Winter 12-1-2010

Offering Free Version Sofaware Products in the Presence of Network Externalities in E-Commerce

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OFFERING FREE VERSIONS OF SOFTWARE PRODUCTS IN THE PRESENCE OF NETWORK EXTERNALITIES IN E-COMMERCE

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Abstract

Network externalities are commonly observed in many markets in e-commerce, especially in the software market. Many software firms provide free-version products to attract consumers. In this paper, we build a discrete-time dynamical system to investigate why software providers choose to offer commercial versions of products as well as free versions when network externalities are present. We first propose a monopoly model, and then build a duopoly model with two competitors, and one of whom doesn't adopt free-version strategy. Simulation results verify that the presence of network externalities induces firms to provide free-version products.

Key words: network externalities; dynamical system; free-version product; software product; e-commerce.

1. Introduction

The network externality can be defined as a change in the utility that a user derives from consumption of a product when the number of other users consuming the same product changes. This phenomenon was first identified in telecommunication network, and then on many other products, such as operating systems, computer games, typewriter keyboards, and online trading platforms in the e-commerce.

Research on network externalities enjoys a high popularity in the past three decades. In the classic article by Katz and Shapiro [1], a static model based on the maximization of profit function and the fulfilled expectations equilibrium was developed to analyze markets in which consumption externalities are present. Following the seminal work of Katz and Shapiro [1], subsequent researchers have examined product compatibility and standardization [2] [3], pricing strategies [4] [5], technology revolutions [6] [7], online service adoption [8], etc. However, most of the models in the above literatures were developed in a static framework without considering dynamic aspect. These models gave a lot of insights into network externalities, but left aside much empirical evidence in which the dynamic aspect is crucial [9]. Only a few articles discussed network externalities in a dynamic environment [9] [10] [11]. Bensaid and Lesne [9] developed a discrete-time model to study the optimal dynamic monopoly pricing. Lambertini and Orsini [10] reconsidered the role of network externalities in a dynamic spatial monopoly by applying differential equations.

In the software market, many firms provide different versions at different prices via the Internet. And sometimes there are free versions, e.g. the Microsoft Corporation began to provide a free version of office2010 recently. How is it possible that firms are willing to offer free products? What theory explains this economic phenomenon? Based on two-sided network effects, Geoffrey and van Alstyne [4] built a model to illustrate the reasons and results of providing free-version products. Jing [12] investigated how network externalities affect the product line decision of a firm. In this paper, we build a discrete-time dynamical system to investigate the effects of the free-version product on a market where network externalities exist. This problem is discussed first in the monopoly, then in the duopoly setting where two firms adopt different strategies.

The rest of the paper is organized as follows. In section 2, a monopoly dynamical system is described. By analyzing the fixed point of the system, we find out three different market evolution patterns. The effects of parameter variation on the market evolution process are identified from the simulation results. Section 3 extends the model to a competitive environment, and analyzes the advantages of offering free-version product by simulation. Section 4 summarizes and concludes this paper.

2. Monopoly model and analysis

2.1 Dynamic model

Consider a monopoly market over multiple discrete-time periods (t=0, 1, 2…) where the monopolist supplies two versions of products. One version is for free, while the price of the other with more functions is p as a commercial product. The value of the products consists of two parts: intrinsic value and network value. Intrinsic value is the utility a user obtains from the product's inherent features, and network value depends on its network size. Let u_1 denote the intrinsic value of the free version, and u_2 the intrinsic value of the commercial version ($u_2 > u_1$). Similar to the equations in [12], the product with intrinsic value u can obtain a network value of $(e+ku)Q$, where $e,k \geq 0$, and Q is the number of consumers of both versions.

