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Single Machine Scheduling to Maximize Number of Batch of Jobs with Uncertain Processing Times

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Abstract: The paper studies single-machine scheduling to maximize number of batch of jobs with uncertain processing times. Firstly, an expected value model to maximize number of batch of jobs processed is given based on uncertainty theory. Then, the model is transformed into a deterministic integer programming model and its properties are provided. Further, its arithmetic, called Man-computer Alternant Arithmetic, is presented. Finally, a numerical example on the model is given.
Keywords: Integer programming, uncertainty theory, single machine, batch scheduling, Man-computer Alternant Arithmetic

§1 Introduction

Recently, the topic of "single-machine scheduling for finite batches of jobs" becomes more and more popular. In [1–3, 7, 11, 13, 22, 26, 31], the scholars focused on **minimizing weighted completion times** for batches of jobs based on determinate processing times for each job on single-machine. Whereas, the processing times of jobs on single-machine are often uncertain. Therefore, many researches discussed the above question using probability theory [6, 12, 13, 15, 16, 21, 33]. Differed from the above literatures, Zhou [34, 35] presented a new model on single machine scheduling problems to **maximize weighted number of batches of jobs** processed on the machine. Here, the processing time of each job is assumed to be a **known constant**. In this paper, by using uncertainty theory initiated by Liu [18, 20, 23, 24], we shall study single machine scheduling problems to maximize weighted number of batches of jobs with **indeterminate processing times**. It should be pointed out that uncertainty theory has been applied in many places such as uncertain programming (Liu [19], Zhang[36, 37], Gao[9, 10], Peng[14], Li[28]), uncertain risk analysis (Li[29]), uncertain logic (Chen[5]), uncertain process (Yao[17]) etc [4, 8, 27, 30, 32].

The rest of this paper is organized as follows. In Section 2, some basic concepts and results about uncertainty theory are recalled. In Section 3, a new model of uncertain batch scheduling on single machine, by assuming processing times of jobs are uncertain variables with uncertainty distributing, is presented. Then this model is transformed into a deterministic integer programming model, and its properties are provided. Further, a arithmetic on this model, called Man-computer Alternant Arithmetic, is constructed. Finally, a numerical example on the model is examined. At last, a brief summary is given.

§2 Preliminaries

In this section, we will introduce some basic concepts and results about uncertainty theory.

Definition 2.1. (Liu [18]). The uncertainty distri-

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bution Φ of an uncertain variable ξ is defined by

$$\Phi(x) = \mathcal{M}\{\xi \leq x\}$$

for any real number x, and we use $\xi \sim \Phi(x)$ to denote ξ has uncertainty distribution Φ.

Liu [20] gave some types of uncertainty distributions to describe uncertain variables. In the following we only state zigzag uncertainty distribution since the paper only use it.

Definition 2.2 [20]. An uncertain variable ξ is called zigzag if it has a zigzag uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x < a \\ (x-a)/2(b-a), & \text{if } a \leq x \leq b \\ (x+c-2b)/2(c-b), & \text{if } b \leq x \leq c \\ 1, & \text{if } \ x > c \end{cases} \tag{1}$$

denoted by $\mathbb{Z}(a, b, c)$.

Definition 2.3.(Liu [20, 23]) An uncertainty distribution Φ of ξ said to be regular if its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in [0, 1]$. It is said to be inverse uncertainty distribution of ξ.

If ψ is regular, uncertainty distribution ψ is continuous and strictly increasing at each point x satisfying $0 < \psi(x) < 1$. Also, inverse uncertainty distribution ψ^{-1} is continuous and strictly increasing in $(0, 1)$.

Definition 2.4.(Liu [20, 23, 25]) Let ξ be an uncertain variable. Then the expected value of ξ is defined by

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \geq r\}\mathrm{d}r - \int_{-\infty}^0 \mathcal{M}\{\xi \leq r\}\mathrm{d}r$$

provided that at least one of the two integrals is finite.

