1987

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ABSTRACT

This paper describes a prototype interactive design tool, PSROUTE, that can be used by designers of data communication networks for selecting primary and secondary routes for every pair of communicating nodes in a network. The objective in selecting routes is to minimize the mean delay faced by messages. The problem was previously modeled as a mathematical programming problem (Pirkul and Narasimhan 1987). PSROUTE allows the user to interactively specify various network and traffic parameters and observe the effects on the solution. The solution can be examined in detail so that the designer can modify the parameters based on his judgment and then use PSROUTE to repeatedly execute the solution procedure until a satisfactory solution is reached.

1. INTRODUCTION

The past decade has witnessed a tremendous increase in the usage of computer networks. These networks have become an important part of industrial, governmental, financial, academic and service institutions. They support an increasing number of services including automatic teller machines, point-of-sale processing at supermarkets, reservation systems, inventory management in complex manufacturing facilities involving several plants and many suppliers, electronic funds transfer and numerous other activities. This has been borne out by the emergence of commercially available value-added networks such as DATAPAC (Cashin 1976; McGibbon, Gibbs and Young 1978), TELENET (Auerbach 1978), TRANSPAC (Despres and Pichon 1980), and TYMNET (Rajaram 1978; Tymes 1981).

The design of computer communication networks is a complex process (Ahuja 1982). Network designers generally follow a hierarchical strategy of designing a backbone network and a number of access networks (Ahuja 1982; Boorstyn and Frank 1977). The access networks link users to nodes of the backbone network. This paper describes an interactive design tool called PSROUTE that helps designers in solving one of the critical problems in the design of backbone data communication networks, namely, that of primary and secondary route selection for each pair of communicating nodes. In backbone networks, which are almost always packet switched networks, messages (packets) are sent from the source node to the destination node via intermediate links and nodes along routes which are either predetermined or determined dynamically as in the ARPA network (Bertsekas 1980; Cantor and Gerla 1974; Frank and Chou 1971). If a link is busy, messages routed via that link wait at the network node from which that link emanates. Route selection is a significant factor in determining the response time experienced by users and has a major impact on utilization of network resources - node buffers and link capacities.

We describe the prototype interactive design tool PSROUTE (Primary and Secondary Routes selection tool). PSROUTE is a user-friendly system that prompts the user to specify network parameters via menus. This system helps the designer experiment with perturbations (such as adding or dropping network nodes and/or links, changing link capacities, adding new communicating node pairs, changing the traffic patterns, etc.) and observe the resulting solutions. The operation of PSROUTE is demonstrated through an example session.

Given the nodes, links, link transmission speeds and external traffic (messages) characteristics, we study the problem of route selection to minimize the mean delay encountered by messages at network nodes. Predetermined routing is used in SNA networks (Ahuja 1979) among other network architectures. We assume that for each source/destination pair we have a set of route pairs. Each route pair consists of a primary route and a secondary route. The secondary route is link-disjoint from the primary route. Given a subset of possible route pairs, we select for each source/destination pair of nodes a route pair. Messages are transmitted from the source node to the destination node through intermediate nodes and links along the primary route of the selected route pair. The secondary route of each route pair is used in case the primary route is not available. The primary route will be unavailable when one or more links and/or nodes along that route fail. Messages entering the network encounter delay due to the finite transmission speed of the links and the resultant queueing at intermediate nodes. The choice of routes strongly affects the delay encountered by messages, and our objective is to select route pairs which minimize the average delay of messages in the network.

Figure 1. The Example Network

This problem is combinatorial in nature, as can be illustrated by a simple example. Suppose that there are r route pairs for each source/destination pair and that there are c such pairs. Then there are (r) possible choices for selecting a route pair for each source/destination pair of nodes. The interested reader is referred to Gavish and Hantler (1983) and Narasimhan, Pirkul and De (1986) for a version of the problem where only primary routes have to be selected.

The primary and secondary route pair selection problem is formulated as a nonlinear integer programming model. The parameters of the model and the decision variables are briefly discussed in Section 2; the complete model is presented in Appendix A. In Section 3, an overview of PSROUTE is presented. An example session is presented in Section 4.