Now we turn to the demand side. Each consumer demands at most one unit of product. Assume that the total consumer population is 1. The utility consumers obtained from two versions of products can be denoted respectively by

$$\begin{cases} T_{u1}(t) = u_1 + (e+ku_1)Q(t), \\ T_{u2}(t) = u_2 + (e+ku_2)Q(t) - p. \end{cases} \quad (1)$$

In this dynamic system, there are three states for the consumers: $\{s_0, s_1, s_2\}$. Consumers in the state s_0 use neither of the two versions, while consumers in the state s_1 use the free version, and those in the state s_2 have bought the commercial version. In each time period, consumers will decide whether to change their states or not. This process can be illustrated by Fig.1.

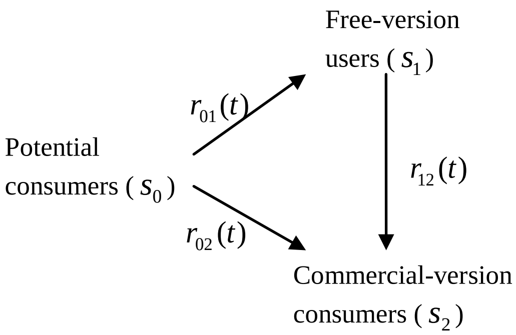


Fig.1 States transfer paths

$r_{ij}(t)(0 \leq r_{ij}(t) \leq 1, t = 1, 2, ...)$ is the rate of consumers transferring from state *i* to state *j* in time period *t*. In this paper, we only consider software products, and the state transfer paths of consumers can only be s_0 to s_1, s_0 to s_2, or s_1 to s_2. This process is irreversible. If we denote the quantity of free-version users and commercial-version consumers in time period *t* by $x(t), y(t)$ respectively, then

$$Q(t) = x(t) + y(t). \quad (2)$$

Therefore, this dynamical system can be described by

$$\begin{cases} x(t+1) = x(t) + (1-x(t)-y(t))r_{01} - x(t)r_{12}, \\ y(t+1) = y(t) + (1-x(t)-y(t))r_{02} + x(t)r_{12}. \end{cases} \quad (3)$$

For simplicity, assume that the transfer rate $r_{ij}(t)$ is proportional positively to the utility difference between state *i* and *j*:

$$r_{01}(t) = \alpha T_{u1}, \quad (4)$$

$$r_{02}(t) = \begin{cases} 0 & T_{u2} \leq 0, \\ \alpha T_{u2} & T_{u2} > 0, \end{cases} \quad (5)$$

$$r_{12}(t) = \begin{cases} 0 & T_{u2} - T_{u1} \leq 0, \\ \alpha(T_{u2} - T_{u1}) & T_{u2} - T_{u1} > 0. \end{cases} \quad (6)$$

Here $r_{ij}(t)$ belongs to the interval [0, 1], so Eq.(7) is the precondition of Eqs.(4), (5) and (6).

$$\begin{cases} 0 \leq \alpha T_{u1} \leq 1, \\ 0 \leq \alpha T_{u2} \leq 1, \\ 0 \leq \alpha(T_{u2} - T_{u1}) \leq 1. \end{cases} \quad (7)$$

2.2 Equilibria and market evolution

We first consider the equilibria of system (3), and then discuss the market evolution patterns for different parameter values. The fixed points of the map (3) can be obtained by setting $x(t+1) = x(t)$ and $y(t+1) = y(t)$, that is

$$(1 - x(t) - y(t))r_{01} - x(t)r_{12} = 0, \quad (8)$$

$$(1 - x(t) - y(t))r_{02} + x(t)r_{12} = 0. \quad (9)$$

$T_{u1}(t) > 0$, thus $r_{01}(t) > 0$. Hence, there exist three equilibra in the market evolution as follows.

(1) If $r_{02}(t) > 0, r_{12}(t) > 0$.

From Eq.(9), we can obtain

$$x(t) = 0,$$
$$1 - x(t) - y(t) = 0$$

Hence, the Nash equilibrium is

$$\begin{cases} x^* = 0, \\ y^* = 1. \end{cases}$$

(2) If $r_{02}(t) = 0, r_{12}(t) = 0$.

$Q(t) = x(t) + y(t)$ increases with time t, thus T_{u2} and $T_{u2} - T_{u1}$ increase with time t, so $r_{02}(t)$ and $r_{12}(t)$ are increasing functions.