Definition 2.5. (Liu [24]) The uncertain) variables $\xi_1, \xi_2, \cdots, \xi_m$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^m (\xi_i \in B_i)\right\} = \min_{1\leq i\leq m} \mathcal{M}\{\xi_i \in B_i\}$$

for any Borel sets $B_1, B_2, \cdots, B_m$ of real numbers.

Theorem 2.1.(Liu [20, 23]) Let ξ and η be independent uncertain variables with finite expected values. Then for any real numbers a and b, we have

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta].$$

Theorem 2.2.(Liu [20, 23]) The zigzag uncertain variable $\xi \sim \mathbb{Z}(a, b, c)$ has a expected value

$$E[\xi] = \frac{a + 2b + c}{4}.$$

Theorem 2.3. (Liu [20, 23])Let ξ be uncertain variable with uncertainty distribution Φ. If the expected value exists, then

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha)d\alpha.$$

§3 Single Machine Scheduling to Maximize Number of Batch of Jobs with Uncertain Processing Times

§3.1 Problem Statement

Firstly, we give the following hypothesis:

(i) There are n independent jobs need to process in one machine in turn. And these jobs are divided into m batches

$$G^1, G^2, \cdots, G^m.$$

Each batch $G^k (k = 1, 2, ..., m)$ has $n_k (k = 1, 2, ..., m)$ sub-jobs, respectively. And $w^k (k = 1, 2, ..., m)$ is the weight (or profit) of $k-$th $(k = 1, 2, ..., m)$ batch, respectively, where

$$\sum_{k=1}^m n_k = n.$$

(ii) The processing time of $j-$th $(j = 1, 2, ..., n_k)$ job of $k-$ batch $(k = 1, 2, ..., m)$ on the machine is a uncertain variable $\xi_j^k (k = 1, 2, ..., m, j = 1, 2, ..., n_k)$, respectively, with uncertainty distributing $\Phi_j^k (j =$

$1, 2, ..., n_k, k = 1, 2, ..., m, j = 1, 2, ..., n_k)$ respectively, and the consignment time of G^k is $d^k(k = 1, 2, ..., m)$, respectively.

(iii) There is no delay time between two connective jobs.

(iv) The machine process only one job at each time, and each job should be processed one time on the machine.

In this paper, with the above hypothesis, we will focus on the question to seek a processing order such that the weighted number of batches of jobs is maximum according to the requested consignment time of each batch job.

§3.2 Model

Note that the above problem is related to uncertain variable, therefore we can deal with it by using uncertainty theory. According to the hypothesis and goal of the above problem, it is easily seen that the optimization solution can be reduced to find a order of batches $\{G^1, G^2, ..., G^m\}$. Now, we assume that the processing order of n jobs on the machine is $\{G^{x_1}, G^{x_2}, ..., G^{x_m}\}$, where $x = (x_1, x_2, ..., x_m)$ is an element of D, the set of all sequences of $\{1, 2, ..., m\}$.

We introduce the following symbols and parameters:

(i) Let

$$t^{x_k} = \sum_{j=1}^{n_{x_k}} \xi_j^{x_k} (k = 1, 2, ..., m)$$

denotes the uncertain processing time of batch $G^{x_k}(k = 1, 2, ..., m)$, and

$$\eta_i(x) = \sum_{k=1}^{i} t^{x_k} = \sum_{k=1}^{i}\sum_{j=1}^{n_{x_k}} \xi_j^{x_k}$$

denotes the uncertain completion time of batch $G^{x_i}(i = 1, 2, ..., m)$, respectively.

(ii) Let

$$T_i(\eta_i(x)) = \begin{cases} 1, & if\ E[\eta_i(x)] \le d^{x_i} \\ 0, & otherwise \end{cases}$$

denotes the truth value of batch $G_{x_i}(i = 1, 2, ..., m)$. That is, $T_i(\eta_i(x))$ values 1 if $G_{x_i}(i = 1, 2, ..., m)$ is completed in requested consignment times and 0 if not.

(iii) Let

$$T(x) = \sum_{i=1}^{m} w^{x_i} T_i(\eta_i(x))$$

denotes the weighted number of batches of jobs processed on the machine.

