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A Triangular Assessment Method

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Abstract
A Triangular Assessment Method (Research in progress, abbreviated to MTC, from the initials of the Spanish name: *Método del Triángulo de Calificaciones*) is described. The proposed method is an improvement of the well known Analytic Hierarchy Process (AHP), which is a pairwise comparison technique developed by Saaty and often applied in complex decision making. The MTC basically consists of comparing criteria and alternative levels of each criterion in trios instead of in pairs. This overcomes some of the drawbacks of the AHP, such as the large number of pairs that must be analyzed when there are numerous criteria and alternatives.

Keywords
Saaty, Multi-criterion, AHP, criteria, alternatives

1. Introduction
The aim of the proposed Triangular Assessment Method (abbreviated to MTC, from the initials of the Spanish name: *Método del Triángulo de Calificaciones*) is to decrease the large number of pairwise comparisons required in the AHP (Saaty 1980, 1990, Ishizaka & Labib 2009) for making decisions involving multiple criteria and alternative levels, as well as minimizing the cognitive skew that may be produced in the repetitive process involved in the comparisons (Pérez-Rodríguez & Rojo-Alboreca 2009). The latter aspect is essential as the first step in the decision-making process is to perceive the decision; if this perception is distorted the next decision will also be distorted, leading to a loss of accuracy in the final decision (Treisman & Kanwisher 1998).

2. Objectives
- To develop a logic-based method.
- To minimize cognitive skews (left-right, upwards-downwards).
- To include independently evaluated repetitions to prevent any effect due to memory of previous perceptions.
- To minimize the time involved in decision making.
- To increase the reliability of the results.

3. Description of the Triangular Assessment Method (MTC)

Comparison of two criteria, e.g. criteria 1 and 2, provides an estimate of the magnitude of the importance of one criterion relative to the other. The same applies to comparison of criterion 2 and another criterion 3. Repetition of the process and quantification of the relation between criteria 1 and 3 would complete the triad of pairwise comparisons for criteria 1-2-3. However, it is possible that the quantification will not be accurate as under- and overestimations may be made in the relative evaluation of some of the initial relationships, thus producing an illogical result (see Figure 1). Thus, for example, if criterion 1 is assigned a greater weight than criterion 2, and criterion 2 than criterion 3, then logically criterion 3 cannot have a greater weight than criterion 1. However, this could occur with the pairwise assessment method, so that the relationship between criterion 3 and criterion 1 could have been obtained directly, without having to consider it specifically. Nonetheless, it is difficult to determine the weight of each criterion in the AHP, as it is not known whether the relationship between each pair has been over or underestimated, so that the relationship cannot be established as the associated error would be unacceptably large.

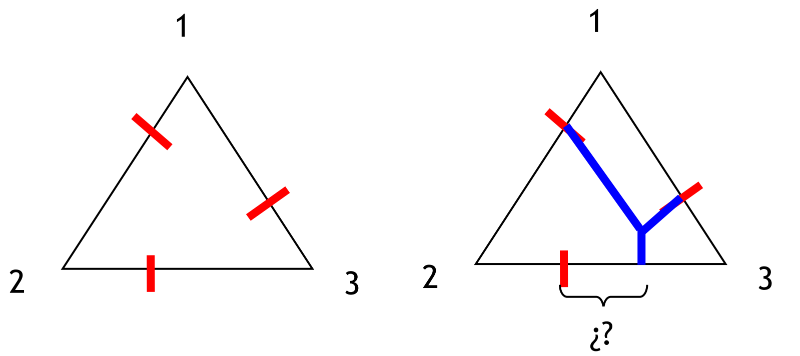

Figure 1: Diagram illustrating the possible error involved in paired assessment of 3 criteria by the AHP projected on a random side.

