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Abstract: Collusion between suppliers in reverse auctions may damage buyers' benifits and then lower efficiency of resource allocation. Under first-price sealed-bid reverse auction mechanism, suppliers' equilibrium bidding strategy and buyers' revenue loss was analyzed considering collusion. Then some general conclusions were given: suppliers' bidding strategy is relevant to some factors such as valuation costs, the number of suppliers and Cartel; the number of suppliers and Carter can lead to buyers revenue loss. Finally, considering two different kinds of distribution function, the strategies of improving buyer's revenue were discussed.

Keywords: reverse auction; collusion; equilibrium bidding

1. INTRODUCTION

Electronic reverse auction (ERA)is an online, real-time dynamic auction between a buying organization and a group of pre-qualified suppliers who compete against each other to win the business to supply goods or services[1].Notwithstanding its practical significance and advantages such as procurement time savings and cost savings,ERAs is an emerging research area in recent years.Today, ERAs have become a common method to source a broad range of products and services. As the electronic reverse auction have some advantages the buyers get so much favor from it.Reverse auction theory has gradually enriched,however the majority of the existing research is focusing on the advantages ,risk, application conditions of reverse auction and relations among buyers and suppliers,without considering suppliers collusion case.Collusion among suppliers can reduce the adverse effects in reverse auction procurement practice.

The collusion in auction firstly be researched by some scholars such as Robinson, Graham and Marshall, McAfee and McMillan. Robinson pointed out that the sharing of information led to instability of the cartel in the first price sealed auction, which explained why the first price sealed auction is frequently used in practice [2]. Graham and Marshall extended the standard model, considering collusion existing among the bidders and the auctioneer's strategic response to collusion, The main result shows that setting a higher reserve price by the auctioneer is the optimal strategy[3].McAfee and McMillan characterized coordinated bidding stategies in two cases:a weak cartel,in which the bidders cannot make side-payments;and a strong cartel ,in which the cartel members can exclude new entrants and make transfer payments[4]. The main result shows that when a strong cartel existed and highest valuation exceeded the reserve price, configuration is valid.Then it pointed out that the auctioneer can improve or hidden the reserve price, interfere the cartel's strategy to prevent collusion. Lyk-Jensen according to introduce the first price or second price pre-auction knockout mechanism and assume all bidders participanting a cartel ,it shows that the the collusioni is stable, but can not achieve the efficiency,but efficiency can be achieved by allowing the sharing of information and budget balance of the pre-auction knockout[5].Marshall and Marx did further reserch on the collusion in the first and second price auction[6].The Jing Huirong and Li Chuanzhao compared cartel representative choice and profit distribution in the first and the second price sealed pre-auction knockout in the strong cartel mechanism , got effective measures to respond to cartel's collusion based on the auction action strategies[7].Wang Wenju and Yang Yingmei combined the collusion between tenders to tenders' publicly reserved price strategy ,studied the interests of the allocation mechanism of the collusionin the second-price sealed bid auctions,then got the results that reducing the public reserve price can prevent collusion[8]. Thomas got the impact factors on collusion under the repeated auction.:the number of suppliers, the expected cost of suppliers, the cost variability of buyers[9]. Ohashi investigated bidder qualification procedure to improve the transparency, and got the result that improved transparency can reduce procurement costs and undermine the stability of the collusion[10]. Pesendorfer under the assumptions that buyers can sell more than one contract and the suppliers' valuation was private, then it got the collusion that the collusion equilibrium is to conceal its offer[11].

Based on the above studies, this article attempts to determine the strategy of cartel under the conditions of "strong" cartel, a single static inseparable single item only a cartel in the reverse auction then according to the analysis of the suppliers' bidding strategy as well as the buyer's revenue losses.Impact factors of bidding strategy and measures to improve the income of buyers through different numerical analysis will be got.