Summarize (i)-(iii) we obtain a new model for uncertain batch scheduling on single machine:

$$\begin{cases} \max\limits_{x=(x_1,x_2,...,x_m)} \sum\limits_{i=1}^{m} w^{x_i} T_i(\eta_i(x)) \\ s.t. \\ \quad T_i(\eta_i(x)) = \begin{cases} 1, if \quad E[\eta_i(x)] \le d^{x_i} \\ \quad 0, \text{otherwise} \end{cases} \\ \quad x = (x_1, x_2, ..., x_m) \in D \end{cases}$$

§3.3 Property

Theorem 3.3.1 If there exists a $k_0 \in \{1, 2, ..., m\}$ such that the batch G^{k_0} satisfies the following conditions:

(1) $E[t^{k_0}] = \min\{E[t^k] | k \in \{1, 2, ..., m\}\}$;

(2) $w^{k_0} = \max\{w^k | k \in \{1, 2, ..., m\}\}$;

(3) $d^{k_0} = \min\{d^k | k \in \{1, 2, ..., m\}\}$;

(4) $E[t^{k_0}] \le d^{k_0}$.

Then there exists a $x = (x_1, x_2, ..., x_m)$, a solution of model (5), such that $j_0 \in \{1, 2, ..., m\}$, $x_{j_0} = k_0$ and $T_{j_0}(\eta_{j_0}(x)) = 1$.

Proof It is evident that Model (5) has at least one solution. Suppose $x = (x_1, x_2, ..., x_m)$ is a solution of model (5), and $k_0 = x_{j_0}$ $(j_0 \in \{1, 2, ..., m\})$. If $T_{k_0}(\eta_{k_0}(x)) = 1$, then conclusion of the Theorem 3.3.1 is true; or else, if $T_{k_0}(\eta_{k_0}(x)) = 0$, then we assert that $x^* = (x_{j_0}, x_2, ..., x_{j_0-1}, x_1, x_{j_0+1}, ..., x_m)$ is also a solution of model (5) (Note that x^* obtained by exchanging x_{j_0} and x_1 in $x = (x_1, x_2, ..., x_{j_0-1}, x_{j_0}, x_{j_0+1}, ..., x_m)$). In fact, because

$E[t^{k_0}] = \min\{E[t^k] | k \in \{1, 2, ..., m\}\}$ we have

$$\begin{aligned} & E[\eta_i(x^*)] \\ & = E[t^{x_{j_0}}] + \sum_{k=2}^{j_0-1} E[t^{x_k}] + E[t^{x_1}] + \sum_{k=j_0+1}^{i} E[t^{x_k}] \\ & \leq E[\eta_i(x)], i = 1, 2, ..., m. \end{aligned}$$

Then it follows from $d^{k_0} = \min\{d^k | k \in \{1, 2, ..., m\}\}$ that $T_i(\eta_i)(x^*) = 1$ if $T_i(\eta_i)(x) = 1$, for every $i \in \{1, 2, ..., m\}$. Thus by $w^{k_0} = \max\{w^k | k \in \{1, 2, ..., m\}\}$, we have

$$\begin{aligned} T(x^*) &= \sum_{i=1}^{m} w^{x^i} T_i(\eta_i(x^*)) \\ &\geq \sum_{i=1}^{m} w^{x^i} T_i(\eta_i(x)) = T(x), \end{aligned}$$

which means that x^* is a solution of model (5) with the desired condition.

Note 3.3.1 The above theorem shows that when G^{k_0} satisfies the given conditions (1)-(4), then G^{k_0} should be priority processing.

Theorem 3.3.2 For two batches scheduling $x = (x_1, x_2, ..., x_m)$ and $x^* = (y_1, y_2, ..., , y_m)$, where

$$x_{k_1} = y_{k_2}, x_{k_2} = y_{k_1}, x_j = y_j,$$

$$j = 1, 2, ..., k_1 - 1, k_1 + 1, ..., k_2 - 1, k_2 + 1, ..., m.$$

If they satisfies the following conditions:

(1) $d^{x_{k_1}} \leq d^{x_{k_2}}$,

(2) $T_{k_1}(\eta_{k_1}(x)) = T_{k_1+1}(\eta_{k_1+1}(x)) = ... = T_{k_2-1}(\eta_{k_2-1}(x)) = 0$,

(3) $T_{k_2}(\eta_{k_2}(x)) = 1$,

then $T(x^*) \geq T(x)$.