2. INPUT PARAMETERS AND DECISION VARIABLES USED IN PSROUTE

Figure 1 shows an example network where primary and secondary route selection has to take place. The route selection problem was formulated as a
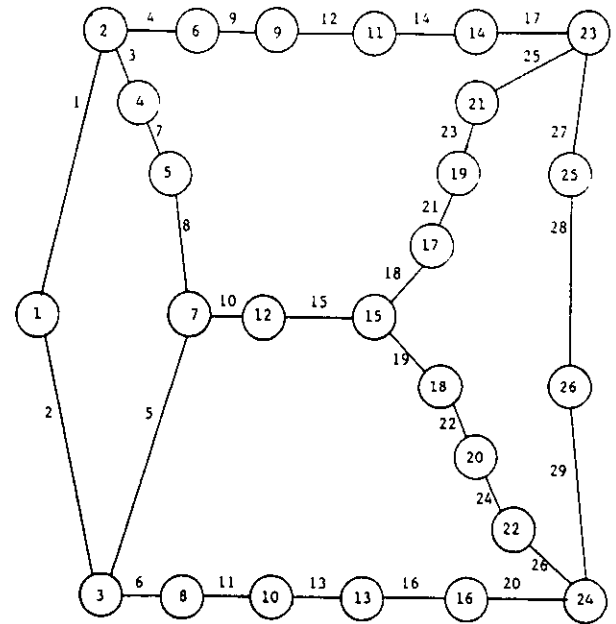
nonlinear zero-one programming problem by Pirkul and Narasimhan (1987). In order to model the end-to-end delay in the backbone network, we make assumptions which are commonly used in modeling the queueing phenomena in such networks. These assumptions include unlimited buffers to store messages at nodes, Poisson arrival process of messages to the network, and exponential message length distributions. Other assumptions include negligible link propogation delay and no message processing delay at nodes. These assumptions have proved robust in the area of computer networks (Ahuja 1982; Gavish 1983; Kleinrock 1975; Tanenbaum 1981). These queueing assumptions are detailed in Pirkul and Narasimhan (1987) and are also listed in Appendix A.

The primary routes for each route set for a source/destination pair of nodes were generated by using a k-shortest path algorithm (Dantzig 1967; Minieka 1978). For each primary route, a secondary route, was generated which was link disjoint from that primary route. To ensure that each primary route has a secondary route the network has to be at least two-connected (Tanenbaum 1981).

The queueing and transmission delay of messages is modeled as a network of independent M/M/1 queues, in which links are treated as servers whose service rate is proportional to the link capacity. The customers are messages whose waiting area is the network nodes. The queueing and transmission delay in every link is a nonlinear function of the link capacity and the traffic on the link. This expression is used for estimating the expected end-to-end delay in the network, which is the weighted sum of the expected delays of the links in the routes.

The interactive system provides a systematic way of specifying and changing values of the following input parameters:

< The index set of the communicating source/-destination pairs in the network; p represents an element of the set. This could be a subset of all the node pairs.

Route parameters

R The index set of primary and secondary candidate route pairs; r represents an element of the set. This can be provided by users or generated by a route generation algorithm or a combination of these techniques. A route pair is characterized by two disjoint ordered sets of links (from the source to the destination).

S_p The index set of candidate route pairs for source/destination p. Out of these one route pair will be selected. We assume that if $p \neq q$ then $S_p \cap S_q = \phi$.

Link parameters

L The index set of links in the network; ℓ represents an element of the set.

Q_ℓ The capacity of link ℓ

Traffic parameters

a_r ($= a_p$) The message arrival rate of the unique source/destination pair associated with route pair r, where $r \epsilon S_p$.

$1/\mu$ The mean of the exponential distribution from which the message lengths are drawn.

Decision variables

The essential decisions to be made by the network designer are to determine for every route pair r in the candidate set, whether to designate it for routing or not. The following variables are used in the model to represent these decisions:

$$x_r = \begin{cases} 1 & \text{if route pair r } (r\epsilon R) \text{ is selected for} \\ & \text{message routing} \\ 0 & \text{otherwise} \end{cases}$$

Delay expression

The objective in selecting routes is to minimize the mean delay a message faces in the backbone network. Delay in the network is represented as a weighted average of delay on all the links of the network (Kleinrock 1975). The weight for each link is dependent on the number of messages in the routes which have been selected for routing which utilize that link and the average message length. The delay on each link is a nonlinear function of the primary traffic on the link and the link capacity and is estimated by using results from queueing theory.

