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Yugang Yu

George Q. Huang

Zunmao Ren

Liang Liang

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Yugang YU[1], George Q. Huang[2], Zunmao REN[3], Liang LIANG[1]
([1]School of Business, University of Science and Technology of China, Hefei, 230026, China)
([2]Department of Industrial and Manufacturing Systems Engineering,
University of Hong Kong, Hong Kong, P R China. Email: gqhuang@hku.hk)
([3]Department of Industrial Engineering, Shanghai Jiaotong University, Shanghai, 200030 China)

Abstract

In this paper, we propose a model to study influence of pricing and deteriorating rate on the supply chain level net profit and total inventory where genetic algorithm is used for determine the optimal solution. A one-vendor and multi-retailer supply chain for a single deteriorating finished product and raw materials is analyzed. Under the proposed strategy, the vendor buys a non-deteriorating materials to vendor a deteriorating finished product, delivers the finished product to all retailers by common replenishment periods based on VMI (vendor managed inventory) being implemented. All retailers who buy the finished product sell the finished product on their markets. In all of these markets, the finished product in different markets has substitution each other since consumers may have opportunity to buy the finished product from different retailer and Cobb-Douglas demand function is introduced to describe this market attribute. After developing an integrated product-inventory-marketing model for deteriorating product, genetic algorithm is conducted to calculate the optimal pricing and inventory policies. Finally we present the results of a detailed numerical study that analyses the market and deteriorating rate related parameters influence on the supply chain level net profit and inventory level.

1. Introduction

A lot of research efforts have been made and several integrated inventory models have emerged in order to minimize the inventory cost of deteriorating products for both vendor and buyers. Goyal (1977) [6] proposes a joint economic lot size (JELS) model where the objective is to minimize the total relevant costs for a single vendor, single buyer system. Banerjee (1986) [1] generalizes Goyal's model by incorporating a finite production rate for the vendor and gives the optimal joint production or order quantity. Goyal (1988) [8] extends Banerjee's model again by relaxing the lot-for-lot production assumption and argues that the economic production quantity will be an integer multiple of the buyer's purchase quantity and shows that its model provides a lower or equal joint total relevant cost as compared to Banerjee's model. Kohli and Park (1994) [12] investigate joint ordering policies as a method to reduce transaction costs between a single vendor and a homogeneous group of buyers. They present expressions for optimal joint order quantities assuming all products are ordered in each joint order. Lu (1995) [15] considers a one-vendor multi-buyer integrated inventory model and gave a heuristic approach for joint replenishment policy. Banerjee and Banerjee (1992) [2] consider an EDI-based vendor-managed inventory (VMI) system in which the vendor makes all replenishment decisions for his/her buyers to improve the joint inventory cost. But most of these previous works on integrated vendor-buyer inventory systems do not consider raw material procurement decisions except for the work by Woo, Hsu, and Wu (2001) [22]. Their model is extended to become a three-level supply chain in which one raw material is considered. However, they do not consider marketing policies and deteriorating products in VMI Vendor Managed Inventory.

Several researchers have integrated marketing policies and deteriorating products into inventory decisions for infinite time horizon. Kotler (1971) [13] first incorporates marketing policies into inventory decisions and discussed the relationship between economic ordering quantity and price decisions. Ladany and Sternleib (1974) [14] study the effect of price variations on demand and consequently on EOQ (economic order quantity). Goyal and Gunasekaran (1995) [10] extend the previous work considering perishable goods in a multi-stage inventory model. Although both of the advertisement and price are considered in their papers, they are treated as input parameters and the impacts of advertisement and price are only analyzed by sensitive analysis in their numerical example, without considering the substitution of finished products where Cobb-Douglas demand function is considered. Moreover, they do not consider supply chain context, only limited to a vendor or manufacturer. However, in reality, there are significant differences between deteriorating rates among raw materials, and between finished products. In such circumstances, the raw materials' deteriorating rates can be neglected when compared with that of the finished product. However, this issue has not yet been widely considered in previous VMI works. Moreover, it has been a challenge to calculate the optimal solution for deteriorating product as reported in the literature. Both Goyal and Gunasekaran (1995) [10] develop a computer program for finding their optimal solution by using an exhaustive search method and consume a large amount of computer resources. Fortunately genetic algorithms have been demonstrated successful in providing good solutions to many complex optimization problems and thus received increasing attentions. Their uses have been well documented in the literatures, such as that of Goldberg (1989) [5], Michalewicz(1994) [16] and Fogel (1994) [4], for a wide variety of optimization problems. GA (genetic algorithm) has also been applied to supply chain optimization.