Proof Since $x_j = y_j, j = 1, 2, ..., k_1 - 1$, we have $T_i(\eta_i(x^*)) = T_i(\eta_i(x)), i = 1, 2, ..., k_1 - 1$. Also, from $x_{k_2} = y_{k_1}$, $d^{x_{k_1}} \leq d^{x_{k_2}}$ and $T_{k_1}(\eta_{k_1}(x)) = T_{k_1+1}(\eta_{k_1+1}(x)) = ... = T_{k_2-1}(\eta_{k_2-1}(x)) = 0$, we have $T_j(\eta_j(x^*)) \geq T_j(\eta_j(x)), j = k_1, ..., k_2 - 1$. Note that $T_j(\eta_j(x^*)) = T_j(\eta_j(x)), j = k_2, ..., m$. Therefore, it follows from the senses of $T(x)$ and $T(x^*)$ that $T(x^*) \geq T(x)$.

Note 3.3.2 From the senses of $T(x)$ and $T(x^*)$, we know that if $T(x^*) \geq T(x)$ then we need to select x^* as the solution of Model (5). If not we reserve x as the solution of Model (5).

Theorem 3.3.3 For two batches scheduling $x = (x_1, x_2, ..., x_m)$ and $x^* = (y_1, y_2, ..., , y_m)$, where

$$x_{k_1} = y_{k_2}, x_{k_2} = y_{k_1}, x_j = y_j,$$

$$j = 1, 2, ..., k_1 - 1, k_1 + 1, ..., k_2 - 1, k_2 + 1, ..., m.$$

If they satisfies the following conditions:

(1) $d^{x_{k_1}} \leq d^{x_{k_2}}$,

(2) $E[t^{x_{k_2}}] \leq E[t^{x_{k_1}}]$,

(3) $T_{k_1}(\eta_{k_1}(x)) = 0$,

then $T(x^*) \geq T(x)$.

Proof Note that

$$T_j(\eta_j(x^*)) = T_j(\eta_j(x)), j = 1, 2, ..., k_1 - 1.$$

Since $d^{x_{k_1}} \leq d^{x_{k_2}}$ and $E[t^{x_{k_2}}] \leq E[t^{x_{k_1}}]$, we have

$$T_{k_1}(\eta_{k_1}(x^*)) \geq T_{k_1}(\eta_{k_1}(x)) = 0,$$

$$T_j(\eta_j(x^*)) \geq T_j(\eta_j(x)), j = k_1 + 1, ..., k_2 - 1,$$

$$T_{k_2}(\eta_{k_2}(x^*)) = 0.$$

$$T_j(\eta_j(x^*)) = T_j(\eta_j(x)), j = k_2 + 1, ..., m.$$

Thus, it follows from the senses of $T(x)$ and $T(x^*)$ that $T(x^*) \geq T(x)$.

Theorem 3.3.4 Suppose two batches scheduling $x = (x_1, x_2, ..., x_m)$ and $x^* = (y_1, y_2, ..., , y_m)$, where

$$x_{k_1} = y_{k_2}, x_{k_2} = y_{k_1}, x_j = y_j,$$

$$j = 1, 2, ..., k_1 - 1, k_1 + 1, ..., k_2 - 1, k_2 + 1, ..., m,$$

satisfies the following conditions:

(1) $T_{k_1}(\eta_{k_1}(x)) = 1$,

(2) $T_{k_2}(\eta_{k_2}(x)) = 0$.