In case a link fails, the traffic on primary routes using it has to be redirected to its respective secondary routes. The model provides for sufficient capacity for backup routes to carry traffic in case of a single link failure. In real life, multiple link failures could occur but the model does not capture this. Capturing such multiple link failures in the model would very likely make it intractable. Furthermore, reserving capacities for all possible combinations of link failures would result in very low utilizations which might not be desirable when the network is operating normally. The objective function of the model captures the "steady state" average queueing and transmission delays without considering additional delays due to link failures. This is an approximation that provides us with an estimate of the actual mean delay. If link failures are infrequent and links do not stay unavailable for significant amounts of time, this approximation will provide us estimates that are close to the actual

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mean delay. PSROUTE also calculates the effects of single link failures on the mean delay, thereby providing us with an indication of possible increases in the mean delay. The route selection described above has been formulated as a nonlinear zero-one mathematical programming problem (Pirkul and Narasimhan 1987) and also appears in Appendix A.

The model underlying the design tool PSROUTE belongs to the class of NP-Complete problems (Garey and Johnson 1979). This classification implies that the computation time for an algorithm which searches for the optimal solution would increase exponentially with the size of the problem, which for realistically sized problems becomes impractical. We therefore study a Lagrangian relaxation of this problem and resort to a heuristic solution procedure that takes advantage of the information provided by the relaxation (Pirkul and Narasimhan 1987). The Lagrangian relaxation scheme has been applied successfully to many combinatorial optimization problems during the last decade. These include the traveling salesman problem (Held and Karp 1970), location problems (Erlenkotter 1978; Geoffrion and McBride 1978; Pirkul, Narasimhan and De 1987), distributed computer system design problems (Gavish and Pirkul 1986; Pirkul 1986, 1987), and communication network design problems (Gavish 1982, 1983; Pirkul, Narasimhan and De 1986). For a survey of Lagrangian relaxation see Fisher (1981, 1985).

The Lagrangian relaxation technique involves relaxing a set of constraints and multiplying them by a set of multipliers and adding this dualized set of constraints to the objective function. The relaxed problem is solved to get a bound. The multipliers are updated iteratively via a subgradient optimization method using the bound and the value of the best feasible solution. The subgradient optimization method terminates when either a prespecified number of iterations are performed or the gap between the bound and the best feasible solution value is within a user specified tolerance. The heuristic solution method is an integral part of, and is executed for, every iteration of the subgradient optimization scheme. The heuristic solution is essentially a manipulation of the solution vector of the bound to yield a feasible solution to the original problem. For details on the subgradient optimization procedure, the reader is referred to Fisher (1981) and Held, Wolfe and Crowder (1974). Also refer to Pirkul and Narasim-

han (1987) for technical details of the solution procedure which is used by PSROUTE.

3. OVERVIEW OF PSROUTE

The PSROUTE design tool described above is implemented on an IBM 3081-D computer. The candidate routes for the primary and secondary route selection problem can be read from a file, generated by a route generation algorithm, or could be entered interactively by the user. As mentioned in Section 2, we are currently determining primary candidate routes by running k-shortest path algorithms; the secondary routes were generated manually after examining the primary candidate route. Work is currently under way on developing a PSROUTE module which would automatically generate a secondary candidate route, which is link-disjoint with its primary route.

For every pair of nodes which communicate, PSROUTE designates one of the route pairs in the candidate set as the selected route to carry messages from the source node to the destination node without violating any capacity constraints on the links of the network. The secondary route has space reserved on network links but no traffic is sent on it until its primary route fails. In doing so, PSROUTE attempts to minimize the mean delay experienced by messages in the primary routes as they traverse the network.

PSROUTE consists of several FORTRAN-77 modules. It is designed to be self-instructive and guides the designer by appropriate prompts and questions. When a choice or decision is to be made, either a "yes" or "no" question or a menu of alternatives appears. The network designer can change an input parameter, for example, the capacity of a particular link, and observe its effect on the delay in the network. The results displayed include the delay in the network, the utilizations of various links, the primary path and secondary path chosen for a particular node pair. As mentioned in the previous section, the time taken to run the algorithm is a function of the maximum number of iterations specified and the tolerance between the bound and the feasible solution values. The run time is also a function of the network size (number of nodes, links, communicating node pairs) and the number of candidate routes. For the first solution, the run time is of the order of a few minutes. For subsequent runs where a few parameters are

updated, the run time is of the order of a few seconds. This is because the best dual multipliers of one run are the starting multipliers for the next run.

4. SAMPLE SESSION

PSROUTE is illustrated using the example network of Figure 1. The following conventions hold in this example: all prompt or output messages which emanate from the system are shown in mixed upper and lower case letters. The designer's responses consist only of upper case letters and are underlined. The session has been displayed in various figures to highlight different features.