2. THE DESCRIPTION OF THEQUESTION AND MODEL HYPOTHESIS.

Buyers use the electronic reverse auction as a tool for supporting e-sourcing, Because suppliers are symmetrical and homogeneous,Competitive bidding will reduce their profit and collusion will increase their profit, which promotes the cartel formation. The first questions faced by the cartel is how to choose cartel representatives to participate the formal auction and rational allocation of profit they received. In the first price sealed auction, all cartel members make a contract: each member quotes a specified price, then any one in the cartel who win the bid should pay to the other members lots of the revenue. But the cartel members may change their tactics, they are likely to reduce the price, so they can get the bid with non-profit-distributing which will obtain additional profits.So a reasonable strategy should be designed within the cartel,The lowest private valuation will win the bid. In order to make more members not compulsory to join the cartel, this mechanism must satisfy the participation constraint condition:If the members does not participate in the cartel ,his profit will be zero,for non-cartel members to participate in the cartel can get a certain transfer payments.So it attracts more non-cartel members participate in the cartel. In short, an equilibrium strategy is as follows:all the cartel members are independently reported its private real valuation, which is a dominant strategy.

In this paper,we consider the bidders can make side-payments which is proposed in McAfee and McMillan[4]in a single static first price sealed reverse auction.Equilibrium bid strategies of cartel representatives who will participate in the reverse auction and buyer's expected revenue loss will be studied.For the convenience of research, this paper make the assumptions as follows:

1) Collusion exists among suppliers in the auction, the number of suppliers is $n(n \geq 3)$,the cartel's number is $k(1 \leq k \leq n)$.

2) buyer and suppliers are risk neutral and maximize their expected revenue in the auction.

3) the valuation of the suppliers i is $v_i(i = 1,2\Lambda\ n)$,which is private information and each v_i $(v_i \in [\underline{v}, \overline{v}])$ is independent and identically distributed in the cumulative distribution function $F(\bullet)$, this function is a continuous monotonically increasing function,and the distribution is common knowledge to buyer and suppliers.

4) The bidding strategy b_i of supplier i is monotonically increasing.

3. EQUILIBRIUM BIDDING STRATEGY OF THE CARTEL IN THE REVERSE AUCTION.

In the first price sealed reverse auction， assuming that suppliers who has the lowest quotation will win the subject and get the payment.The suppliers' expected revenue function is

$$U_i = (b_i - v_i)\Pr(b_i \le b_j, j \ne i)$$

In which $\Pr(\bullet)$ is the lowest probability in all bid b_i ,Obviously, this probability is $[1-F(b(v_i))]^{n-1}$,

That is

$$U_i(v_i, b(v_i)) = (b_i - v_i)[1-F(b(v_i))]^{n-1}$$

When the cartel members k equal to the number of the total number of suppliers n , the cartel's optimal bidding strategy: cartel representatives directly quoted price of $\overline{v}$,the other members quoted price which is higher than $\overline{v}$. If there is no collusion, suppliers quoted their true costs v_i , then the cartel representatives should transfer payment to the union $u = \overline{v} - v_i$,then the proceeds will be distributed in the union.

When the cartel members k is less than the number of total number of suppliers n , cartel representative consider other cartel members' action.The following proposition will be got:

Proposition3.1 Considering the cartel exists, Influence factors in equilibrium price of suppliers in the first price sealed reverse auction : private valuation of suppliers (v_i), suppliers bidding strategy ($F(\bullet)$), the number of suppliers (n), the cartel numbers(k).

Proof: In the reverse auction, for non-cartel members, he win the bid when his bid is the lowest bid the expected revenue and payment is differernt regardless of the existing of the cartel , . Actually the number of participating in the reverse auction is $n-k+1$ .When a supplier's private cost is v_i ,his bidding strategy is b_i ,so his expected revenue is

$$U_i(v_i, b(v_i)) = (b_i - v_i)[1-F(b(v_i))]^{n-k}$$

The first order conditions optimization is

$$\frac{\partial U_i(v_i, b(v_i))}{\partial b_i} = 0$$

That is

$$\frac{\partial U_i(v_i, b(v_i))}{\partial b_i} = [1-F(b(v_i))]^{n-k} + (n-k)(b_i - v_i)[1-F(b(v_i))]^{n-k-1}[F(b(v_i))]' = 0$$