If $w^{x_{k_1}} \leq w^{x_{k_2}}$, $d^{x_{k_1}} \leq d^{x_{k_2}}$ and $E[t^{x_{k_2}}] \leq E[t^{x_{k_1}}]$, then $T(x^*) \geq T(x)$. If not we compare $T(x^*)$ with $T(x)$ by calculating $\sum_{j=k_1}^{k_2} T_j(\eta_j(x^*))$ and $\sum_{j=k_1}^{k_2} T_j(\eta_j(x))$. In fact, the relation $T(x^*)$ and $T(x)$ is equivalent to the relation between

$\sum_{j=k_1}^{k_2} T_j(\eta_j(x^*))$ and $\sum_{j=k_1}^{k_2} T_j(\eta_j(x))$.

Proof Since $T_{k_1}(\eta_{k_1}(x)) = 1$ and $T_{k_2}(\eta_{k_2}(x)) = 0$ hold , $T(x^*) \geq T(x)$ is evident if $w^{x_{k_1}} \leq w^{x_{k_2}}$, $d^{x_{k_1}} \leq d^{x_{k_2}}$ and $E[t^{x_{k_2}}] \leq E[t^{x_{k_1}}]$.

It is easily seen that the case of if not holds by the following fact

$$T_j(\eta_j(x^*)) = T_j(\eta_j(x)), j = 1, 2, ..., k_1 - 1,$$

$$T_j(\eta_j(x^*)) = T_j(\eta_j(x)), j = k_2 + 1, ..., m.$$

§3.4 Man-computer Alternant Arithmetic

Now we design a arithmetic (called Man-computer Alternant Arithmetic) of the model (5) according to Theorem 3.3.1-3.3.4.

Step 1 Calculating execrated values $E[t^k]$ of $t^k(k = 1, 2, ..., m)$, respectively, by Theorem 2.1 and 2.2.

Step 2 From small to big, $d^1, d^2, ..., d^m$ is arranged as $d^{x_1^1}, d^{x_2^1}, ..., d^{x_m^1}$. Then select $x^1 = (x_1^1, x_2^1, ..., x_n^1)$ as the initial approximate solution of the model (5).

Step 3 Suppose $x^1 = (x_1^1, x_2^1, ..., x_{j-1}^1, x_j^1 = k_0, x_{j+1}^1, ..., x_m^1)$ and k_0 satisfies conditions of Theorem 3.3.1. Then we use $x^2 = (k_0, x_1^1, x_2^1, ..., x_{j-1}^1, x_{j+1}^1, ..., x_m^1)$ to substitute x^1, by Theorem 3.3.1. Repeat the above step for $\{x_1^1, x_2^1, ..., x_{j-1}^1, x_{j+1}^1, ..., x_m^1\}$ entail to the Theorem 3.3.1 does not work.

Step 4 Suppose that we have obtained $x^2 = (x_1^2, x_2^2, ..., x_m^2)$ by Step 3. By Theorem 3.3.2, we arrange the order of x_i^2 $(i = 1, \ldots, m)$ in x^2 and obtain a new sequence denoted as $x^3 = (x_1^3, x_2^3, ..., x_m^3)$, repeat the above procedure such that $T_i(\eta_i(x^3)) = 1, i = 1, 2, ..., j_0, T_i(\eta_i(x^3)) = 0, i = j_0 + 1, ..., m$.

Step 5 Suppose that we have obtained $x^3 = (x_1^3, x_2^3, ..., x_m^3)$ by Step 4 such that

$$T_i(\eta_i(x^3)) = 1, i = 1, 2, ..., j_0,$$

$$T_i(\eta_i(x^3)) = 0, i = j_0 + 1, ..., m.$$

By using Theorem 3.3.3, we arrange elements of $\{x_{j_0+1}^3, ..., x_m^3\}$ and obtain a new sequence denoted as $x^4 = (x_1^4, x_2^4, ..., x_m^4)$ such that $T(x)$ is a increasing function.

Step 6 Suppose that we have obtained $x^4 = (x_1^4, x_2^4, ..., x_m^4)$ by Step 5 such that

$$T_i(\eta_i(x^4)) = 1, i = 1, 2, ..., p_0,$$

$$T_i(\eta_i(x^4)) = 0, i = p_0 + 1, ..., m.$$

By using Theorem 3.3.4 again and again, we arrange every two elements of $x_r^4, x_s^4, r \in \{1, 2, \ldots, p_0\}, s \in \{p_0 + 1, \ldots, m\}$ and obtain a new sequence denoted as $x^5 = (x_1^5, x_2^5, ..., x_m^5)$ such that $T(x)$ is a increasing function.