This problem can be resolved by comparing the criteria directly in triads rather than in pairs. This is the basis of the MTC (Pérez-Rodríguez & Rojo-Alboreca 2009). Thus, if we are obliged to determine the magnitude of the relationship between each of the three pairs simultaneously, i.e. of the relationships 1-2, 2-3 and 3-1, the evaluation will be more complex because the decision maker has to compare three criteria at same time, but it can be assumed that the inconsistency in this triad will be zero. These relationships may be assimilated geometrically into an equilateral triangle, in which each vertex represents a criterion. In the MTC, the magnitude of the relative weights of the pairs that form the sides in the triangle can be calculated by projecting lines perpendicular to the sides of the triangle from a point within the triangle (independently of discrete scales).

When the number of criteria is greater than 3, a series of triangles is developed. For this purpose, considering a set of criteria C_1, C_2, C_3,… C_n, the series of comparisons for a triangle with vertices C_a, C_b and C_c is defined by the subindices *a*, *b* and *c*, which are governed by the following series of *n* triangles of comparison for *n* criteria:

$$
\begin{cases}
a = i \in [1, n] \in \mathrm{N} \\
b = \begin{cases} i+1 > n \Rightarrow b = (i+1) - n \\ i+1 \le n \Rightarrow b = i+1 \end{cases} \\
c = \begin{cases} i+2 > n \Rightarrow c = (i+2) - n \\ i+2 \le n \Rightarrow c = i+2 \end{cases}
\end{cases}
$$

For example, assuming 8 criteria (*n* = 8), the following triangles are formed: 1-2-3; 2-3-4, 3-4-5, 4-5-6, 5-6-7, 6-7-8, 7-8-1 and 8-1-2. As can be observed, 8 triads of criteria are obtained. Much less time is therefore required for the decision making process, as 8 triangles are evaluated instead of 28 pairs. This frees up more time for each qualification so that the final decision will be more reliable.

In the MTC, consideration of the pairs in the 8 triangles provides the results of the comparison of 8·3=24 pairs, so that not all of the relations required to adapt this system to a square matrix of relations are obtained. However, some pairs are repeated and the repetition is produced at random on different sides of the triangle (the position of the pair being compared changes), thus minimizing any effect of the visual memory of the anterior pair, so that the new repetition will be independent of the previous comparison. Taking this into account, we can easily obtain the errors associated with the relationships considered only once, as these are within the triangles whose sides are formed by duplicated relationships. To obtain the relationships between the pairs that have not been evaluated, a triangle is formed, with the two known relationships (which have been obtained in duplicate because they have been repeated) forming two of the sides and the unknown relationship forming the third side. This procedure also reveals the errors associated with the non-evaluated pairs directly, as the same relationship can be obtained with different triangles depending on which sides are chosen. The procedure for calculating the errors is illustrated in Figure 2.