In this paper, we propose a model to study the influence of pricing and deteriorating rate on the net profit and total inventory cost of the supply chain. GA is used for determining the optimal solution from the proposed model. A one-vendor and multi-retailer supply chain for a single deteriorating finished product is analyzed, also considering the raw materials. Under the proposed strategy, the vendor buys non-deteriorating materials for producing deteriorating finished products, and delivers the finished product to all retailers by common replenishment periods based on VMI (vendor managed inventory). All retailers who buy the finished product sell the finished product on the markets. In the markets, the finished product sold by different retailers has substitution each other since consumers may have opportunity to buy the finished product from different retailers and Cobb-Douglas demand function is introduced to describe this market attribute. After developing an integrated product-inventory-marketing model for deteriorating product, genetic algorithm is conducted to calculate the optimal pricing and inventory policies.

This paper will be organized as follows. Section 2 introduces assumptions and notation. In Section 3, we analyze the system and develop an integrated product-inventory -marketing model for deteriorating product. Section 4 presents a genetic algorithm for calculating the optimal pricing and inventory policies. Section 5 gives a detailed numerical study to analyze the influence of parameters such as market and deteriorating rate on the net profit and inventory level of the supply chain. Section 6 concludes the paper by closing remarks.

2. Assumptions and Notations

The supply chain system considered in this paper includes only one vendor and only one type of finished product are considered. This vendor provides the finished product to multiple retailers and buys multiple raw materials in order to produce the product.

2.1 Assumptions

The following assumptions are used to derive the mathematical model:

(1) Demand rate is determined by the retail price and the attributes of the markets and is a function of the retail price and a concave function with respect to each retail price; finished products in different retailer have substitution each other since the product is procured from the same vendor. And the units from the raw materials are immediately available.

(2) Common replenishment cycle is performed by the vendor and all retailers in the supply chain based on VMI.

(3) Deteriorating rate for the finished product is deterministic and deteriorated finished product is replaced and repaired.

(4) The deteriorating cost per unit is different when they are in the different owner. For the deteriorating finished product occurred in retailer i, the deteriorating cost for each finished product is its retail price. However, the deteriorating cost for the vendor is its production price.

2.2 Notations

M	Number of retailers considered;
i=1,2,…,m	Index of retailers;
p_i	A decision variable, retail price for retailer i ($/unit);
D_i	Demand for retailer i per unit time, a function of p_i, ($/unit/time);
a_i	A positive number which is determined by retailer i's market;
β_{ij}	Substitution elasticity for retailer i's demand with respect to retailer j's price;
α_i	Price elasticity;
H_{bi}	Holding cost of the finished product for retailer i, ($/unit/time);
ζ_i	Transportation cost for retailer i ($/unit);
p_0	Production and raw material cost per unit finished product ($/unit);
C	A decision variable, Common replenishment cycle for the vendor and all retailers;
n	Integral number of production batches per raw materials procurement cycle, which is a decision variable, and n>1;
A	Ordering cost per raw materials order for the vendor;
S	Fixed cost for the vendor per common replenishment cycle;
T_i	Ordering cost for per retailer i's order;
M	Usage rate of raw material for producing each finished product;
P	Production rate for the vendor, which is a known constant;
H_{vm}	Holding cost per unit finished product;
t_{vi}	Production time for the vendor to satisfy the requirements of retailer i in the common replenishment cycle;
H_{vp}	Holding cost of the finished product for the vendor, ($/unit/time);
θ	Deteriorating rate of the finished product;(percentage/units/time)
Q_i	Lot size for retailer i;
TC_{bi}	Total inventory cost for retailer i; per unit time, ($/time);
H_p	Holding cost of the finished product for the vendor in the common replenishment cycle C, ($);
TC_{vp}	Total inventory cost of the finished product for the vendor per unit time, ($/time);
HC_{vm}	Holding cost of the raw material for the vendor in the common replenishment cycle C, ($);
TC_{vm}	Total inventory cost of the raw material for the vendor per unit time, ($/time);