Therefore, if

$$r_{02}(t)=0,\ r_{12}(t)=0,$$

then

$$y(t)=0.$$

Subsequently, we can obtain

$$x(t)=1.$$

Hence, the Nash equilibrium is

$$\begin{cases} x^*=1, \\ y^*=0. \end{cases}$$

(3) If $r_{02}(t)>0,\ r_{12}(t)=0$.

From Eqs.(8) and (9), it can be deduced that Nash equilibrium satisfies

$$\begin{cases} x^*+y^*=1, \\ x^*>0, \\ y^*>0. \end{cases} \quad (10)$$

Three equilibria are totally different from each other, and the market evolution processes are also different. Figs.2-4 show different market evolution patterns under the above three conditions.

Pattern 1 in Fig.2 is the most ideal pattern to the firm. The quantity of free-version users increases at first, then more and more users of the free version purchase the commercial version with the increasing of commercial version's total utility. Therefore, $x(t)$ decreases to zero. Pattern 2 in Fig.3 is the worst pattern as no one will purchase the commercial version, that's because its total utility isn't big enough. In pattern 3, there are consumers who will purchase the commercial version, but no consumers will transfer from free-version user to commercial-version consumer as $r_{12}(t)=0$ during all periods.

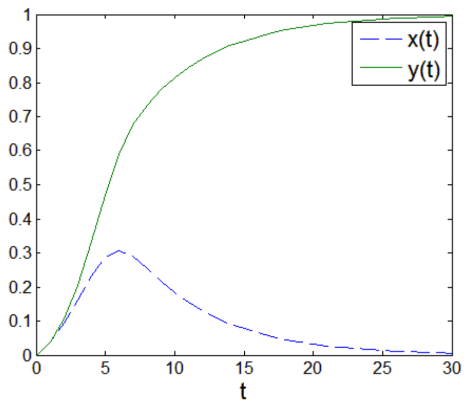


Fig.2 Market evolution pattern 1
($\alpha=0.2,\ k=1,\ e=1,\ u_1=0.2,\ u_2=1,\ p=0.8$)

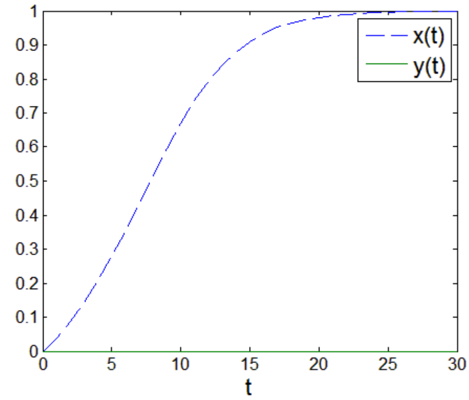


Fig.3 Market evolution pattern 2
($\alpha=0.2,\ k=1,\ e=1,\ u_1=0.2,\ u_2=1,\ p=3.2$)

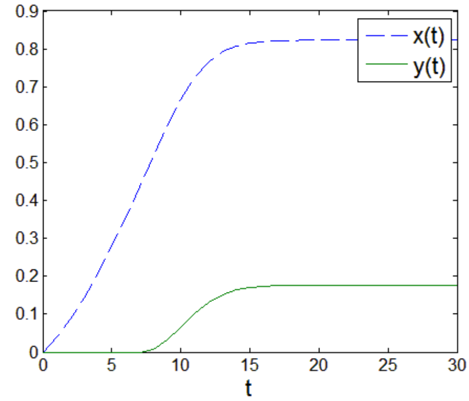


Fig.4 Market evolution pattern 3
($\alpha=0.2,\ k=1,\ e=1,\ u_1=0.2,\ u_2=1,\ p=1.8$)

2.3 Numerical simulations

In the following, we present some numerical simulations to show the effects of parameter variations on the market evolution process. We present two main cases. In the first one, the price of the commercial version is variable while the rest parameters are all constant. In the second one, the intrinsic value of the free version is the only variable.

