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Abstract: Based on uncertainty theory, a two-stage supply chain model is presented, where the customers' demands are characterized as uncertain variables. The objective is to minimize the combined costs incurred in the manufacturing and logistics phases. When these uncertain variables are linear, the objective function and constraints can be converted into crisp equivalents, then can be solved by traditional methods. An example is given to illustrate the model and the converting method.

Keywords: Uncertain demand; Manufacturing; Logistics; Two-stage; Supply chain.

§1 Introduction

A supply chain is generally viewed as a network of some business entities including suppliers, plants, distribution centers and customers, who are organized to covert raw materials into specified products and distribute these products to customers. The supply chain design is to determine how to choose the entities and distribute goods to satisfy the demands of customers with minimum total cost. It provides an optimal platform to manage supply chain effectively and plays an important and strategic role in supply chain management.

In recent years, the supply chain design has been studied widely in the field of supply chain management. In 1974, Geoffrion and Graves [7] studied a multi-commodity single-period distribution system and solved it by Benders Decomposition. In [4], Cohen et al. presented a dynamic, nonlinear mixed integer programming model, in which the operation of a network of suppliers, manufacturers and customers is considered. Syarif et al. [19] formulated the logistic chain network problem as a 0-1 mixed integer linear programming model, and proposed the spanning tree-based genetic algorithm for optimal solution. In order to improve the efficiency of genetic algorithm, the fuzzy logic controller concept is hybridized in the proposed method for auto-tuning the GA parameters[18]. Yan et al. [22] applied the mixed integer programming modelling techniques to supply chain design with consideration of bills of materials.

The majority of these researches studied the supply chain design problem with deterministic parameters. In practice, however, it is usually difficult to foretell the demands of customers and the different operating costs to be crisp values, namely, these parameters are uncertain. Traditionally, in literatures, these uncertain parameters in supply chain design are considered as random variables and have been modelled by probability distribution. Cohen and Lee [5] studied the whole supply chain design by four stochastic sub-models. The optimal solution for each sub-model is solved individually under some assumptions, and heuristic procedure was developed for the optimal solution when these four sub-models are integrated. Gutierrez et al. [8] proposed a robust optimization framework to seek network configurations for network design in random environment and modified the Benders decomposition algorithm [3] to solve it. Alonso-Ayuso et al. [1] proposed a Branch

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and Fix Coordination approach for solving two-stage stochastic supply chain design problems. Santoso et al. [20] proposed a stochastic programming model for solving supply network design problems of a realistic scale, and gave a solution algorithm integrating a sampling strategy with an accelerated Benders decomposition scheme. As known, the probability distribution is usually derived from history record.

However, when these historical data is not available or unreliable, the probabilistic framework is not suitable. In this case, uncertain parameters can be given based on experience by a leader's subjective judgement. These parameters are often given by vague and imprecise phrase, namely, they are fuzzy numbers. In this case, fuzzy set theory provides the appropriate framework to cope with this kind of uncertainty. A few researchers studied the supply chain with fuzzy parameters. Petrovic et al. [16, 17] developed a fuzzy isolated inventory model to determine the order-up-to level for each individual site independently on the serial supply chain. And a simulation approach was developed to evaluate the performance of the entire supply chain. Wang [21] developed a fuzzy decision methodology to determine inventory strategies and evaluate the performances of a supply chain with fuzzy parameters. In a real market, the decision makers focus on different points to configure their supply chain network.some the decision makers want to minimize the expected cost, some the decision makers concern on the chance with which the total cost is less than a given cost, and the others set some confident levels as an appropriate safety margin to constrain the total cost of supply chain network. In [9], Ji presented a model to maximize the credibility degree by dependent chance programming [10, 11] for supply chain network design, in which the demands of customers and the costs of purchasing, transportation and distribution are fuzzy numbers, and designed an effective genetic algorithm based on fuzzy simulation to solve it.