JTC	Joint total inventory cost for the supply chain system per unit time, ($/time);
π	Total net profit per unit time, ($/time).

3. Model

The net profit of the supply chain level that we studied is equal to the net revenue minus the total inventory cost. In this section, firstly we calculate the net revenue; secondly the total inventory cost is put forward; and lastly the integrated model is proposed.

3.1 Net Revenue

The product demand D_i for retailer i is a general function of $(p_1 \quad p_2 \quad \cdots \quad p_m)$ and downward sloping with respect to its own price p_i. That is

$$\frac{\partial D_i}{\partial p_i} < 0 \,. \tag{1}$$

Since all retailers offer the substitutable products, the other retailer's retailer price will influence the demand volume of retailer i, we assume that

$$\frac{\partial D_i}{\partial p_j} > 0 \qquad \text{i=1,}\cdots\text{,m and j} \neq \text{i} \,. \tag{2}$$

This represents a common definition of substitutable products, which goes back to Samuelson (1947) [19] and see also Vives (1990) [21]. In other words, each retailer can expect its sales volume to go up, whenever one of another retailer increases its price.

Assume product demand rate for each retailer at the retail price p has a Cobb-Douglas form (Nicholson, 1989) [17], that is

$$D_i(p) = a_i p_i^{-\alpha_i} \prod_{j\neq i} p_j^{\beta_{ij}} \qquad \text{i=1,}\ldots\text{,m} \,, \tag{3}$$

where $p = |p_1 \quad p_2 \quad \ldots \quad p_m|^T$, $\alpha_i > 1$, $a_i > 0$, $\beta_{ij} \geq 0$ for all i and $i \neq j$.

The product is sold by the system at the retail price p_i per unit, which yields net revenue of:

$$\text{net revenue}=\sum_{i=1}^{m} D_i \cdot (p_i - p_0 - \zeta_i) \,. \tag{4}$$

3.2 Total Inventory Cost

In our model, we assume that the vendor purchases raw material outside to produce its finished product. The procurement lot size of raw material is assumed to be an integral multiple of the usage of each production batch. This policy has also been considered in some previous works by Goyal (1977) [7], Goyal and Deshmukh (1992) [9] and Goldberg (1989) [5] etc, which are more general than the lot-for-lot procurement policy adopted in other works such as that of Sarker and Parija (1994) [20] and Nori and Sarker (1996) [18]. The product is then delivered to multiple retailers at a common replenishment cycle. We assume that all enterprises to cooperate the supply chain wide profit function.

In this section we calculate the joint total inventory cost for the vendor and all retailers. The inventory levels for all retailers and vendor are shown in Fig 1.

The change of inventory level for retailer i, notified by $I_i(t)$, during common order cycle C can be described by the following equations:

$$I_i^{'}(t) = -\theta I_i(t) - D_i \qquad 0 \leq t \leq C \tag{5}$$

with the boundary conditions

$$I_i\,(C) = 0 \,. \tag{6}$$

The solution of the differential equation of Equation (5) - (6) is

$$I_i(t) = \frac{D_i}{\theta}\left(e^{\theta(C-t)} - 1\right) \qquad 0 \leq t \leq C \,. \tag{7}$$