The PSROUTE session begins as shown in Figure 2 with a menu of choices displayed by PSROUTE. Specification of code 7 will let the network designer read the problem data from a data file. Other options are available to go to other menus for displaying and modifying other parameters associated with the network or with the iterations. After the data has been read from a data file, the designer enters option 5 to execute the subgradient optimization algorithm. After every 20 iterations PSROUTE prints out the iteration number, run time, the solution and the gap between the feasible solution and the bound, and prompts the user as to whether he is satisfied with the solution or wants the solution procedure to iterate further (this feature is not shown in Figure 2 for lack of space). When the procedure stops, the designer enters option 6 to display the current solution.

The designer may then examine the solution (Figure 3) to either accept it and stop or change some parameters and rerun the solution procedure. One can examine the details of the solution including the run time, the number of subgradient iterations, and the gap between the best feasible solution and the bound. Figure 3 also illustrates how the results menu provides information about the average delay in the network for a message. Specific links can be examined to check their utilizations and the routes using them. The average utilization of links with the minimum, mean and maximum traffic loads are listed. In the example shown, the three most utilized links are listed. Link 17, with a traffic load of 70 percent, is the most utilized link and the primary and secondary routes using it are listed offline for the designer's examination. Also shown are the effect of link failure on the average delay; in the example given,

failure of links 17, 10 or 2 cause the three greatest increases in the delay. The primary and secondary routes for a particular node pair can also be examined. The user, armed with this solution, may want to experiment by changing capacities of links, adding new candidate pairs of routes or new nodes, etc. The solution procedure can then be executed with the new parameters. This process can be repeated until a satisfactory solution is obtained. We now illustrate how such modifications to the network are easily accomodated by PSROUTE.

If the designer wants to modify the network link and node parameters, Figure 4 illustrates how this can be done. In the example session, since link 17 is the most utilized link, the designer doubles its capacity to reduce delay in the network. The network designer could have similarly increased the capacity of the five most or ten most utilized links and observed the effect of such parameter changes on the solution. Similar options are available for adding, dropping, or displaying nodes and/or links.

The designer then executes the solution procedure after the parameters have been modified. Figure 5 shows the results of running the solution procedure with the modifications to the network parameters. The average delay in the network has decreased from 38.4 to 37.3 milliseconds.

Figures 6 and 7 illustrate additional menus available to the designer in PSROUTE. Figure 6 displays options for modifying the network traffic parameters; an addition of a communicating source/destination pair of nodes is illustrated with the system prompting the designer for the node numbers and the traffic parameters from the source node to the destination node.

Figure 7 shows how the designer can execute a k--shortest path algorithm to generate primary candidate routes for any or all communicating node pairs. He can also interactively enter/modify the primary and/or secondary candidate routes for any source/destination pair of nodes.

5. CONCLUSIONS

An interactive design tool, PSROUTE, for primary and secondary route selection in backbone data communication networks has been presented. The design problem is that of selecting a primary route and a secondary route for every pair of nodes which communicate. It is assumed that the network

```
************************************************************
*                  !!! Welcome to PSROUTE !!!             *
*                                                         *
* You can use this system to select primary and secondary routes *
* in a backbone data communications network to minimize delay    *
* faced by messages.                                      *
************************************************************
```

Please specify the operation you want to perform
(Type 0 for list of options available)

-> <u>0</u>

Code	Operation
0	List this menu
1	Display/Modify network nodes and links
2	Display/Modify network traffic characteristics
3	Specify run parameters (iterations and tolerance)and dual multipliers
4	Specify candidate routes
5	Execute solution procedure
6	Display current solution
7	Read problem data from file
8	Print problem parameters and results
9	Exit PSROUTE system

-> <u>7</u>
Enter file name -> <u>Example.data</u>

The problem data has been read

Please specify the operation you want to perform
(Type 0 for list of options available)

-> <u>5</u>

The solution is found.

Please specify the operation you want to perform
(Type 0 for list of options available)

-> <u>6</u>

Figure 2. Menu of Choices.

designer wishes to minimize the delay experienced by messages in the network and also reserve space on links in case any other link fails. An easy to use interface allows the designer to specify network and traffic parameters. Once the network and execution parameters are specified, PSROUTE executes the solution procedure. The results of the solution procedure can be examined by the designer to either accept them or modify some parameters and repeat the solution procedure until a satisfactory solution is reached.

