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An Algorithm for Warehouse Capacity and Replenishment Scheduling Decisions

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Abstract

In our paper, a Singapore chemical manufacturer procures a kind of raw material from Europe and the material arrives at a fixed quantity with a cycle of about two months according to their long term contract. The material can be stored for only one week in normal conditions. So the manufacturer has to rent a special warehouse to hold the material before it is transported into the workshop, which happens at the beginning of each week. But at that time the manufacturer cannot know the exact demands of its customers each week. So understock and overstock of the material may exist. The manufacturer has to decide the capacity of the special warehouse and the replenishment quantities to the workshop each week.

The total costs make up of three parts: the cost due to the warehouse capacity constraint, the cost due to the workshop inventory constraint and the inventory cost at the warehouse. Because the material is transported and stored in tanks, we build up a newsvendor model with discrete variables. And it is an integer quadratic programming with linear constraints. Then we convert this integer quadratic programming into a binary quadratic programming, and then into an integer linear programming. Thus, existent software, such as ILOG CPLEX can be used to solve the problem.

Keywords: stochastic lead time, discrete newsvendor model, binary integer linear programming

1. Introduction

In real life, due to the different supply contracts (lead time, order quantity etc.), different industrial requirements (emphasis on quick delivery or on transportation economy etc.) and different suppler-buyer relationship, logistic decision making may be complex and diversified and the crusial decision variables may be completely different. Unlike usual papers, ours makes the decisions of the warehouse capacity and replenishment quantity to the workshop jointly.

In our project, a Singapore chemical manufacturer procures a kind of raw material from Europe and the material arrives at a fixed quantity with a cycle of about two months according to their long term contract. The manufacturer has a long term contract with its outside supplier to repleshish the warehouse's inventory at a fixed quantity of two hundred tanks with a period of about two months (We have the distribution of the period's length. At the same time the manufacturer faces uncertain demands. And due to the operational complexity and related high costs, a Just-in-Time system is infeasible in this company and we can only replenish the workshop's inventory from the warehouse at the beginning of every week. A kind of special trucks with the capacity of only one tank is used for the transportation from the warehouse to the workshop. So the transportation fee from the wareshouse to the workshop is unrelated to our decision.

2. Notations and Modeling

We assume that the manufacturer gives an order of fixed number to the warehouse at the beginning of the t^{th} week after the last replenishment, and the preplanned interval between two replenishments of the warehouse is T weeks and the fixed order quantity is R. As we mentioned above the trucks transporting containers from the warehouse to the factory's workshop has a capacity of one tank, so every T weeks a fixed number of R trucks are necessary and the transportation cost is fixed. Thus our decision will not change the transportation cost and it can be ignored in our model. Since all the goods are packaged in tanks until being consumed in the factory and the left inventories in the factory after a week will be discarded, the demands in the factory at the t^{th}week can be modeled as discrete random variables D_t with the probability mass function $P(D_t = j) = p_j^t$, $\sum_{j=0}^{D_m} p_j^t = 1$. If the order quanity of the factory from the warehouse is too large, a holding cost of h_f per tank per week will occur. On the contrary, if the replenishment quantity at the factory is too small, the production will break and a penalty cost of p_f for the shortage per tank per week will occur. We should decide the proper quantites from the warehouse to the factory at the t^{th} week Q_t to optimize the total costs. We also need to decide the capacity of the warehouse due to the uncertainty of lead times from outside supplier and the factory's demands. If the warehouse is too large, a penalty cost of co per tank occurs (this cost is reasonable since the over capacity will last all the T weeks), and if it is too small, the company will have to rent other warehouse to hold the inventory and lead to an incremental cost is cu (this incremental cost may be the additional transportation cost from the temporary warehouse to the factory). The goods arrive at the warehouse i weeks ahead of the plan with a probability of p_i, and the inventory level of the warehouse at the end of t^{th} week is I_t if the replenishment arrives as plans. Define

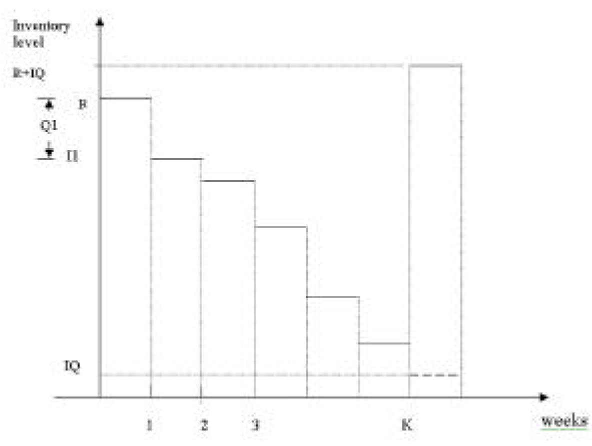

Figure 1: Inventory Level of the Warehouse

$K(Q_w, Q_1, Q_2, ..., Q_T) := max\{k : I_m(k) \leq Q_w\}$ as the week of the warehouse's inventory clearance, then every T weeks, the costs caused by the capacity of the warehouse is