For retailer i, the maximal inventory level, that is, the order quantity of retailer i, is

$$Q_i = I_i(0) = \frac{D_i}{\theta}\left(e^{\theta C} - 1\right) \,. \tag{8}$$

So in the common replenishment cycle C the retailer i's total inventory cost is given as

$$\int_0^C H_{bi} I_i(t)dt = \int_0^C H_{bi}\frac{D_i}{\theta}\left(e^{\theta(C-t)} - 1\right)dt$$

$$= \frac{D_i H_{bi}}{\theta^2}(e^{\theta C} - 1) - \frac{D_i H_{bi} C}{\theta} \,, \tag{9}$$

and total deteriorating cost is

$$p_i(Q_i - D_i C) \,. \tag{10}$$

The retailer i's total inventory cost is given as

$$TC_{bi} = \frac{1}{C}[T_i + \frac{D_i H_{bi}}{\theta^2}(e^{\theta C} - 1) - \frac{D_i H_{bi} C}{\theta} + p_i(Q_i - D_i C)] \,. \tag{11}$$

From Fig.1., it can be seen that the finished product inventory level notified by $I_{vi}(t)$ for the vendor follows the differential equation:

$$I_{vi}^{'}(t) = P - \theta I_{vi}(t) \qquad 0 \leq t \leq t_{vi} \,, \tag{12}$$

with boundary conditions:

$$I_{vi}(0) = 0 \,. \tag{13}$$

From Equations (12)-(13), we have

$$I_{vi}(t) = \frac{P}{\theta}\left(1 - e^{-\theta t}\right) \qquad 0 \leq t \leq t_{vi} \,, \tag{14}$$

where t_{vi} is determined by $I_{vi}(t_{vi}) = Q_i$. The solution is

$$t_{vi} = -\frac{1}{\theta}\ln[1 - \frac{D_i}{P}(e^{\theta C} - 1)] \,. \tag{15}$$

Then, the vendor's holding cost for the finished product in the common replenishment cycle C is

$$H_p = \int_0^{t_{vi}} H_{vp} I_{vi}(t)dt = \int_0^{t_{vi}} H_{vp}\frac{P}{\theta}\left(1 - e^{-\theta t}\right)dt$$

$$= \frac{H_{vp}P}{\theta^2}(\theta t_{vi} + e^{-\theta t_{vi}} - 1) \,. \tag{16}$$

And the vendor's decaying cost for the finished product in the common replenishment cycle C is