ACKNOWLEDGEMENTS

```
                    *******************
                    * Display Results *
                    *******************

Mean message delay in network -    38.4 milliseconds

Average utilization of link with highest traffic load =    70.00 % (Link #17)
Average utilization of all links in the network       -    52.31 %
Average utilization of link with lowest traffic load  -    18.43 %

The worst increase in delay caused by a single link failure:
Link that fails                Mean Delay
      17                       61.6 milliseconds
      10                       61.0 milliseconds
       2                       59.0 milliseconds

Number of Iterations -  200
CPU Seconds used     -  386.2
Tolerance between bound and best feasible solution =   4.9%

Please enter     If you want

1                List links (nodes) and delay on the Primary and Secondary
                 routes for a particular node pair.
2                List utilization of all links
3                List routes (primary and secondary) using a particular link
4                List m-most utilized links
5                List effects of single link failures on mean delay
6                Return to main menu


Please specify the operation you want to perform
       -> 4

Enter a value for m -> 3

       The 3-most utilized links are :
       Link     Average Utilization
        17            54.0
        14            51.8
         2            51.0


Please specify the operation you want to perform
       -> 6

       >
```

Figure 3. Results of Primary and Secondary Route Selection.

Please select an option:

```
Code            Option
1               Display network nodes
2               Add/Drop network nodes
3               Display network links
4              ·Add/Drop network links
5               Modify link capacities
6               Return to main menu


-> _5_

Which link ? -> _17_
Currently this link has capacity 50000
Enter the new capacity    -> 100000

Link 17 has capacity 100000 assigned to it.
Is the update OK ? (yes/no) -> _YES_
Return to main menu ? (yes/no) -> _YES_
```

Figure 4. Network Nodes and Links.

```
                    *******************
                    * Display Results *
                    *******************


Mean message delay in network -   37.3  milliseconds

Average utilization of link with highest traffic load -   69.40 % (Link # 2)
Average utilization of all links in the network       -   51.12 %
Average utilization of link with lowest traffic load  -   18.31 %


The worst increase in delay caused by a single link failure:
Link that fails              Mean Delay
      17                     61.7 milliseconds
      10                     57.8 milliseconds
      15                     54.5 milliseconds

Number of Subgradient Iterations -   20
CPU Seconds used - 41
Tolerance between bound and best feasible solution - 3.50%

Please enter    If you want

1               List links (nodes) and delay on the Primary and Secondary
                routes for a particular node pair.
2               List utilization of all links
3               List routes (primary and secondary) using a particular link
4               List m-most utilized links
5               List effects of single link failures on mean delay
6               Return to main menu


Please specify the operation you want to perform
       -> 6
```

Figure 5. Results of Primary and Secondary Route Selection.

<u>TRAFFIC PARAMETERS</u>

Please select an option:

Code	Option
1	Display/modify traffic parameters for a node pair
2	Display all communicating node pairs and their traffic parameters
3	Display all destination nodes for a source node
4	Display all source nodes for a destination node
5	Add node pairs
6	Drop node pairs
7	Return to main menu

=> <u>5</u>

What is the source node ? => <u>4</u>
What is the destination node ? => <u>6</u>
What is the average message length ? => <u>40</u>
What is the frequency of messages per second ? => <u>3</u>
Any more additions (yes/no) ?
(if no, go back to main menu
to generate candidate routes for this pair) => <u>NO</u>

Figure 6. Network Traffic Parameters.

<u>CANDIDATE ROUTES</u>

Please select an option:

Code	Option
1	If you want the k-shortest path algorithm executed to generate primary candidate routes for all node pairs
2	If you want the k-shortest path algorithm executed to generate primary candidate routes for a particular node pair
3	If you want to interactively enter the secondary candidate routes for a particular node pair
4	If you want to interactively display/modify the primary or secondary candidate routes for a particular node pair
5	If you want the candidate routes read from an external file
6	Return to main menu

=> <u>2</u>

What is the source node ? => <u>4</u>
What is the destination node ? => <u>6</u>
What is k ? (# of candidate routes needed) => <u>5</u>

After the routes are generated, the main menu will reappear.

Figure 7. Candidate Routes.