Step 7 Report x^5, i.e., the optimal solution of the model (5).

§3.5 Numerical Example

In the section we give a numerical example of the model (4) or (5).

Suppose that we need to process 8 batch $\{G_1, G_2, ..., G_8\}$ of jobs with weights $w^1 = \frac{1}{8}, w^2 = \frac{3}{16}, w^3 = \frac{1}{16}, w^4 = \frac{1}{8}, w^5 = \frac{1}{8}, w^6 = \frac{3}{16}, w^7 = \frac{1}{16}, w^8 = \frac{1}{8}$ on a machine, respectively, and processing times of jobs on the machine are zigzag uncertain variable. Their frondose indexes are given by the following table 1:

Note that $\mathbb{Z}_j^k(a, b, c)$ in the above table denotes zigzag uncertainty distribution of uncertain processing times ξ_j^k for $j-$th job of $k-$th batch. Such as, date of 1$-$ line in the above table tell us, 1$-$th batch contains two jobs, uncertain processing times of its has a zigzag uncertainty distribution $Z_1^1(11, 12, 13)$, 2$-$th job has a zigzag uncertainty distribution $Z_2^1(15, 16, 17)$, and their consignment time is in 48 hour.

From the table 1, we have

Table 1 Indexes of 8 batch of jobs

Name	Distribution of ξ_1^k	Distribution of ξ_2^k	Distribution of ξ_3^k	Consignment time(hour)d^k
1-th batch	$Z_1^1(11,12,13)$	$Z_2^1(15,16,17)$	no	48
2-th batch	$Z_1^2(15,16,19)$	no	no	24
3-th batch	$Z_1^3(43,45,46)$	$Z_2^3(15,18,19)$	no	240
4-th batch	$Z_1^4(34,35,36)$	$Z_2^4(25,28,29)$	$Z_3^4(14,16,17)$	294
5-th batch	$Z_1^5(12,14,16)$	$Z_2^5(15,16,17)$	$Z_3^5(15,18,19)$	95
6-th batch	$Z_1^6(16,18,19)$	$Z_2^3(15,17,18)$	no	96
7-th batch	$Z_1^7(40,45,47)$	$Z_2^7(15,17,18)$	no	250
8-th batch	$Z_1^8(34,36,37)$	$Z_2^8(27,28,30)$	$Z_3^8(15,16,18)$	300

$$t^1(x)=\xi_1^1+\xi_2^1,\ t^2(x)=\xi_1^2,$$
$$t^3(x)=\xi_1^3+\xi_2^3, t^4(x)=\xi_1^4+\xi_2^4+\xi_3^4,$$
$$t^5(x)=\xi_1^5+\xi_2^5+\xi_3^5, t^6(x)=\xi_1^6+\xi_2^6,$$
$$t^7(x)=\xi_1^7+\xi_2^7,\ t^8(x)=\xi_1^4+\xi_2^4+\xi_3^4.$$

Thus we have a new model of uncertain batches scheduling on single machine as follows:

$$\begin{cases} \max\limits_{x=(x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8)} \sum\limits_{i=1}^{8} w^{x_i} T_i(\eta_i(x)) \\ s.t. \\ \quad T_i(\eta_i(x)) = \begin{cases} 1, if \quad E[\eta_i(x)] \le d^{x_i} \\ \quad 0, \text{otherwise} \end{cases} \\ \quad x=(x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8) \in D \end{cases}$$

New we design a Man-computer Alternant Arithmetic of model (7).