The variation of p will impact the market structure and evolution. In order to investigate the effects, it's convenient to take the parameter values as follows: $\alpha=0.2,\ k=1,\ e=1,\ u_1=0.2,\ u_2=1$. From the analysis of section 2.2 and Eqs.(4), (5) and (6), it's easy to draw the conclusion that:

(1) If $p\in(0,1.6)$, market evolution pattern will be the ideal one with all the consumers buying the commercial version finally (similar to the curves in Fig.2).

(2) If $p\geq 3$, market evolution pattern will be the worst one with no consumers buying the

commercial version (similar to the curves in Fig.4).

(3) If $p \in [1.6, 3)$, market evolution pattern will be the second one in which no free-version user will buy the commercial version, as the total utility of the commercial version isn't bigger than that of the free version.

Fig.5 and Fig.6 illustrate market evolution curves for $p \in \{0.5, 1, 1.5, 2, 2.5\}$. The maximum value of free-version users increases with p, while the growth speed of the commercial-version consumers decreases with it.

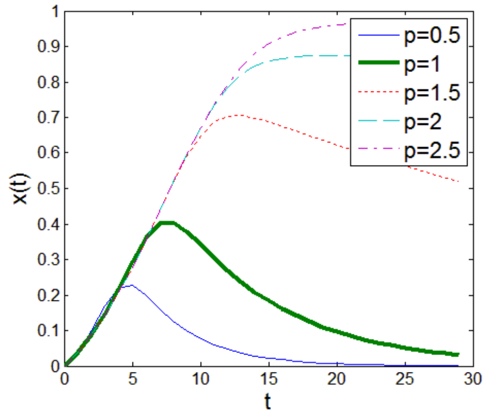


Fig.5 Market evolution curves of $x(t)$

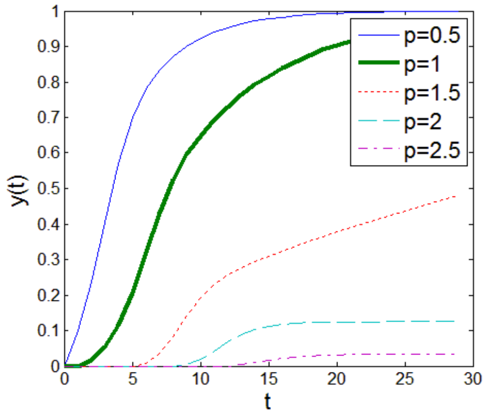


Fig.6 Market evolution curves of $y(t)$

Similarly, we run a simulation based on the variation of parameter u_1, other parameters are set as: $\alpha = 0.2, k = 1, e = 1, p = 0.8, u_2 = 1$. Fig.7, Fig.8, and Fig.9 illustrate market evolution curves for $u_1 \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$.

Fig.9 shows that the growth speed of the number of commercial-version consumers increases with the intrinsic value of the free version at the beginning. That's due to that the population of free-version users is bigger when the intrinsic value is higher, so the network value of the commercial version enlarges, thus the growth speed is faster in the initial periods. However, as time period proceeds, higher intrinsic value of the free version will lead to a lower transfer rate from free version to commercial version, as in Fig.8, and we find that $y(t)$ decreases with u_1 during the interim periods.