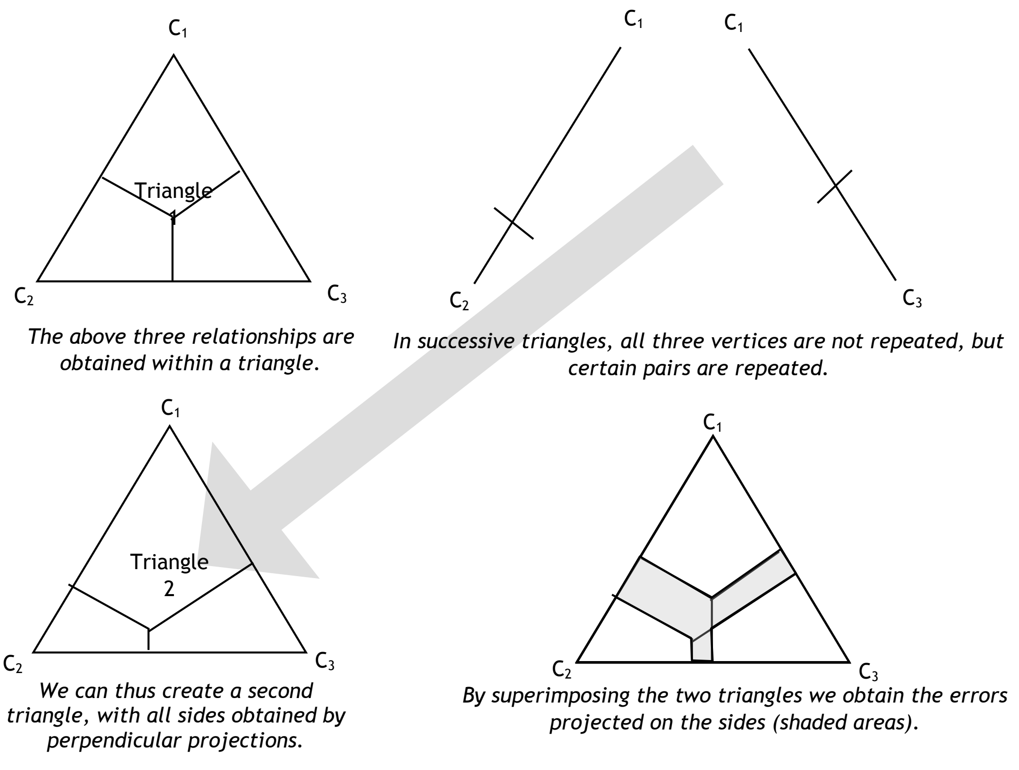

Figure 2: Procedure for obtaining the errors by the MTC, whereby only two pairs of criteria are repeated, and the other pair is estimated by constructing another triangle from the first two pairs.

Therefore, after a more or less laborious procedure for estimating values and errors, all of the relationships are obtained, enabling construction of a matrix and therefore establishing the hierarchies, so that the same procedure for calculating the relative importance of the criteria as used in the AHP can then be applied.

It is essential to take into account that the complicated post perception process may be involved to a certain extent in this process, as the weight of one pair may vary according to the personal perception of the person making the decision; the weight of an element in a pair may differ depending on the other vertex with which it is compared, as this will affect and indeed determine the subsequent perception. It is very difficult to evaluate the importance of a pair in the AHP or a triangle in the MTC, and that the decision-maker evaluates the pair or triad without being conditioned by previous decisions (Luck & Hollingworth 2003). It is useful to know the range of variation of each criterion to enable a sensitivity analysis to be carried out, as the weight of each criterion within the logical limits that mark the range of uncertainty may vary.

Analysis of the uncertainty associated with criteria, based on the repetition of pairs within the triangle (in which one of the vertices is changed and thus the context of the decision is altered), enables the magnitude of the reliability of the pairs to be determined.

If the limits of uncertainty are known, the criteria used in making the decision can be changed, modified or divided to achieve the stated goal. This is very important as the number of criteria greatly affects the time involved in making the posterior decision.

Once the weights of the criteria have been established, the next step is to evaluate the alternative levels considered in the decision making process, under the criteria analyzed. In this step, the process will be exactly the same with the proposed method, and a range of variation will also be obtained, but this time for each alternative level of each criterion. Two variables are thus obtained as a result of the procedure followed in the MTC: the range of weight of the criteria and the respective mean value, and the range of weight of each alternative level of each criterion, also with their respective mean values. These two variables can be plotted against each other on different axes, to produce a rectangle, as shown in Figure 3.

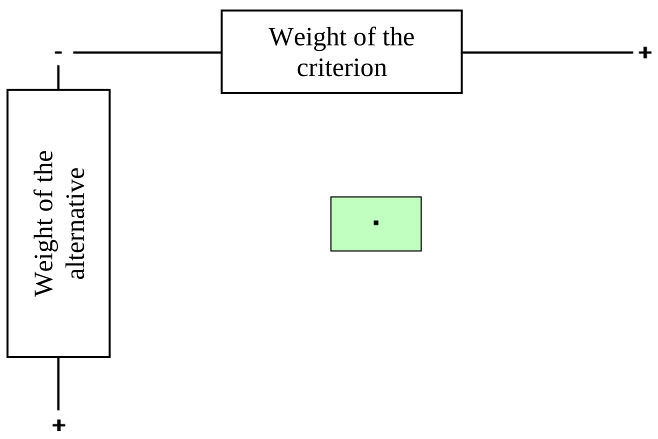

Figure 3: Diagram illustrating the result of the decision making process with the MTC. This consists of a rectangle the sides of which represent the range of weighting of each criterion and of the alternative.