When historical data are not available to estimate a probability distribution, we have to invite some domain experts to evaluate their belief degree that each event will occur. Since human beings usually overweight unlikely events, the belief degree may have much larger variance than the real frequency. Perhaps some people think that the belief degree is subjective probability. However, it showed that it is inappropriate because probability theory may lead to counterintuitive results in this case. Similar paradoxes will appear with fuzzy theory. For example, a variable in fuzzy set theory should be characterized by a membership function. Suppose it is a triangular fuzzy variable $\xi = (0.4, 0.6, 0.9)$. Based on the membership function, the variable is exactly 0.6 with belief degree 1 in possibility measure. However, this conclusion is unacceptable because the belief degree of exactly 0.6 is almost zero. In addition, the variable being exactly 0.6 and not exactly 0.6 have the same belief degree in possibility measure, which implies that the two events will happen equally likely. This conclusion is quite astonishing and hard to accept. In order to deal with these phenomena, uncertainty theory was founded by [12] in 2007 and refined by [13] in 2010. Nowadays uncertainty theory has become a branch of mathematics for modeling human uncertainty, and have been developed and applied widely. Based on uncertainty theory, a two-stage supply chain model is presented in this paper, where the customers' demands are characterized as uncertain variables. The objective is to minimize the combined costs incurred in the manufacturing and logistics phases.

The rest of the paper is organized as follows. For better understanding of the paper, some necessary knowledge about uncertain variable will be introduced in Sect. 2. In Sect. 3, uncertain supply chain model which has been the basis of our work is described, where the customers' demands is characterized as uncertain variables. Subsequently, a two-stage uncertain supply chain model is presented in Sect. 4. In Sect. 5, the objective function and constraints of the two-stage uncertain supply chain model are converted into crisp equivalents when the customers' demands are characterized as linear uncertain variables. Then, the model can be solved by traditional methods. In Sect. 6, a numerical example will be given. Finally, in Sect. 7, some conclusion remarks will be given.

§2 Preliminary

To better understand the proposed models for uncertain supply chain, we will review some necessary knowledge about uncertainty theory in this section.

Definition 1 ([14]) *Let Γ be a nonempty set, τ a σ-algebra over Γ, and $\mathcal{M}$ an uncertain measure, $\mathcal{M}$ satisfies the conditions:*
(1)$\mathcal{M}\{\Gamma\}=1$;
(2)$\mathcal{M}\{\Lambda\}+\mathcal{M}\{\Lambda^c\}=1$ for any event Λ;
(3)$\mathcal{M}\{\bigcup_{i=1}^{\infty}\Lambda_i\}\leq\sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}$ for every countable sequence of events $\{\Lambda_i\}$.
Then the triplet $(\Gamma,\tau,\mathcal{M})$ is called an uncertainty space.

Definition 2 ([14]) *An uncertain variable ξ is a measurable function from an uncertainty space $(\Gamma,\tau,\mathcal{M})$ to the set of real numbers, i.e., for any Borel set B of real numbers, the set*

$$\{\xi\in B\}=\{r\in\Gamma|\xi(r)\in B\} \tag{1}$$

is an event.

Definition 3 ([14]) *The uncertainty distribution Φ of an uncertain variable ξ is defined by*

$$\Phi(x)=\mathcal{M}\{\xi\leq x\} \tag{2}$$

for any real number x.

Definition 4 ([14]) *Let ξ be an uncertain variable with regular uncertainty distribution Φ. Then the inverse function Φ^{-1} is called the inverse uncertainty distribution of ξ.*

Definition 5 ([14]) *Let ξ be an uncertain variable. Then the expected value of ξ is defined by*

$$E[\xi]=\int_0^{+\infty}\mathcal{M}\{\xi\geq r\}\mathrm{d}r-\int_{-\infty}^{0}\mathcal{M}\{\xi\leq \mathrm{r}\}\mathrm{d}r \tag{3}$$

provided that at least one of the two integrals is finite.

Theorem 1 ([14]) *Let ξ be an uncertain variable with regular uncertainty distribution Φ. If the expected value exists, then*

$$E[\xi]=\int_0^1\Phi^{-1}(\alpha)\mathrm{d}\alpha \tag{4}$$

Lemma 1 ([14]) *Let ξ and η be independent uncertain variables with finite expected values. Then for any real numbers a and b, we have*

$$E[a\xi+b\eta]=aE[\xi]+bE[\eta]. \tag{5}$$

Definition 6 ([14]) *An uncertain variable ξ is called linear if it has a linear uncertainty distribution*

$$\Phi(x)=\begin{cases}0, & if\quad x\leq a,\\ (x-a)/(b-a), & if\quad a\leq x\leq b\\ 1, & if\quad x\geq b\end{cases} \tag{6}$$