$$co \sum_{i=0}^{K} (Q_w - I_m(i))p_i + cu \sum_{i=K+1}^{L_m} (I_m(i) - Q_w)p_i,$$

the warehouse's inventory cost is

$$\sum_{t=1}^{T} I_t t h_w + R h_w \sum_{t=0}^{T} t p_t,$$

the second part is caused by the advance of the warehouse's replenishment and is unrelevant to our decision of $Q_w, Q_1, Q_2, ..., Q_T$, so we can ignore it;
in each week the costs caused by the imbalance of the factory's demand and the replenishment quantity is

$$p_f \sum_{D_i=Q_i+1}^{D_m} (D_i - Q_i)p_{D_i}^i + h_f \sum_{D_i=0}^{Q_i} (Q_i - D_i)p_{D_i}^i$$

so our problem is

$$Min. \quad co \sum_{i=0}^{K} (Q_w - I_m(i))p_i + cu \sum_{i=K+1}^{L_m} (I_m(i) - Q_w)p_i +$$

$$\sum_{t=1}^{T} I_t t h_w + \sum_{i=1}^{T} [h_f \sum_{D_i=0}^{Q_i} (Q_i - D_i)p_{D_i}^i$$

$$+ p_f \sum_{D_i=Q_i+1}^{D_m} (D_i - Q_i)p_{D_i}^i]$$

s.t.

$$I_t = R - \sum_{i=1}^{t} Q_i, \tag{1}$$

$$\sum_{t=1}^{T} Q_i = R, \tag{2}$$

$$I_m(t) = R + I_{T-t} = 2R - \sum_{i=1}^{T-t} Q_i, \tag{3}$$

$$I_m(K) \leq Q_w < I_m(K+1) \tag{4}$$

$$Q_t \geq 0, t = 1, 2, ..., T, Q_w \geq 0 \tag{5}$$

Constraint (1) expresses the relationship between the planned inventory level and the transportation quantity to the factory. Constraint (2) states that the replenishment of the warehouse is used for T weeks' demands for the factory. Constraint (3) is used for calculating the maximum inventory level at the warehouse if the replenishment arrives at the warehouse t weeks ahead of the schedule. Constraint (4) is the definition of the cutoff level K.

In order to express the sum of K(a random variable) variables in a convinient way, we make the following conversion. Define

$$x_t^{(K)} = \begin{cases} 1 & \text{if } t \leq K \\ 0 & \text{if } t > K \end{cases},$$

then $K = \sum_{t=1}^{T} x_i^{(K)}$,

$$x_t^{(T-K)} = \begin{cases} 1 & \text{if } t \leq T-K \\ 0 & \text{if } t > T-K \end{cases},$$

$T - K = \sum_{t=1}^{T} x_t^{(T-K)}$, and $x_t^{(K)}$ and $x_t^{(T-K)}$ have to satisfy the constraint (6) $x_{T+1-i}^{(T-K)} + x_i^{(K)} = 1, \forall i = 1, 2, ..., T$

$$x_t^{(T-K-1)} = \begin{cases} 1 & \text{if } t \leq T-K-1 \\ 0 & \text{if } t > T-K-1 \end{cases},$$

$T-K-1 = \sum_{t=1}^{T} x_t^{(T-K-1)}$, and obviously, there is constraint (7) $\sum_{t=1}^{T} x_t^{(T-K-1)} + 1 = \sum_{t=1}^{T} x_t^{(T-K)}$

$$x_i^t = \begin{cases} 1 & \text{if } i \leq Q_t \\ 0 & \text{if } i > Q_t \end{cases},$$

then $Q_t = \sum_{i=1}^{Q_m} x_i^t$, the problem becomes:

$$co\sum_{t=0}^{T}(\sum_{i=1}^{M} x_i^w - 2R + \sum_{i=1}^{T-t}\sum_{j=1}^{Q_m} x_j^i)p_t x_t^K +$$

$$cu\sum_{t=0}^{T}(2R - \sum_{i=1}^{T-t}\sum_{j=1}^{Q_m} x_j^i - \sum_{i=1}^{M} x_i^w)p_t(1 - x_t^K) +$$