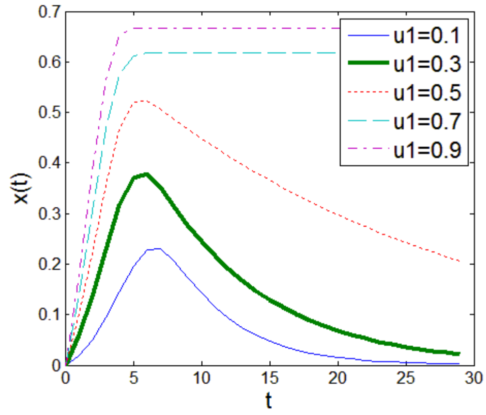


Fig. 7 Market evolution curves of $x(t)$

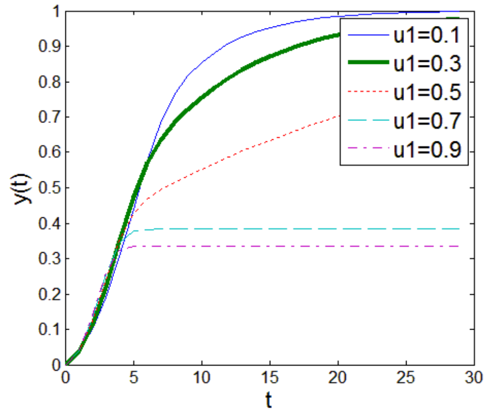


Fig. 8 Market evolution curves of $y(t)$

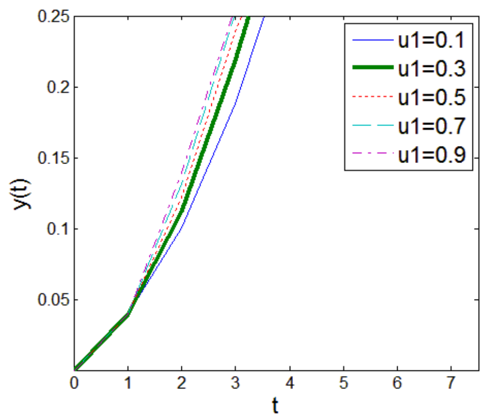


Fig. 9 Market evolution curves of $y(t)$

3. Duopoly model and simulations

3.1 Duopoly model

In this section, we consider a duopoly setting with a competitor who only provides one version of its product at the price of p'. And its intrinsic value is u'. In order to describe the system clearly, we represent the free-version product of the first firm with A', the commercial version with A, and the product of the second firm with B. As a competitor is taken into consideration, a fourth state of the system appears. We denote the new state which represents product B's consumers by s_3. Fig.10 is the state transfer paths in competitive environment.

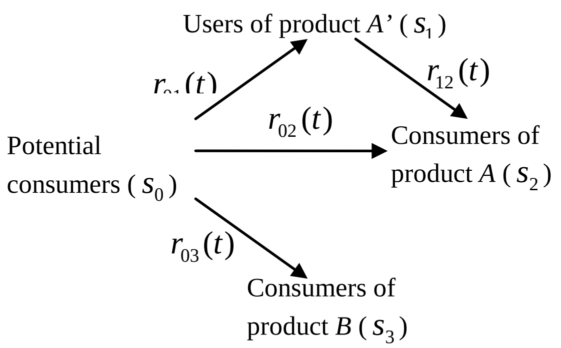


Fig.10 States transfer paths

$r_{03}(t) = \alpha T_{u3}$ $(0 \le \alpha T_{u3} \le 1)$ is the rate of consumers who purchase product B in time period *t*. The user quantity of product B in time period *t* is denoted by $z(t)$. For simplicity, network externalities between products offered by different firms are not considered in this paper. Therefore, the utility consumers obtained from different products can be described respectively as

$$\begin{cases} T_{u1}(t) = u_1 + (e + ku_1)Q(t), \\ T_{u2}(t) = u_2 + (e + ku_2)Q(t) - p, \\ T_{u3}(t) = u' + (e + ku')z(t) - p'. \end{cases} \quad (11)$$

Then the discrete-time dynamical system is described as

$$\begin{cases} x(t+1) = x(t) + (1 - x(t) - y(t) - z(t))r_{01} \\ \qquad - x(t)r_{12}, \\ y(t+1) = y(t) + (1 - x(t) - y(t) - z(t))r_{02} \\ \qquad + x(t)r_{12}, \\ z(t+1) = z(t) + (1 - x(t) - y(t) - z(t))r_{03}. \end{cases} \quad (12)$$

3.2 Numerical simulations

The main purpose of this section is to analyze the advantages of offering free-version product by simulation. Fig.11 shows the market evolution process with parameters are set as follows: $\alpha = 0.2$, $k = 1, e = 1, p = p' = 0.9, u_1 = 0.2, u_2 = u' = 1.$ Assume that $y(t) = z(t), u_2 = u', p = p'$ and $x(t) > 0$ in time period *t*, then $T_{u2}(t) > T_{u3}(t)$, thus $r_{02}(t) > r_{03}(t)$, so $y(t+1) > y(t)$, which means that the existence of the free-version product A' can promote the growth speed of the commercial-version product A.