denoted by $\mathcal{L}(a,b)$ where a and b are real numbers with $a<b$. The inverse uncertainty distribution of linear uncertain variable $\mathcal{L}(a,b)$ is

$$\Phi^{-1}(\alpha)=(1-\alpha)a+\alpha b \tag{7}$$

Theorem 2 ([14]) *Assume that ξ_1 and ξ_2 are independent linear uncertain variables $\mathcal{L}(a_1,b_1)$ and $\mathcal{L}(a_2,b_2)$, respectively. Then the sum $\xi_1+\xi_2$ is also a linear uncertain variable $\mathcal{L}(a_1+a_2,b_1+b_2)$, i.e.,*

$$\mathcal{L}(a_1,b_1)+\mathcal{L}(a_2,b_2)=\mathcal{L}(a_1+a_2,b_1+b_2). \tag{8}$$

The product of a linear uncertain variable $\mathcal{L}(a, b)$ *and a scalar number* $k > 0$ *is also a linear uncertain variable* $\mathcal{L}(ka, kb)$, *i.e.,*

$$k\mathcal{L}(a, b) = \mathcal{L}(ka, kb) \quad (9)$$

Theorem 3 ([14]) *Let* ξ *thicksim*$\mathcal{L}(a, b)$ *be a linear uncertain variable. Then its expected value is*

$$E[\xi] = \int_0^1 ((1-\alpha)a + \alpha b)\mathrm{d}\alpha = (\mathrm{a}+\mathrm{b})/2. \quad (10)$$

§3 Problem statement

The supply chain model was originally proposed by McDonald and Karimi [15] in 1997. This model was aimed at determining the optimal allocation of an enterprise's limited resources so as to satisfy the uncertain product demands in the most cost-effective way. The supply chain network considered in the model consists of multiple production sites, potentially located globally, manufacturing multiple products. The demand for these products exists at a set of customer locations. The planning horizon, in keeping with the midterm nature of the model, ranges from around 1 to 2 years. Each production site is characterized by one or more single stage semi-continuous processing units having limited capacity. The various products, which are grouped into product families, compete for the limited capacity of these processing units. The decision making process at the tactical level can be decomposed into two distinct phases: the manufacturing phase and the logistics phase. The manufacturing phase focuses on the efficient allocation of the production capacity at the various production sites with an aim to determining the optimal operating policies. Subsequently, in the logistics phase, the post-production activities such as demand satisfaction and inventory management are considered for effectively meeting the customer demand. The variables of supply chain model are characterized as following.

Sets

$\{i\}$	set of products
$\{f\}$	set of product families
$\{j\}$	set of processing units
$\{s\}$	set of production sites
$\{t\}$	set of time periods
$\{c\}$	set of customers

Parameters

FC_{fjs}	fixed production cost for family f on unit j at site s
v_{ijs}	variable production cost for product i on unit j at site s
p_{is}	price of raw material i at site s
$t_{ss'}$	transportation cost from site s to $s^{'}$
t_{sc}	transportation cost from site s to c
h_{ist}	inventory holding cost for product i at site s in period t
ζ_{is}	safe stock violation penalty for product i at site s
μ_{ic}	revenue per unit of product i sold to customer c
R_{ijst}	rate of production of product i on unit j at site s in period t
$\beta_{i'is}$	yield adjusted amount of product i consumed to produce $i^{'}$ at site s
λ_{if}	0-1parameter indicating whether product i belongs to family f
H_{jst}	production capacity of unit j at site s in period t
MRL_{fjs}	minimum run length for family f on unit j at site s
d_{ict}	demand for product i at customer c in period t
I^L_{ist}	safety stock for product i at site s in period t

Variables

Y_{fjst}	binary variable indicating whether product family f is manufactured on unit j at site s in period t
P_{ijst}	production amount of product i on unit j at site s in period t

RL_{ijst}	run length of product i on unit j at site s in period t
FRL_{ijst}	run length of product f on unit j at site s in period t
C_{ist}	consumption of product i at site s in period t
$W_{iss't}$	intersite shipment of product i from site s to $s^{'}$ in period t
A_{ist}	amount of product i available for supply at site s in period t
I_{ist}	inventory of product i at site s in period t
S_{isct}	supply of product i from site s to customer c in period t
I^{-}_{ict}	shortage for product i at customer c in period t
I^{Δ}_{ist}	deviation below safety of product i at site s in period t

