds a certain value T*: this discovery also significantly changes the principles of the company's dividend policy.

CONCLUSIONS

The Brusov–Filatova–Orekhova (BFO) theory of capital cost and capital structure as well as the theory by Nobel Prize winners Modigliani and Miller (perpetuity limit of BFO theory) consider the case of constant income, while in practice profit of the company is, of cause, variable. Recently we have generalized the latter for the case of variable profit, and here we have generalized for the first time the Brusov–Filatova–Orekhova theory for the case of variable income.

This generalization significantly expands the applicability of this modern capital structure theory, valid for companies of any age, in practice, in particular, in corporate finance, in business valuation, in investments, in banking, in ratings, etc.

We have derived the generalized BFO formula for WACC and this consists a main theoretical result of a current paper.

From this formula and as well from using this formula in MS Excel, we show that the role of the discount rate shifts from WACC to WACC–g (where g is the growth rate) for financially dependent companies and from k_0 to k_0–g (for financially independent companies). While WACC increases with g, the actual discount rates WACC–g and k_0–g decrease with g and, accordingly, the company value, V, increases with g. For the cost of equity ke, the slope of the curve ke (L) increases with g. Since the cost of equity determines the economically justified amount of dividends, this should change the company's dividend policy. It turns out that at the rate g < g* the slope of the curve ke (L) becomes negative, which can significantly change the company's dividend policy principles. This means the qualitatively new effect discovery in corporate finance.

Bringing existing financial models closer to real practice is the most important challenge in precision finance. Over the past couple of years, the two main theories of the capital structure (Brusov–Filatova–Orekhova (BFO) and Modigliani–Miller (MM)) have been adapted to the established financial practice of the functioning of companies, taking into account the real conditions of their work. They are generalized to the case of variable income, to paying income tax with arbitrary frequency, to the advance payments of income tax etc. Taking these effects into account significantly changed the results of both theories and brought both theories closer to reality. This allows the company's management to make the right management decisions, reduces the risks of losses and bankruptcy and, in general, makes the economy more stable and less susceptible to the risks of global crises.

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