That is

$$[1-F(b(v_i))] + (n-k)(b_i - v_i)[F(b(v_i))]' = 0$$

Suppliers will bid their true value when meeting Bayesian Nash equilibrium,that is

$$b^*(v) = v$$

Assume that the supplier's valuation $\overline{v}$,his bid is $\overline{v}$, the boundary conditions $b(\overline{v}) = \overline{v}$ .Above equation satisfies the first-order linear differential equations and boundary conditions,Suppliers' equilibrium bidding function is

$$b^*(v) = v + \frac{\int_v^{\bar{v}} [1-F(u)]^{n-k} du}{[1-F(v)]^{n-k}} \qquad (1)$$

Proposition3.1 shows that: 1) Supplier's equilibrium bidding function $b^*(v)$ is interrelated with supplier's bidding strategy v, the number of suppliers n, the the cartel scale k ,and his equilibrium bidding is higher than v ; 2) In order to guarantee the revenue,suppliers set reasonable evaluation on the project , and the higher the evaluation is, the higher equilibrium bidding strategy is; 3) In order to get more profit,suppliers would form cartel to obtain additional income, obviously cartels scale will affect suppliers' equilibrium bidding strategy; 4) the more suppliers to participate in the auction ,the lower of the bidding strategy is ,then suppliers will get less profits, the buyer will get higher profit. Therefore, the buyer would consider the number of suppliers on their profit, he will attract more suppliers to participate in the bidding.

Suppliers equilibrium bidding function has the following properties:

Property 3.1.1,Supplier's equilibrium bidding improves considering the existence of cartel in the reverse auction.

Proof:Known from literature[8],non-collusive conditions, in the standard first price sealed auction suppliers'equilibrium bidding function is:

$$b^*(v) = v + \frac{\int_v^{\bar{v}} [1-F(u)]^{n-1} du}{[1-F(v)]^{n-1}} \qquad (2)$$

(1) subtracted (2) ,that is

$$\int_v^{\bar{v}} [\frac{1-F(u)}{1-F(v)}]^{n-k} [\frac{F(u)-F(v)}{1-F(v)}]^{k+1} du$$

Because $u > v, 0 < F(u) < 1, \frac{1-F(u)}{1-F(v)} > 0, F(u)$ is the monotone increasing function

We can get

$$F(u) > F(v), \frac{F(u)-F(v)}{1-F(v)} > 0 .$$

That is （1）>(2)

That is supplier's equilibrium bidding improves considering the existence of cartel in the reverse auction.

Property3.1.2:Supplier's equilibrium bidding improves considering the improving scale of cartel when fixing the same number of suppliers in the reverse auction.

Proof:Fixed the number of suppliers n ,when the cartel scale increases, from k to $k+1$ .The equilibrium bidding is

$$b^*(v) = v + \frac{\int_v^{\bar{v}} [1-F(u)]^{n-k-1} du}{[1-F(v)]^{n-k-1}} \qquad (3)$$

The analysis process is the same as above,we can obtain (1) <(3).

That is supplier's equilibrium bidding $b^*(v)$ improves with the increasing scale of cartel k.

Property 3.1.3:Suppliers' equilibrium bidding reduced with the increasing number of suppliers when fixed the same scale of cartel in the reverse auction,and then it will closer to the cost valuation.

Proof: By (1), when the number of suppliers increased from n to $n+1$ and fix the cartel scale k, the function of the equilibrium bidding is

$$b^*(v) = v + \frac{\int_v^{\bar{v}} [1-F(u)]^{n+1-k} du}{[1-F(v)]^{n+1-k}} \qquad (4)$$

It is easy to obtain (1)> (4).

The equilibrium bidding $b^*(v)$ is reduced when the number of suppliers n increase, but it's higher than the cost valuation v.

From the analysis above, we can get that fixing the cartel scale, the more the suppliers, the lower the cartel the lower the bidding strategy is,the greater the probability of winning the bid. When the the suppliers' bid is lower and closer to the true cost,his transfer to the other members of the cartel is smaller.Therefore, in order to achieve greater gains, Cartel representatives will make efforts to expand the scale of the cartel, which indirectly harms the interests of buyer. But in a certain extent more suppliers will improve the competition among suppliers, may increase the buyer's income.Therefore, the number of suppliers, the cartel scale will affect the buyer's income.