$$p_0(P\sum_{i=1}^{m} t_{vi} - \sum_{i=1}^{m} Q_i) \,. \tag{17}$$

Now the total inventory cost per unit time of the vendor is

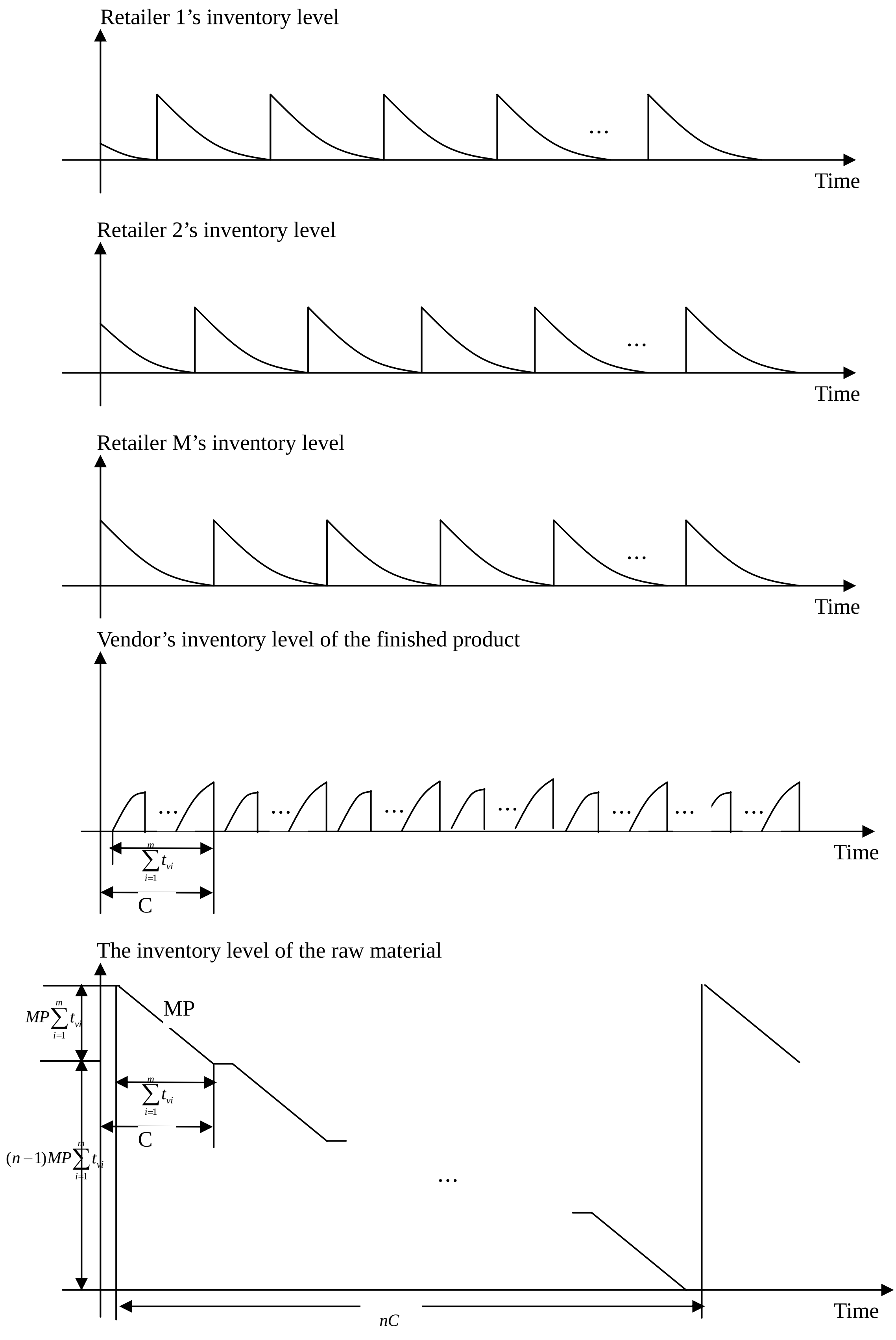

Fig. 1. The inventory level for all retailers and the vendor

$$TC_{vp} = \frac{1}{C}[S + \sum_{i=1}^{m} \frac{H_{vp}P}{\theta^2}(\theta t_{vi} + e^{-\theta t_{vi}} - 1) + p_0(P\sum_{i=1}^{m} t_{vi} - \sum_{i=1}^{m} Q_i)]. \quad (18)$$

Assume that the raw material's replenishment cycle for the vendor is integral multiple of the finished product's replenishment cycle. The holding cost of the vendor in its replenishment cycle nC is

$$HC_{vm} = H_{vm}[\frac{n}{2}\sum_{i=1}^{m} t_{vi}\sum_{i=1}^{m}(MPt_{vi}) + \sum_{j=1}^{n-1}(jC\sum_{i=1}^{m}(MPt_{vi})]$$
$$= \frac{nMPH_{vm}}{2}(\sum_{i=1}^{m} t_{vi})^2 + \frac{n(n-1)CMPH_{vm}}{2}\sum_{i=1}^{m} t_{vi}. \quad (19)$$

The total inventory cost per unit time of the raw material for the vendor is

$$TC_{vm} = \frac{1}{nC}(A + HC_{vm}). \quad (20)$$

Therefore, the joint total inventory cost for the vendor and all retailers per unit time is

$$JTC = TC_{vm} + TC_{vp} + \sum_{i=1}^{m} TC_{bi}. \quad (21)$$

3.3 Supply Chain Level Net Profit

From the above analysis, by the net revenue described in Equation (4) minus the total inventory cost described in Equation (21), we can get the following model, notified by SCLNP.