The objective function (11) of the deterministic supply chain planning model, captures the combined costs incurred in the manufacturing and logistics phases. The manufacturing phase costs include fixed and variable production charges, cost of raw material purchase and transportation charges incurred for the intersite shipment of intermediate products. The subsequent logistics phase costs are comprised of the transportation charges incurred for shipping the final product to the customer, inventory holding charges, safety stock violation penalties and penalties for lost sales. The decisions made in the manufacturing phase establish the location and timing of production runs, length of campaigns, production amounts and consumption of raw materials. Specifically, P_{ijst}, RL_{ijst}, FRL_{fjst}, A_{ist}, C_{ist}, $W_{iss't}$, Y_{fjst} constitute the manufacturing variables, and uniquely define the production levels and resource utilizations in the supply chain,

$$\begin{aligned}&\min \textstyle\sum_{f,j,s,t} FC_{fjs}Y_{fjst} + \sum_{i,j,s,t} v_{ijs}P_{ijst}\\&+\textstyle\sum_{i,s,s',t} t_{ss'}W_{iss't} + \sum_{i,s,c,t} t_{sc}S_{iss't}\\&+\textstyle\sum_{i,s,t} h_{ist}I_{ist} + \sum_{i,s,t} \zeta_{is}I^{\Delta}_{ist}\\&+\textstyle\sum_{i,c,t} \mu_{ic}I^{-}_{ict} + \sum_{i,s,t} p_{is}C_{ist}.\end{aligned} \tag{11}$$

These manufacturing variables are constrained by the manufacturing constraints given by (12)-(18). The production amount of a particular product is defined in terms of the rate of production and the campaign run length by (12),

$$P_{ijst}{=}R_{ijst}RL_{ijst}. \tag{12}$$

The input-output relationships between raw materials and final products, accounting for the bill-of-materials, are given through (13),

$$\mathrm{C}_{ist} = \sum_{i'} \beta_{i'is} \sum_{j} P_{ijst}. \tag{13}$$

Redundancy in the intersite shipment of intermediate products is eliminated by (14), which forces the products shipped to a particular site in a particular period to be consumed in the same period,

$$\mathrm{C}_{ist} = \sum_{s'} W_{iss't}. \tag{14}$$

The allocation of products to product families is achieved through (15). Grouping of products into product families is typically done to account for the relatively small transition times and costs between similar products,

$$\mathrm{F}RL_{fjst} = \sum_{\lambda_{if}=1} RL_{ijst}. \tag{15}$$

(16) models the capacity restrictions while (17) provides upper and lower bounds for the family run lengths,

$$\mathrm{F}RL_{fjst} = \sum_{\lambda_{if}\leq 1} H_{jst}, \tag{16}$$

$$\mathrm{M}RL_{fjs}Y_{fjst} \le FRL_{ijst} \le H_{jst}Y_{fjst}. \qquad (17)$$

The amount available for supply in the logistics phase following the manufacturing phase is defined through (18). The decisions made in the logistics phase, termed the logistics variables, are I_{ist}, S_{isct}, I_{ict}^{-}, I_{ist}^{Δ}. The corresponding logistics constraints are given by (19)-(22),

$$\mathrm{A}_{ist} = I_{ic(t-1)} + \sum_{j} P_{ijst} - \sum_{s'} W_{iss't}. \qquad (18)$$

The corresponding logistics constraints are given by (19)-(22). The linking between the manufacturing and logistics phases is captured by (19). The inventory level, which is determined by the amount available for supply and the actual supplies to the various customers, is defined by (19),

$$\mathrm{I}_{ist} = A_{ist} + \sum_{c} S_{isct}. \qquad (19)$$

No overstocking is permitted at the customer (20) and the customer shortages are carried over time (21),

$$\mathrm{E}[\sum_{s,t' \le t} S_{isct'} - \sum_{t' \le t} d_{ict'}] \le 0, \qquad (20)$$

$$\mathrm{E}[I_{ic(t-1)}^{-} + d_{ict} - \sum_{c} S_{isct}] \le I_{ict}^{-} \le E[\sum_{t' \le t} d_{ict'}], \qquad (21)$$

where $d_{ict'}$ and d_{ict} are all uncertain variables.
(22) models the violation of the safety stock levels. Establishing of safety stock targets for the inventory level can be viewed as an aggregate deterministic attempt to buffer against unpredicted contingencies such as demand variations and production rate fluctuations,

$$\mathrm{I}_{ist}^{L} - I_{ist} \le I_{ist}^{\Delta} \le I_{ist}^{L}. \qquad (22)$$