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APPENDIX A: THE MODEL UNDERLYING PSROUTE

The model underlying the interactive design tool, PSROUTE, is presented here. The model is a nonlinear zero-one programming problem (Pirkul and Narasimhan 1987). We assume that the number of nodes, the communicating source/destination pairs of nodes, and the set of candidate route pairs for each source/destination pair are known. In order to model the end-to-end delay in the backbone network, we make queueing assumptions which are commonly used in modeling such networks (Kleinrock 1975). We assume that link capacities are known, that nodes have unlimited buffers to store messages waiting for the links, that the arrival process of messages to the network follows a Poisson distribution and that message lengths follow an exponential distribution. We further assume that the propagation delay in the links is negligible and that there is no message processing delay at the nodes. The queueing and transmission delay of messages is modeled as a network of independent M/M/1 queues (see Kleinrock 1975) in which links are treated as servers whose service rate is proportional to the link capacity. The customers are messages whose waiting area is the network nodes.

The queueing and transmission delay in link ℓ is $1/(\mu Q_\ell - a_\ell)$, where a is the arrival rate of messages on primary routes using link ℓ, Q is the capacity of link ℓ and $1/\mu$ is the expected message length. This expression is used for estimating the expected end-to-end delay in the network, which is the weighted sum of the expected delays of the links in the routes.

We use the following additional notation to complement the notation used in Section 2:

PR_ℓ The index set of primary routes that pass through link ℓ.

SR_ℓ The index set of secondary routes that pass through link ℓ.

LP_r The index set of links used by the primary route of pair r.

LS_r The index set of links used by the secondary route of pair r.

The additional decision variables are:

$$y_r^\ell = \left\{ \begin{array}{ll} 1 & \text{if route r is selected on link } \ell, \text{ where } \ell \epsilon LP_r \\ 0 & \text{otherwise.} \end{array} \right\}$$

In terms of the above notations, the total bit arrival rate on link ℓ equals:

$$\sum_{r \epsilon pr_\ell} a_r y_{r\ell} / \mu.$$

This gives us an expression for the expected network delay as

$$1/T \sum_{\ell \epsilon L} [\sum_{r \epsilon PR_\ell} (a_r y_{r\ell})/(\mu Q\ell - \sum_{r \epsilon PR\ell} a_r y_{r\ell})]$$

where T = y (a) is the total arrival rate of messages to the network. The route selection problem can now be formulated as follows:

Problem-P

$$Z_P = \text{Min.} \left\{ 1/T \sum_{\ell \in L} \left[\sum_{r \in PR_\ell} (a_r y_{r\ell})/(\mu Q\ell - \sum_{r \in PR\ell} a_r y_{r\ell}) \right] \right\} \tag{1}$$

subject to:

$$(1/\mu) \sum_{r \in PR_\ell} a_r y_{r\ell} \leq Q_\ell , \qquad \forall \ell \in L \tag{2}$$

$$(1/\mu) \sum_{r \in PR_\ell - \{PR_\ell \cap PK_k\}} a_r y_{r\ell} +$$

$$(1/\mu) \sum_{r \in PK_k \cap SR_\ell} a_r y_{rk} \leq Q_\ell , \qquad \forall k \in L-\{\ell\}, \, \forall \ell \in L \tag{3}$$

$$\sum_{r \in SP} x_r = 1 , \qquad \forall \text{-}p \in \pi \tag{4}$$

$$x_r \leq y r_\ell \, , \forall \, P \in LP , \qquad \forall \text{-}r \in R \tag{5}$$

$$xr, yr\ell \in \{0,1\}, \qquad \forall \text{-}\ell \in LPr, \forall \text{-}r \in R \tag{6}$$

The objective function minimizes the "steady state" average queueing and transmission delay for messages. Constraint set (2) ensures that the flow on each link does not exceed its capacity under conditions of no link failure. Constraint set (3) reserves space on each link in case any other single link failure occurs. The effect of link k's failure on link ℓ is two-fold: first, the primary routes common to links k and ℓ do not need space reserved on link ℓ; second, the primary routes on link k whose secondary routes use link ℓ need space to be reserved on link ℓ. Although we reserve space for secondary routes, we do not include them in the objective function. The objective function as stated does not capture the effect of link failure on network delay. This approximation is acceptable for systems where link failures are infrequent and/or links do not stay off-line for significant amounts of time. The effect of failed links on the average delay is estimated after the problem is solved by calculating the delay due to a link failure. Attempting to capture the effect of link failure in the model would make it significantly more complicated and very likely make it intractable. Constraint set (4) ensures that one and only one route pair is selected for each source/destination pair of communicating nodes. Constraint set (5) ensures that if a route is selected, then all segments of its primary route are also selected. Constraint set (6) enforces integrality conditions on the decision variables.

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