Step of Man-computer Alternant Arithmetic:

Step 1 By using Theorem 2.1 and 2.2 we gained the execrated values of $t^k, k=1,2,...,8$ are as follows, respectively:

$$E[t^1]=28, E[t^2]=16.5, E[t^3]=62.25, E[t^4]=78.25,$$
$$E[t^5]=47.5, E[t^6]=34.5, E[t^7]=61, E[t^8]=80.25.$$

Step 2 From small to big, $48,24,240,245,95,96,250,300$ is arranged as $24,48,95,96,240,245,250,300$. Thus we choose $x^1=(2,1,5,6,3,7,4,8)$ as initialization approximate solution of the model (7). Thus we have the

Table 2 Indexes of scheduling x^1

Name	$\eta_i(x^1)$	d^i	$T_i(x^1)$
2-th batch	16.5	24	1
1-th batch	44.5	48	1
5-th batch	92	95	1
6-th batch	216.5	96	0
3-th batch	188.75	240	1
7-th batch	249.75	250	1
4-th batch	328	294	0
8-th batch	408.25	300	0

Table 3 Indexes of scheduling x^2

Name	$\eta_i(x^1)$	d^i	$T_i(x^1)$
2-th batch	16.5	24	1
1-th batch	44.5	48	1
5-th batch	92	95	1
3-th batch	154.25	240	1
7-th batch	215.25	240	1
6-th batch	249.75	96	0
4-th batch	328	294	0
8-th batch	408.25	300	0

Table 4 Indexes of scheduling x^2

Name	$\eta_i(x^1)$	d^i	w^i	$E[t^i]$	$T_i(x^1)$
2-th batch	16.5	24	3/16	16.5	1
1-th batch	44.5	48	2/16	28	1
5-th batch	92	95	2/16	47.5	1
3-th batch	154.25	240	1/16	62.25	1
7-th batch	215.25	240	1/16	61	1
4-th batch	293.5	294	2/16	78.25	1
6-th batch	328	96	3/16	34.5	0
8-th batch	408.25	300	2/16	80.25	0

indexes of scheduling x^1 given by table 2. **Step 3** We can verify that $k_0 = 2$ satisfies conditions of Theorem 3.3.1. Thus $x^2 = x^1 = (2, 1, 5, 6, 3, 7, 4, 8)$.
Step 4 We again and again use Theorem 3.3.2 to arrange the order of x_i^2 $(i = 1, \ldots, m)$ in x^2 and obtain a new sequence $x^3 = (2, 1, 5, 3, 7, 6, 4, 8)$. Their indexes are given by table 3.
Step 5 By using Theorem 3.3.3, we arrange elements of $\{6, 4, 8\}$ and obtain a new sequence denoted as $x^4 = (2, 1, 5, 3, 7, 4, 6, 8)$ such that $T(x)$ is a increasing function. The indexes of x^4 are given by table 4.

Note that Theorem 3.3.3 cannot be used for the above x^4.
Step 6 Note that $w^6 = 3/16 > w^5 = 2/16$. We exchange place of 6 and 5 in x^4 to get a new $x_1^5 = (2, 1, 6, 3, 7, 4, 5, 8)$ such that $T(x^5) > T(x^4)$ by using Theorem 3.3.4. For indexes of scheduling x^5 we see table 5. Note that Theorem 3.3.4 cannot be used for the above x^5.
Step 7 By the above process, we get the optimal solution of the model (7) as $x^5 = (2, 1, 6, 3, 7, 4, 5, 8)$.

§4 Conclusions

In the paper, based on Liu's uncertainty theory, an expected value model to maximize number of batch of jobs processed was given. Then the model was transformed into a deterministic integer programming model and its properties were provided. The so called Man-computer Alternant Arithmetic on this model was established. The availability of the model and its arithmetic were checked by a numerical example.

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Table 5 Indexes of scheduling x^5

Name	$\eta_i(x^1)$	d^i	w^i	$E[t^i]$	$T_i(x^1)$
2-th batch	16.5	24	3/16	16.5	1
1-th batch	44.5	48	2/16	28	1
6-th batch	89	96	3/16	34.5	1
3-th batch	151.25	240	1/16	62.25	1
7-th batch	212.25	240	1/16	61	1
4-th batch	290.5	294	2/16	78.25	1
5-th batch	338	95	2/16	47.5	0
8-th batch	418.25	300	2/16	80.25	0

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