From the analysis above and Fig.11, we can conclude that under the conditions of $p = p'$ and $u_2 = u'$, the growth speed of $y(t)$ is faster than that of $z(t)$, and the final market share of product A is bigger than that of product B.

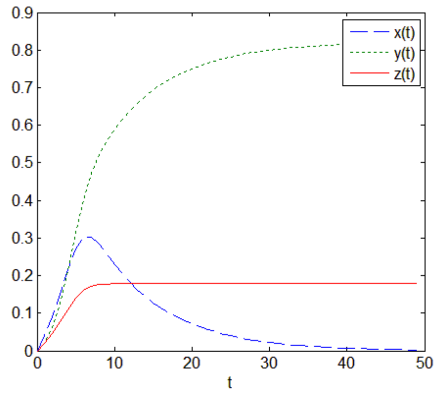
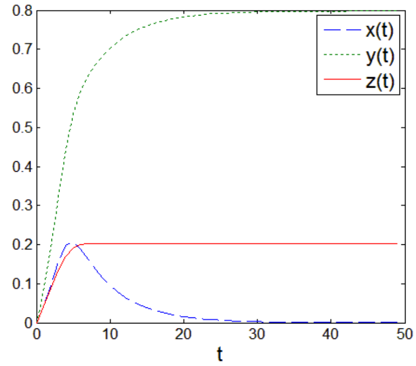
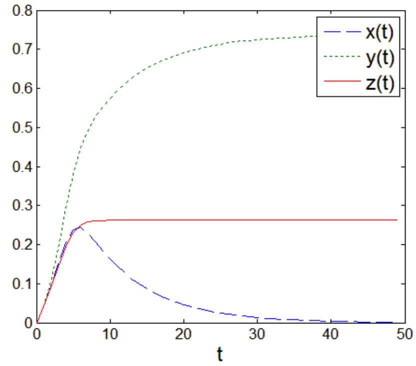


Fig.11 Market evolution processes

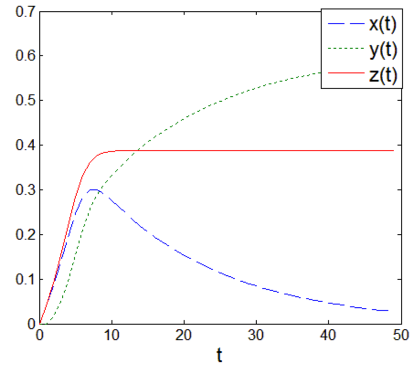
Finally, we run a simulation based on the variations of parameter p and u_2. Fig.12 shows the simulation results for $p \in \{0.6, 0.8, 1.0, 1.2\}$, with $\alpha = 0.2, k = 1, e = 1, p' = 0.8, u' = u_2 = 1, u_1 = 0.2$. Fig.13 shows the results of simulation for $u_2 \in \{0.7, 0.8, 0.9, 1.0\}$, and other parameters are $\alpha = 0.2, k = 1, e = 1, p' = p = 0.8, u' = 1, u_1 = 0.2$. We can observe from Fig.12 and Fig.13 that the final market share of product A decreases with its price, and if the final market share of product A equals that of product B, the price of A will be higher than that of B. And the final market share of product A increases with the intrinsic value of it. Similarly, if the final market share of product A equals that of product B, the intrinsic value of A will be lower than that of B. Therefore, the presence of network externalities induces the firm to provide free-version product.