4. BUYER EXPECT REVENUE ANALYSIS WHEN CARTEL EXISTS.

Define the loss of buyer's expected revenue $\Delta R = R_1 - R_2$ .In which R_1 is the buyer expected revenue with non-collusion, R_2 is the buyer's expected revenue when cartel exists. The optimal auction mechanism can make buyer to maximize his expected revenue.

Under the condition of non-collusion, the buyer's expected payment is

$$R_1 = n\int_v^{\bar{v}} [vF'(v) + F(v)][1-F(v)]^{n-1} dv$$

Just as bidding payment equivalence theorem in the literature [13].

Riley [12] considered that the buyer's expected payment is n times of the supplier's expected payment . That is

$$n\int_v^{\bar{v}} U(v,b)F'(v)dv = n\int_v^{\bar{v}} (b-v)[1-F(v)]^{n-1} F'(v)dv$$

in which

$$U(v,b) = (b-v)[1-F(v)]^{n-1}$$

When the collusion exists. Essentially, the number of supplier is $n-k+1$,that is

$$R_2 = (n-k+1)\int_v^{\bar{v}} [vF'(v) + F(v)][1-F(v)]^{n-1} dv$$

The the buyer's expected revenue loss is

$$\Delta R = R_1 - R_2 = (k-1)\int_v^{\bar{v}} [vF'(v) + F(v)][1-F(v)]^{n-1} dv.$$

5. NUMERICAL ANALYSIS.

In order to get more intuitive explanation of influence factors on supplier's equilibrium bidding strategy b and buyer's expected revenue ΔR , such as the number of suppliers (n), the private cost valuation (v) and the cartel scale (k). Let $\underline{v}=0, \bar{v}=1, v\in[0,1]$, take the following two kinds of valuation distribution function,and it's monotonically increasing value interval $[0,1]$.All meet the assumptions.

Case 1: The supplier valuation uniform distribution and the valuation of the distribution function is:

$$F(v_i) = v_i$$

The suppliers'equilibrium bidding function is

$$b_1^* = v + \frac{1-v}{n-k+1}$$

The buyer's expected revenue loss is

$$\Delta R = \frac{2(k-1)}{n(n+1)}$$

Case 2: Supplier valuation obey polynomial distribution and the valuation of the distribution function is:

$$G(v_i) = -v_i^2 + 2v_i$$

The suppliers'equilibrium bidding function is

$$b_2^* = v + \frac{1-v}{2(n-k)+1}$$

The buyer's expected revenue loss is

$$\Delta R = \frac{(k-1)(4n-1)}{n(2n+1)(2n-1)}$$

Respectively let $n=3, k=1,2$; $n=10, k=1,2,8$; $n=15, k=1,2,8,14,15$; $v=0.2,0.5,0.9$.

By calculating the equilibrium bidding function and the buyer's expected revenue loss is as shown in Table 1 and Table 2 below.

Table 1. Bidding Strategies of Suppliers between Collusion and Non-collusion

	v	$b_1^* = v+(1-v)/(n-k+1)$			$b_2^* = v+(1-v)/(2(n-k)+1)$		
		$n=3$	$n=10$	$n=15$	$n=3$	$n=10$	$n=15$
$k=1$	0.2	0.4667	0.2800	0.2533	0.3600	0.2421	0.2276
	0.5	0.6667	0.5500	0.5333	0.6000	0.5263	0.5172
	0.9	0.9333	0.9100	0.9067	0.9200	0.9053	0.9035
$k=2$	0.2	0.6000	0.2889	0.2571	0.4667	0.2471	0.2296
	0.5	0.7500	0.5556	0.5357	0.6667	0.5294	0.5185
	0.9	0.9500	0.9111	0.9067	0.9333	0.9059	0.9037
$k=8$	0.2	-----	0.4667	0.3000	-----	0.3600	0.2533
	0.5	-----	0.6667	0.5625	-----	0.6000	0.5333

	0.9	-----	0.9333	0.9125	-----	0.9200	0.9067
$k=14$	0.2	-----	-----	0.6000	-----	-----	0.4667
	0.5	-----	-----	0.7500	-----	-----	0.6667
	0.9	-----	-----	0.9500	-----	-----	0.9333
$k=15$	0.2	-----	-----	1.0000	-----	-----	1.0000
	0.5	-----	-----	1.0000	-----	-----	1.0000
	0.9	-----	-----	1.0000	-----	-----	1.0000