$$\pi = \sum_{i=1}^{m} D_i p_i - \sum_{i=1}^{m} D_i p_0 - \sum_{i=1}^{m} D_i \zeta_i - JTC$$
$$= \sum_{i=1}^{m} D_i p_i - \sum_{i=1}^{m} D_i p_0 - \sum_{i=1}^{m} D_i \zeta_i - \frac{1}{C}\{\frac{A}{n} + S + \sum_{i=1}^{m} T_i$$
$$+ \frac{(n-1)CMPH_{vm}}{2}\sum_{i=1}^{m} t_{vi} + \frac{MPH_{vm}}{2}(\sum_{i=1}^{m} t_{vi})^2$$
$$+ \sum_{i=1}^{m} \frac{H_{vp}P}{\theta^2}(\theta t_{vi} + e^{-\theta t_{vi}} - 1) + \sum_{i=1}^{m}[\frac{D_i H_{bi}}{\theta^2}(e^{\theta C} - 1) - \frac{D_i H_{bi} C}{\theta}]$$
$$+ p_0(P\sum_{i=1}^{m} t_{vi} - \sum_{i=1}^{m} Q_i) + \sum_{i=1}^{m} p_i(Q_i - D_i C)\}, \quad (22)$$

$$\text{and } \sum_{i=1}^{m} t_{vi} \le P \quad \text{(capacity constraint)}, \quad (23)$$

where Q_i, D_i and t_{vi} are determined by Equations (8), (15) and (3) respectively.

In the integrated inventory model, the decision variables are retail price $(p_1 \quad p_2 \quad \cdots \quad p_m)$, common replenishment cycle C, the integral multiple number n to maximize the net profit π. A closed form analytical solution cannot be obtained for the objective function to calculate the optimal decision variables. So to this kind of function, both Goyal and Gunasekaran in 1995 and Luo in 1997 are to develop a computer program for finding their optimal solution by using an exhaustive search method. In their method, for all each possible combination of decision variables is calculated and this may consume a large amount of computer resources since there are much more continuous variables $(p_{b_1} \quad p_{b_2} \quad \dots \quad p_{b_m})$ in our model. So the genetic algorithm for the model is developed in the following section.

4. Genetic Algorithm

Genetic algorithms have demonstrated a considerable success in providing good solutions to many complex optimization problems and received more and more attentions. They have been well documented by numerous pieces of literature, such as that of Goldberg (1989) [5], Michalewicz (1994) [16] and Fogel (1994) [4], and applied to a wide variety of optimization problems. In this section, a genetic algorithm for solving the optimal solution for model SCLNP to maximize the supply chain level profit is designed as follows:

Step 0: Input parameters pop-size, pm, pc, pr, M, and N, where pm, pc and pr is the percentage of mutation, crossover and reproduction respectively.

Step 1: Initialize pop-size chromosomes (the first generation).

Step 2: Evaluate the pop-size chromosomes by fitness function.

Step 3: Selection.

Step 4: Alter the chromosomes by crossover and mutation operations.

Step 5: Evaluate the pop-size chromosomes by fitness function.

Step 6: Selection.

Step 7: Repeat the 2nd to 6th steps till the third termination condition is satisfied.

Step 8: Report the best chromosome as the optimal solution.

5. Numerical Example

This section presents a numerical example for model SCLNP. The related input parameters are given in Table 1. As an illustration, the case of m=3 are discussed. The unit time is one year and the monetary unit is U.S. dollar.

The optimal decisions for all retailers and the vendor are shown in Table 2 and the sensitivity analysis for the parameters that associate with market and deteriorating, i.e., a_i, α_i, β_{ij} and θ are shown from Table 3-6.