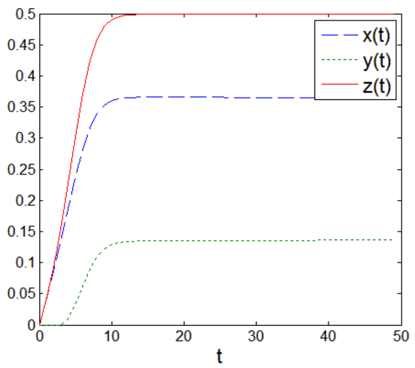
(a) p =0.6



(b) p =0.8

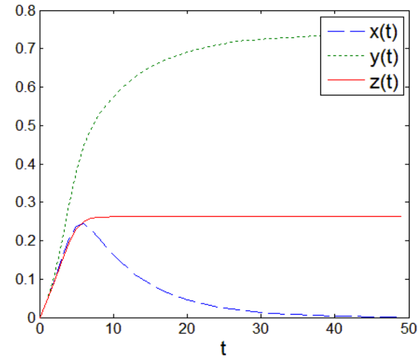


(c) p =1.0

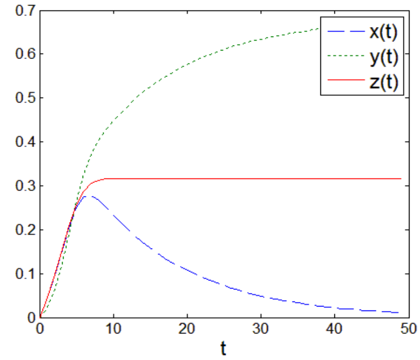


(d) p =1.2

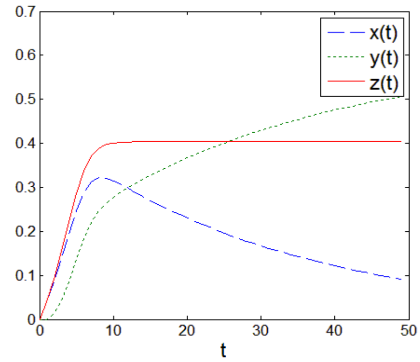
Fig.12 Market evolution processes



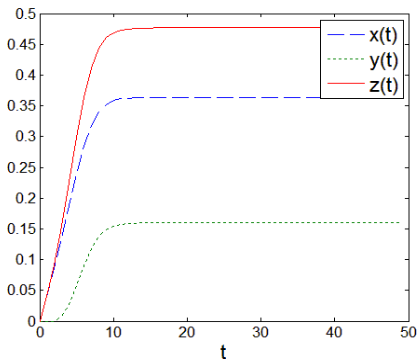
(a) u_2 =1



(b) u_2 =0.9



(c) u_2 =0.8



(d) u_2 =0.7

Fig.13 Market evolution processes

From the simulations above, when the price of the commercial version is not too high and the intrinsic value of it is not too low, the market share of the commercial version from the first company is bigger than that of the product offered by the competitor who doesn't adopt free-version strategy, and the profitability of the first firm is also much stronger. That's because the commercial version benefits from the network externalities engendered by the free version. However, if the price of the commercial version is too high or the intrinsic value of it is too low, a lot of users will continue to use the free version.

4. Conclusions

In this paper we proposed and analyzed a discrete-time dynamical system to investigate the effects of the network externalities generated by free-version product. We first built a monopoly model with no competition. Through fixed points analysis and simulations, we found out three market evolution patterns. The impacts of the parameter variations on market structure and evolution were simulated in experiments .After that, the model was extended to a competitive environment. Simulation results show that, the firm who adopt free-version strategy will get a higher market growth speed and a bigger market share, because its commercial-version product can gain a larger network value through the free-version product. Furthermore, the firm who adopt free-version strategy may still obtain bigger market share and more profits even though the price of its commercial-version product is higher and its the intrinsic value is lower. Therefore, it is almost always profitable for software providers to offer free-version products in the market with network externalities. Future research will focus on the optimal pricing strategy of multi-version products in the monopoly and duopoly in the e-commerce.

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