This rectangle represents and shows the entire area in which the weight of the alternative under the criterion considered may vary as a result of the repetitions carried out. In successive repetitions, we will obtain more than one rectangle, which if superimposed will produce more than one delimited area, of different surface area and in different positions. These rectangles thus provide information about how the repetitions affect the decision over time. In the case of group decisions the degree of uniformity can be evaluated.

Finally, to facilitate the application of methodology, we are developing software that you can see the interface in the Figure 4.

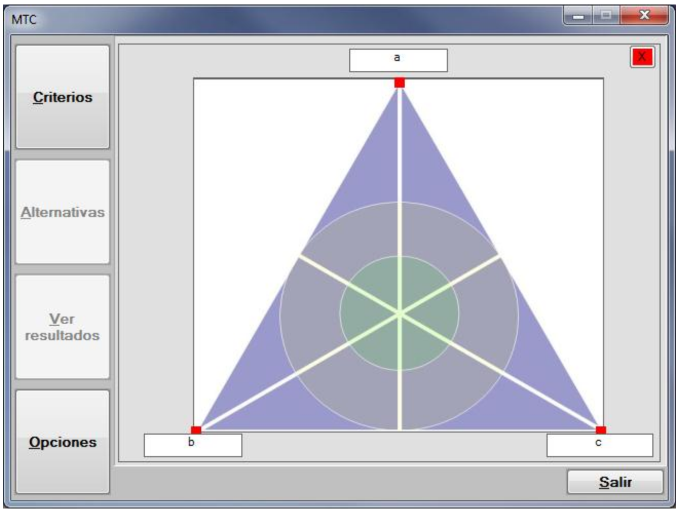

Figure 4: This figure showed the MTC interface. In it you can see the triangle area that it is compose of three criteria (a, b and c), that the software shows you automatically.

4. Preliminary conclusions

- With the MTC, the number of decisions to be taken for the triads is much less than the number of pairs that must be assessed in the AHP, especially when there are numerous criteria and alternatives.
- In addition to the value or weight of each alternative, the MTC also reveals the range of uncertainty associated with each, which enables determination of the degree of validity of the decision to be made.
- The variation in the weight of the criteria or of the alternative levels can also be determined with the MTC by changing the context in which the assessment is made (i.e. by changing one of the vertices of the triangle).

5. Current lines of research

- Analysis of the increased/decreased concentration of the person making the decision, with the aim of decreasing the influence of the environment and minimizing the cognitive skew that may be produced in the repetitive process of the making the decision.
- Analysis of the prior evaluation capacity of the criteria for selection of those that should be maintained for the decision making process, thus eliminating the least relevant criteria (i.e. those with a greater range of uncertainty). Also addition and/or elimination of criteria.
- Creation of three dimensional logic maps with successive repetitions.

6. Acknowledgements

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References

Ishizaka, A. and A. Labib (2009) "*Analytic Hierarchy Process and Expert Choice: Benefits and Limitations*", UK: ORInsight, 22(4), pp. 201–220.

Luck, S.J. and A. Hollingworth (eds.) (2003) *Visual memory*, USA: Oxford University Press, *Oxford series in visual cognition*, 338 pp.

Pérez-Rodríguez, F. and A. Rojo-Alboreca (2009) "*El Método del Triángulo de Calificaciones (MTC): una alternativa para la toma de decisiones dentro del método AHP*". 5º Congreso Forestal Español. Ávila (España), 21-25 setember 2009. (Ref.: 5CFE01-566), pp. 1-10 [In Spanish].

Saaty, T.L. (1980) *The Analytic Hierarchy Process*, New York: McGraw-Hill, 287 pp.

Saaty, T.L. (1990) *Decision making for Leaders. The Analiytic Hierarchy Process for decision in a complex World*, Pittsburgh: University of Pittsburgh, RWS Publications, 292 pp.

Treisman A.M. and N.G. Kanwisher (1998) "*Perceiving visually presented objects: recognition, awareness and modularity*", USA: Current Opinion in Neurobiology 8, pp. 218-226.