Table 1: Input parameters

Parameters	Value	Parameters	Value
P	60000	M	0.95
p_0	40	θ	0.02
A	5000	ζ_i	3
S	2000	α_i	1.45
T_i	1000	a_i	20000000
H_{bi}	80	H_{vm}	15
H_{vp}	40	β_{ij}	0.01

Table 2: optimal decisions of the base example

(p_1, p_2, p_3)	$\sum_{i=1}^{3} D_i$	N	C	JTC	π
(154.95,154.95,154.95)	44276.81	3	0.04771	279384.71	4677414.84

The parameter a_i represents the attribute of customer markets of retailer i; in which the larger of a_i, the larger of demand volume of customer i. From Table 3, it can be seen that with the increasing of a_i, the demand of the vendor which is the sum of the demands of its retailers is increasing constantly. The demand volume increases from 10205.57 at $a_i = 0.5$ to 59968.11 at $a_i = 3.5$ by 487 %. P$_1$, P$_2$ and P$_3$ decrease at first when the capacity of the vendor is full before $a_i \le 2.5$. However when the capacity of the vendor is used up after $a_i \ge 3$, with the increasing of a_i, all prices begin to go up and JTC increases by 2.67 from 1251180.68 at $a_i = 0.5$ to 334708.00 at $a_i = 3.5$ whereas π increases by 7.39 from 1109766.82 at $a_i = 0.5$ to 8202461 at $a_i = 3.5$. When the replenishment cycle of the finished product decreases constantly since when the demand of the vendor goes up rapidly with the increase of a_i before $a_i \le 2.5$. With the increase of a_i after $a_i \ge 3$ where the capacity of the vendor is used up, the increasing of the retail price deduces the penalty of the deteriorating cost per unit finished product goes up and such the replenishment cycle of the common replenishment cycle decrease too.

Table 3: Sensitivity analysis where a_i is changed

a_i (107)	(p_1, p_2, p_3)	$\sum_{i=1}^{3} D_i$	n	C	JTC	π
0.5	164.00,164.00,164.00	10205.57	2	0.11972	125180.67	1109766.82
1	158.46,158.46,158.46	21441.64	2	0.08067	185896.66	2289635.59
1.5	156.32,156.32,156.32	32791.98	2	0.06371	235410.31	3480610.46
2	154.95,154.95,154.95	44276.81	3	0.04771	279384.71	4677414.84
2.5	154.18,154.18,154.18	55742.76	3	0.04168	319706.79	5877672.20
3	166.45,166.45,166.45	59968.11	3	0.03986	334251.51	7066984.50
3.5	185.40,185.40,185.40	59968.23	3	0.03965	334708.00	8202461.33

Table 4: Sensitivity analysis where α_i is changed

α_i	(p_1, p_2, p_3)	$\sum_{i=1}^{3} D_i$	N	C	JTC	π
1.15	451.92,451.92,451.92	59969.97	3	0.03753	341299.39	24181502.3
1.25	275.92,275.92,275.92	59968.39	3	0.03923	336838.44	13571187.0
1.35	186.70,186.70,186.70	57216.98	3	0.04089	325475.18	7896685.77
1.45	154.95,154.95,154.95	44276.81	3	0.04771	279384.71	4677414.84
1.55	135.74,135.75,135.75	32750.91	2	0.06385	234812.22	2801737.00
1.65	122.90,122.90,122.90	23559.22	2	0.07696	195104.79	1687361.87
1.8	110.82,110.82,110.82	13763.88	2	0.10289	145784.39	787662.38

a_i represents the elasticity of markets of retailer i; in which the larger a_i is, the more price sensitivity for the market of retailer i. From Table 4, it can be seen that with the increasing of a_i, the price of the finished product reduced rapidly; the price is 451.92 at $\alpha_i = 1.15$ whereas the price is 110.28 at $\alpha_i = 1.15$ by 341.10. Accordingly n, $\sum_{i=1}^{3} D_i$, JTC and π reduce and the finished product replenishment cycle C is increasing.

From Table 5, with the increasing of β_{ij}, P$_1$, P$_2$, P$_3$, $\sum_{i=1}^{3} D_i$, JTC and π go up while C decreases and n keeps unchanged. Something needs to be noted that when β_{ij} is equal to 0.05 P3 becomes Infinity. Likewise, when β_{ij} is equal to 0.08, P2 and P3 both become Infinity. The infinity price of P_i means D_i becomes zero, which leads to the decreasing of retailers. It can be seen that when β_{ij} is equal to 0.08 the prices of three retailers are 547.78, Infinity and Infinity respectively. That is, only one retailer should be left while another two should be canceled.