Table 2. Expected Revenue Loss Analysis of Buyers between Collusion and Non-collusion

	$\Delta R=2(k-1)/n(n+1)$			$\Delta R=2(k-1)(4n-1)/n(2n+1)(2n-1)$		
	$n=3$	$n=10$	$n=15$	$n=3$	$n=10$	$n=15$
$k=1$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$k=2$	0.1667	0.0182	0.0083	0.1048	0.0098	0.0044
$k=8$	-----	0.1273	0.0583	-----	0.0684	0.0306
$k=14$	-----	-----	0.1238	-----	-----	0.0569
$k=15$	-----	-----	0.2333	-----	-----	0.0613

(1) When $F(\bullet)$ is the same, the number of suppliers is the same, then the suppliers' private cost valuation is different, there is no collusion exists, the higher suppliers' private cost valuation is , the higher their equilibrium bidding strategy is. Expected revenue of the supplier (ie balanced bid minus cost valuation) With the rise of the private cost valuation is getting smaller and smaller, for example, when $n=15$, respectively, by 0.0533,0.0333, down to 0.0067. When the collusion exists, the more the number of collusion is, the higher the equilibrium bid is, when $n=15$ $v=0.5$,the bid is 0.5357,0.5625,0.7500. The suppliers expected revenue is increasingly high, buyer's loss is also increasing correspondingly.

(2) When $F(\bullet)$ is the same,the number of suppliers is different, the suppliers of private cost valuation is the same, there is no collusion, the more the number of suppliers is ,the lower equilibrium bid is,for example, when $v=0.5$, the bid is 0.6667 , 0.5500 down to 0.5333. When the collusion exists, the more the number of collusion, that is the greater cartel scale is , the higher the balanced bid, increased from 0.6667 to 0.7500. Buyer's loss is increasing correspondingly.

(3) When $F(\bullet)$ is the same and there is no collusion, under the ideal conditions, buyer's expected revenue loss is 0.When all suppliers participate in the collusion , the equilibrium bid is the highest value of 1.0000, as was set before .the maximum loss of the buyer's is 0.2333. So the buyer must take measures to reduce its loss of revenue.

(4)For the two different equilibrium bidding function $F(\bullet)$,the buyer's expected revenue loss and the suppliers' equilibrium bidding function are different but the above conclusions are also fit , that is not irrelevant to $F(\bullet)$.In order to facilitate the study,independent uniform distribution is also assumed in the reverse auction.

6. CONCLUSIONS

Supplier's bid dominant strategy in the cartel is all cartel members independently report their private real value in the first price sealed reverse auction.The factors that affect the suppliers' equilibrium bidding strategy is as follows: the suppliers' cost valuation, the suppliers' distribution function of the the valuation,the number of suppliers, the cartel scale.The suppliers'equilibrium bidding improved when the collusion exists and the more the number of the cartel, the higher the suppliers'equilibrium bidding;the higher the number of all suppliers, the lower the suppliers' equilibrium bidding,and it is closer to his cost valuation.Collusion lead to buyers’revenue loss, and the loss of revenue is relevant to and the number of all suppliers and the scale of the cartel. Therefore the buyer may take measures to control the size of the cartel and increase the number of suppliers to improve his revenue.

This article is in ideal conditions :the "strong" cartel mechanism, in single static inseparable single item reverse auction only considering a cartel exists.The equilibrium bidding strategies is studied under the collusive and non-collusive conditions and buyer's loss of revenue considering the existence of only a cartel and then numerical analysis is showed. Only two different distribution functions is considered in this article, conclusions merely reflects the function of the impact of changes ,other factors and the presence of multiple cartels,and the universality of the formula needs do further research;how to extend the model to multi-items and dynamic reverse auction need further research.

Compared with the traditional auction, in reverse auction the buyer is facing large-scale item and highly competitive,and more complicated process. With the growing popularity of e-commerce, network provides more information to the suppliers at the same time it's convenient to collusion among suppliers. how to effectively reduce the occurrence of collusion is also one of the focus of future research in the network environment.

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