$$\sum_{t=1}^{T}(R - \sum_{i=1}^{t}\sum_{j=1}^{Q_m} x_j^i)th_w + \sum_{t=1}^{T}[h_f \sum_{D_t=0}^{Q_m}(\sum_{j=1}^{Q_m} x_j^t - D_t)p_{D_t}^t x_{D_t}^t$$

$$+p_f \sum_{D_t=0}^{Q_m}(D_t - \sum_{j=1}^{Q_m} x_j^t)p_{D_t}^t(1 - x_{D_t}^t)]$$

s. t. constraints (6) and (7),

$$x_t^K, x_i^w, x_i^t = 0, or\ 1, \forall i = 1, 2, ..., Q_m, \forall j = 1, 2, ..., M,$$

$$\forall t = 1, 2, ..., T;$$

$$x_{i+1}^w \leq x_i^w, \forall i = 1, 2, ..., M-1;$$

$$x_{i+1}^t \leq x_i^t, \forall t = 1, 2, ..., M-1, \forall i = 1, 2, ..., Q_m - 1;$$

$$x_{t+1}^K \leq x_t^K, \forall t = 1, 2, ..., L_m;$$

$$\sum_{t=1}^{T}\sum_{i=1}^{Q_m} x_i^t = R;$$

$$2R - \sum_{t=1}^{T}(\sum_{i=1}^{Q_m} x_i^t)x_t^{(T-K)} \leq \sum_{i=1}^{M} x_i^w;$$

$$\sum_{i=1}^{M} x_i^w < 2R - \sum_{t=1}^{T}(\sum_{i=1}^{Q_m} x_i^t)x_t^{(T-K-1)}.$$

This is a quadratic binary problem. In the following paragraph, we convert it to linear binary problem. When the two factors have the same subscription, such as, x_i^t and x_j^t, it is easy to see that $x_i^t \cdot x_j^t = x_i^t$, if $i \leq j$. When the two factors have different subscription, we need the following theorem.

Theorem x, y are binary variables, then $x \cdot y$ is equivalent to the a variable and a group of constraints: z, subject to $z \leq x$, $z \leq y$, $z \geq x + y - 1$, $z \geq 0$.
The proof is easy from the following table,

Table 1: Convert Multiplication to Linear Constraints

x	0	0	1	1
y	0	1	0	1
$x+y-1$	-1	0	0	1
$x \cdot y$	0	0	0	1
z	0	0	0	1

This theorem provides a general method to convert a quadratic binary programming to a linear programming. However, this conversion will increase the number of decision variables and constraints. So it is only suitable to small scale problems. Define

$$x_{it}^{wK} := x_i^w \cdot x_t^K,$$

$$x_{ijt}^{fK} := x_j^i \cdot x_t^K,$$

$$x_{tjD_t} := x_j^t \cdot x_{D_t}^t,$$

$$y_{ij} := x_i^{(K)} \cdot x_j^{(T-K)}$$

$$z_{ij} := x_i^{(K)} \cdot x_j^{(T-K-1)}$$

then,

$$Min.(co+cu)\sum_{t=0}^{T}\sum_{i=1}^{M} p_t x_{it}^{wK} - 2R(co+cu)\sum_{t=0}^{T} p_t x_t^K + (co+cu)$$

$$\sum_{t=0}^{T}\sum_{i=1}^{T-t}\sum_{j=1}^{Qm} p_t x_{ijt}^{fK} + 2Rcu - cu\sum_{t=0}^{T}\sum_{i=1}^{T-t}\sum_{j=1}^{Q_m} p_t x_j^i$$

$$-cu\sum_{t=0}^{T}\sum_{i=1}^{M} x_i^w p_t + R\sum_{t=1}^{T} th_w - \sum_{t=1}^{T}\sum_{i=1}^{t}\sum_{j=1}^{Q_m} th_w x_j^i +$$

$$(h_f+p_f)\sum_{t=1}^{T}\sum_{D_t=0}^{Q_m}\sum_{j=1}^{D_t} p_{D_t}^t x_{D_t}^t + (h_f+p_f)\sum_{t=1}^{T}\sum_{D_t=0}^{Q_m}\sum_{j=D_t+1}^{Q_m} p_{D_t}^t x_j^t$$

$$-(p_f+h_f)\sum_{t=1}^{T}\sum_{D_t=0}^{Q_m} D_t p^t_{D_t} x^t_{D_t} - p_f \sum_{t=1}^{T}\sum_{D_t=1}^{Q_m}\sum_{j=1}^{Q_m} p^t_{D_t} x^t_j$$