The other constants are constrained by (23),

$$\begin{aligned} &P_{ijst}, RL_{ijst}, FRL_{ijst}, C_{ist}, W_{iss't}, \\ &A_{ist}, I_{ist}, I_{ict}^{-}, I_{ist}^{\Delta}, S_{isct} \ge 0, Y_{fjst} \in \{0,1\}. \end{aligned} \qquad (23)$$

So, the supply chain model which aims to minimize the combined costs incurred in the manufacturing and logistics phases can be characterized as following,

$$\begin{cases} (11) \\ \text{s.t.} \\ (12)-(23). \end{cases} \qquad (24)$$

§4 Two-stage supply chain model with uncertain customer demand

One of the most popular frameworks for planning under uncertainty is two-stage programming [2, 6]. In this method, the decisions and constraints of the system are classified into two sets. The first-stage variables, also known as design variables, are determined to the resolution of the underlying uncertainty. Contingent on these 'here-and-now' decisions and the realizations of the uncertain parameter, the second-stage or control variables are determined to optimize in the face of uncertainty. These 'wait-and-see' recourse decisions model how the decision maker adapts to the unfolding uncertain events. The presence of uncertainty is reflected by the fact that both the second-stage decisions as well as the second stage costs are probabilistic in nature. The objective is, therefore, to minimize the sum of the first stage costs, which are deterministic, and the expected value of the second-stage costs. The classification of the decisions of the midterm planning model into manufacturing and logistics naturally fits into the two-stage stochastic programming framework as described as following. The midterm production-planning model under demand uncertainty is formulated as the following two-stage uncertain supply chain model. The model of the first-stage is (25),

$$\begin{cases} \min \sum_{f,j,s,t} FC_{fjs} Y_{fjst} + \sum_{i,j,s,t} v_{ijs} P_{ijst} \\ + \sum_{i,s,s',t} t_{ss'} W_{iss't} + \sum_{i,s,t} p_{is} C_{ist} \\ + E_{ict}[Q] \\ subject \;\; to: \\ \qquad (12) - (18). \end{cases} \tag{25}$$

The second-stage of the supply chain can be modeling as (26),

$$\begin{cases} \mathrm{Q} = min \sum_{i,s,c,t} t_{sc} S_{iss't} \\ + \sum_{i,s,t} h_{ist} I_{ist} + \sum_{i,s,t} \zeta_{is} I_{ist}^{\Delta} \\ + \sum_{i,c,t} \mu_{ic} I_{ict}^{-} \\ subject \;\; to: \\ \qquad (19) - (22). \end{cases} \tag{26}$$

In the above models, the manufacturing variables are considered as the first-stage, while the logistics decisions are modeled as the second-stage. The manufacturing decisions are made prior to the realization of the uncertain demand. The logistics decisions, which essentially aim at satisfying the customer demand in the most cost effective way while accounting for inventory management, are postponed to after the demand is realized. The objective function of the model is composed of two terms. The first term captures the costs incurred in the manufacturing phase. The second term quantifies the costs of the logistics decisions and is obtained by applying the expectation operator to an embedded optimization problem. Two-stage uncertain supply chain model provides an effective tool for managing the risk exposure of an enterprise. The uncertainty in the product demand is translated into the uncertainty in the logistics decisions through the second stage inventory management problem. This implies that inventory levels, supply policies, safety stock deficits and customer shortages are contingent on the first-stage, manufacturing decisions and the demands realized.

Suppose that all the uncertain variables in Model (26) can be characterized as linear ones, the model can be converted into crisp equivalent. It is obvious that Model will be a nonlinear programming, and can be solved by many traditional methods. The steps will be introduced in the following.

§5 Crisp equivalents

In order to solve the proposed models by traditional methods, constraints (20) and (21) will be converted into the corresponding crisp equivalents in this section.

In accordance with the property of the expected value of linear uncertain variables, we have the following theorem.