Table 5: Sensitivity analysis where β_{ij} is changed

β_{ij}	(p_1, p_2, p_3)	$\sum_{i=1}^{3} D_i$	N	C	JTC	π
0	150.33,150.33,150.33	41819.58	3	0.0493	270253.37	4218795.06
0.001	150.78,150.78,150.78	42063.13	3	0.04822	271165.11	4262479.02
0.005	152.60,152.60,152.60	43037.41	3	0.04852	274797.88	4441961.47
0.01	154.95,154.95,154.95	44276.81	3	0.04771	279384.71	4677414.84
0.03	165.65,165.65,165.65	49369.98	3	0.04481	297877.96	5757515.32
0.05	170.45,170.45,Infinity*	59967.47	3	0.03565	315691.32	7328893.33
0.08	547.78,Infinity,Infinity	59967.47	3	0.02734	310124.20	29533606.3

* When the optimal P_i calculated by GA is larger than 10^9 we give Infinity to P_i there.

From Table 6, with the increasing of θ, JTC arises to 513343.84 at $\theta = 2$ by 86.15% comparing with that of by from 275764.28 at $\theta = 0.001$, and π is decreased by 5.19% from 4680974.66 at $\theta = 0.001$ to 37037420.10 at $\theta = 2$. The decrease of π by 5.19% is relatively small comparing with the increase of the joint inventory cost JTC, 86.15% since in this example the inventory cost is relatively small compare with π With the increase of θ, at first, the demand of the vendor $\sum_{i=1}^{3} D_i$ rises and all retailers' prices goes down before $\theta = 1$, but then both of them drop down from $\theta = 1$ to $\theta = 2$. C and n increase constantly with the increase of deteriorating rate θ; which means that with the increase of the deteriorating volume for the finished product and the decrease of the demand of the vendor the common replenishment cycle C will be ascended and the replenishment cycle for the raw material increases too since there is no deterioration for the raw material.

Table 6: Sensitivity analysis where θ is changed

θ	(p_1, p_2, p_3)	$\sum_{i=1}^{3} D_i$	N	C	JTC	π
0.0001	155.00,155.00,155.00	44259.98	3	0.04843	275764.28	4680974.66
0.005	154.98,154.98,154.98	44263.03	3	0.04819	276657.44	4680092.53
0.01	154.97,154.97,154.97	44267.44	3	0.04802	277568.71	4679197.14
0.02	154.95,154.95,154.95	44276.81	3	0.04771	279384.71	4677414.84
0.05	154.89,154.89,154.89	44300.97	3	0.04665	284752.15	4672134.04
0.1	154.80,154.80,154.80	44339.44	3	0.04496	293482.22	4663540.94
1	154.64,154.64,154.64	44405.83	4	0.02986	416114.48	4541142.59
2	155.30,155.31,155.30	44130.84	5	0.02312	513343.84	4442923.33

6. Conclusion

In this paper, an integrated model for a one-vendor, multi-buyer supply chain where the Cobb-Douglas demand function is modeled. The vendor and all buyers to place the replenishment orders at common replenishment cycle based on vendor management inventory. The vendor buys raw material that is no decay to produce a finished product by a finite production rate and transport them to its retailers. All retailers sell the finished product in its market and customers can buy the product from different retailers. So the finished product sold by different retailers has the attribute of substitution. The problem of determining the optimal the common replenishment cycle for the finished product, the replenishment cycle for raw materials and all retailer's price and be modeled as an integrated model in our paper. After modeling the problem, a genetic algorithm is proposed to give the optimal decisions. An extensive numerical study was conducted to understand the influence of various parameters related to market and deteriorating rate. The numerical study revealed that the decrease of the deteriorating rate θ and the increase of the elasticity of its own price α_i and substitution elasticity β_{ij} contribute to the net profit of the supply chain and vise versa. And Here that should be noted from our integrated model that with the increase of the β_{ij} , the whole market is improved, and even the β_{ij} is better enough the number of the retailers may be decreased.

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