$$+p_f \sum_{t=1}^{T}\sum_{D_t=0}^{Q_m} D_t p^t_{D_t}$$

s. t. constraints (6) and (7),

$$x^w_{i+1} \le x^w_i, \forall i = 1,2,...,M-1;$$

$$x^t_{i+1} \le x^t_i, \forall i = 1,2,...,Q_m-1, and\ \ t = 0,1,2,...,T;$$

$$x^K_{i+1} \le x^K_i, \forall i = 1,2,...,T-1;$$

$$x^{wK}_{it} \ge x^w_i + x^K_t - 1, \forall i = 1,2,...,M, and\ \ t = 1,2,...,T;$$

$$x^{wK}_{it} \le x^w_i, \forall i = 1,2,...,M, and\ \ t = 1,2,...,T;$$

$$x^{wK}_{it} \le x^K_i, \forall i = 1,2,...,M, and\ \ t = 1,2,...,T;$$

$$x^{fK}_{ijt} \le x^K_t + x_{ij} - 1, \forall i = 1,2,...,T, j = 1,2,...,Q_m,$$

$$and\ \ t = 1,2,...,T;$$

$$x^{fK}_{ijt} \le x^K_t, \forall i = 1,2,...,T, j = 1,2,...,Q_m,$$

$$and\ \ t = 1,2,...,T;$$

$$x^{fK}_{ijt} \le x_{ij}, \forall i = 1,2,...,T, j = 1,2,...,Q_m,$$

$$and\ \ t = 1,2,...,T;$$

$$y_{ij} \le x^{(K)}_i + x^{(T-K)}_j - 1, \forall i = 1,2,...,T,$$

$$and \forall j = 1,2,...,T;$$

$$y_{ij} \le x^{(K)}_i, \forall i = 1,2,...,T, \forall j = 1,2,...,T;$$

$$y_{ij} \le x^{(T-K)}_j, \forall i = 1,2,...,T, \forall j = 1,2,...,T;$$

$$z_{ij} \le x^{(K)}_i + x^{(T-K-1)}_j - 1, \forall i = 1,2,...,T,$$

$$and \forall j = 1,2,...,T;$$

$$z_{ij} \le x^{(K)}_i, \forall i = 1,2,...,T, \forall j = 1,2,...,T;$$

$$z_{ij} \le x^{(T-K-1)}_j, \forall i = 1,2,...,T, \forall j = 1,2,...,T;$$

$$\sum_{t=1}^{T}\sum_{i=1}^{Q_m} x^t_i = R;$$

$$2R - \sum_{t=1}^{T}\sum_{j=1}^{Q_m} xz_{tj} \le \sum_{i=1}^{M} x^w_i < 2R - \sum_{t=1}^{T}\sum_{j=1}^{Q_m} xz_{tj}.$$

This is a linear binary problem with $Q_mT^2 + Q_m^2T + T^2 + TQ_m + TM + 3T + M$ varialbes and $3Q_mT^2 + 6T^2 + 3MT + Q_mT + M + T + 1$ constraints. Actually the problem could be much smaller because we can ignore many unreasonable values of decision variables if we have some historical data, so many x^t_i, xK_{it} etc. can be replaced by 1s directly. In our project, the warehouse replenishes the inventory about per twenty weeks with a fixed order quantity of $R = 200$, the factory does so per week with a delivery quanity of at most $Q_m = 20$. So the problem has 15960 binary decision variables and 45121 constraints. We use ILOG CPLEX 7.1(Intel mobile CPU1.6G, and 256M RAM) to solve this problem. After the proprocessing of CPLEX, there are 34422 rows and 11841 columns left and it spends about half an hour to get a solution within the 0.01% error of the optimal solution.

When the problem's scale becomes larger, we can get the solution iteratively. First we can split the domains of decision variables into several large intervals (just like in the above, we select 1 as the interval) and in the same method we can decide which interval should be selected to get the optimal solution. Then we can split the cutoff intervals into several smaller intervals with a proper number so that we can handle them, the same method is used for selection of these smaller intervals. Iterate these steps until we can shorten the intervals to 1. Then we can got the optimal solution.

4. A Numerical Example

pl=[0.3, 0.3, 0.2, 0.1, 0.05, 0.05, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

Table 2: Values of Parameters

co	cu	hw	hr	p
0.9	0.5	0.3	0.9	1

pd[t, 21 - t] = 0.6, pd[t - 1, 21 - t] = pd[t + 1, 21 - t] = 0.2,

We can get Qw=200, Cost=1488.32, and

Q=[20, 20, 20, 20, 20, 20, 20, 20, 20, 9, 9, 2, 0, 0, 0, 0, 0, 0, 0, 0]

3. Conclusions

In this project we met an unusual problem as common papers. We need to consider the warehouse capacity and the factory replenishment jointly. Due to the small scale and containerization of the problem, we build e a discrete model. A method of solving quadratic binary programming is designed.

4. References

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