Theorem 4 *Let $d_{ict'}$ and d_{ict} be independent uncertain variables for all i, c, t and t'. Then constraint (20) is equivalent to the following deterministic constraint,*

$$E[\sum_{t' \le t} d_{ict'}] - \sum_{s,t' \le t} S_{isct'} \ge 0. \tag{27}$$

When the parameters $d_{ict'}$ is a linear uncertain variable $\mathcal{L}(a_{ict'}, b_{ict'})$, based on Lemma 1 and Theorem 3, the constraint (20) is converted into the following:

$$(\sum_{s,t' \le t} S_{isct'} - \sum_{t' \le t} (a_{ict'} - b_{ict'})/2) \ge 0. \tag{28}$$

Similarly, the constraint (21) is equivalent to the following deterministic constraint,

$$I_{ic(t-1)}^{-} + E[d_{ict}] - \sum_{c} S_{isct} \le I_{ict}^{-} \le E[\sum_{t' \le t} d_{ict'}]. \tag{29}$$

When the parameters d_{ict} and $d_{ict'}$ are all linear uncertain variable $\mathcal{L}(a_{ict}, b_{ict})$, $\mathcal{L}(a_{ict'}, b_{ict'})$, respectively, based on Lemma 1 and Theorem 3, the constraint (20) is converted into the following:

$$\begin{aligned} & I_{ic(t-1)}^{-} + (a_{ict} - b_{ict})/2 - \textstyle\sum_c S_{isct} \\ & \le I_{ict}^{-} \le \textstyle\sum_{t' \le t} (a_{ict'} - b_{ict'})/2. \end{aligned} \tag{30}$$

The second-stage model (26) of the two-stage supply chain planning can be converted into the following crisp equivalent,

$$\begin{cases} \mathrm{Q} = min \sum_{i,s,c,t} t_{sc} S_{iss't} \\ + \sum_{i,s,t} h_{ist} I_{ist} + \sum_{i,s,t} \zeta_{is} I^{\Delta}_{ist} \\ + \sum_{i,c,t} \mu_{ic} I^{-}_{ict} \\ subject \ \ to: \\ \qquad (19),\ (22),\ (28),\ (30). \end{cases} \tag{31}$$

Then, the two-stage supply chain model with customers' demands can be also converted into crisp equivalent. It can be solved by traditional methods, for example, using LINDO or Matlab toolbox.

§6 Numerical example

In this section, we consider an example to illustrate the modeling idea and test the effectiveness of the proposed models. For simplicity, suppose that there are 1 product, 1 product family, 1 processing unit, 2 production sites, 1 time period and 2 customers. So, there are 27 constant and 2 uncertain variables. The uncertain customers' demands are given in Table 1. Table 1 summarizes the parameters d_{ict} as linear uncertain variables. In Table 2 other parameters in this model are assumed to crisp numbers.

Table 1 Uncertain customers' demands

c	d_{ict}
1	$\mathcal{L}(100, 120)$
2	$\mathcal{L}(90, 110)$

The objective function of the second-stage (26) can be converted into the following form based on data in Tables 1-2,

$$\begin{aligned} &3S_{1111} + 2S_{1121} + 2S_{1211} + 3S_{1221} + 4I_{111} \\ &+5I_{121} + 10I^{\Delta}_{111} + 9I^{\Delta}_{121} + 15I^{-}_{111} + 20I^{-}_{121}. \end{aligned} \tag{32}$$

Table 2 Other parameters related to the model

$FC_{111} = 600$	$FC_{112} = 500$	$\upsilon_{111} = 5$
$\upsilon_{112} = 4$	$p_{11} = 2$	$p_{12} = 2.5$
$t'_{12} = 5$	$t'_{21} = 5$	$t_{11} = 3$
$t_{12} = 2$	$t_{21} = 2$	$t_{22} = 3$
$h_{111} = 4$	$h_{121} = 5$	$\zeta_{11} = 10$
$\zeta_{12} = 9$	$\mu_{11} = 15$	$\mu_{12} = 20$
$R_{1111} = 0.6$	$R_{1121} = 0.4$	$\lambda_{11} = 1$
$H_{111} = 110$	$H_{121} = 120$	$MRL_{111} = 100$
$MRL_{112} = 80$	$I^{L}_{111} = 110$	$I^{L}_{121} = 100$

The constraints can be converted into the following,

$$\begin{aligned} &\mathrm{I}_{111} = A_{111} - S_{1111} - S_{1121} \\ &\mathrm{I}_{121} = A_{131} - S_{1311} - S_{1321} \\ &\mathrm{S}_{1111} + S_{1211} \le 110 \\ &\mathrm{S}_{1121} + S_{1221} \le 100 \\ &110 - S_{1211} - S_{1111} \le I^{-}_{111} \le 110 \\ &100 - S_{1221} - S_{1121} \le I^{-}_{121} \le 100 \\ &110 - I_{111} \le I^{\Delta}_{111} \le 110 \\ &100 - I_{121} \le I^{\Delta}_{121} \le 100 \\ &S_{1111}, S_{1121}, S_{1211}, S_{1221}, I_{111}, I_{121}, \\ &I^{\Delta}_{111}, I^{\Delta}_{121}, I^{-}_{111}, I^{-}_{121}, A_{111}, A_{121} \ge 0. \end{aligned} \tag{33}$$

Up to now, the model (26) can be converted into the crisp one with the objective function (34) and constraints (35). It is a nonlinear integer programming and can be solved by traditional method. In this paper, the software LINDO is applied to solve the model. The optimal objective value is 1360.00, and the optimal logistics plan solution is shown in Table (3).

Table 3 Optimal logistics plan

variable	S_{1111}	S_{1121}	S_{1211}	S_{1221}	I_{111}	I_{121}
value	0	100	110	0	110	100
variable	I^{Δ}_{111}	I^{Δ}_{121}	I^{-}_{111}	I^{-}_{121}	A_{111}	A_{121}
value	0	0	0	0	210	210

The objective function of the first-stage (25) can be converted into the following form based on data in Tables 1-2,

$$\begin{aligned}&600Y_{1111}+500Y_{1121}+5P_{1111}+4P_{1121}\\&+2C_{111}+2.5C_{121}+5W_{1121}+5W_{1211}.\end{aligned} \tag{34}$$

The constraints can be converted into the following,

$$\begin{aligned}
&P_{1111}=0.6RL_{1111}\\
&P_{1121}=0.4RL_{1121}\\
&C_{111}=P_{1111}\\
&C_{121}=P_{1121}\\
&C_{111}=W_{1121}\\
&C_{121}=W_{1211}\\
&FRL_{1111}=RL_{1111}\\
&FRL_{1121}=RL_{1121}\\
&FRL_{1111}\le H_{111}\\
&FRL_{1121}\le H_{121}\\
&80Y_{1121}\le FRL_{1121}\le 120Y_{1121}\\
&100Y_{1111}\le FRL_{1111}\le 110Y_{1111}\\
&210=P_{1111}-W_{1121}\\
&210=P_{1121}-W_{1211}\\
&Y_{1111},Y_{1121}\in\{0,1\},\\
&C_{111},C_{121},W_{1121},W_{1211},P_{1111},P_{1121},FRL_{1111},\\
&FRL_{1121},RL_{1111},RL_{1121},H_{111},H_{121}\ge 0.
\end{aligned} \tag{35}$$

Up to now, the model (26) can be converted into the crisp one with the objective function (34) and constraints (35). It is a nonlinear integer programming and can be solved by traditional method. In this paper, the software LINDO is applied to solve the model. The optimal objective value is 54650.00, and the optimal manufacturing plan solution is shown in Table (4).

Table 4 Optimal manufacturing plan

variable	Y_{1111}	Y_{1121}	C_{111}	C_{121}	W_{1121}
value	0	1	0	4800	0
variable	W_{1211}	P_{1111}	P_{1121}	FRL_{1111}	FRL_{1121}
value	459	0	4800	0	120
variable	RL_{1111}	RL_{1121}	H_{111}	H_{121}	
value	0	120	0	120	

§7 Conclusions

The supply chain planning aims to minimize the combined costs incurred in the manufacturing and logistics phases. In this paper, a two-stage supply chain model with uncertain customer's demands is proposed. When the uncertain variables are all linear, the model can be converted into a crisp equivalent, then can be solved by software LINDO. An example illustrates the effectiveness of the model and the converting method. When the uncertain variables are irregular, uncertain simulation methods or hybrid Intelligent Algorithms will be introduced. In following papers, the uncertain supply chain models will be researched deeply.

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