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ABSTRACT

This research provides a novel method for discovering group-level differences on endogenous variables in a multilevel structural equation modeling context. Furthermore, methods for calculating associated significance values of these group-level differences is described. This builds on current techniques for discovering group-level differences in multilevel regression models by extending this capability to full multilevel structural equation models. The included analysis provides a verification mechanism for the proposed method in a multilevel regression context with other current software. This provides verification that the method can then be extended to multilevel structural equation models.

Keywords

multilevel structural equation modeling, multilevel regression, random effects, group differences

INTRODUCTION

Multilevel modeling (also referred to as multilevel analysis, hierarchical linear modeling, or multilevel regression) is a type of analysis which allows for a focus on nested sources of variability in data (Snijders & Bosker, 1999). Multilevel modeling provides a statistical method that can analyze nested (e.g. group) data by allowing the researcher to analyze individual-level, group-level, and cross-level effects simultaneously in the same model (Hofmann, 1997; Raudenbush & Bryk, 1992). Many areas are naturally suited for multilevel analysis including sociological issues such as individuals within neighborhoods (Leventhal & Brooks-Gunn, 2000), family issues of members within households (Teachman & Crowder, 2002), psychological issues such as individual depression within states (Chen, Subramanian, Acevedo-Garcia, & Kawachi, 2005), educational issues such as students within classrooms and/or schools (Koth, Bradshaw, & Leaf, 2008), and organizational issues such as members within teams (Short, Piccoli, Powell, & Ives, 2005) just to name a few.

Traditional multilevel modeling builds on conventional regression analysis to allow for estimation of effects at multiple levels of a regression model. While this adds a much needed tool for multilevel analysis, the use of regression within a multilevel context is limiting for researchers. Structural equation modeling (SEM) builds on traditional regression by allowing for the simultaneous estimation of both a measurement factor-analytic model and a structural model (Gefen, Straub, & Boudreau, 2000). Recently multilevel SEM (MLSEM) has emerged as a viable technique for combining the advantages of multilevel modeling with that of SEM (B. Muthén & Asparouhov, 2011).

MLSEM provides tremendous opportunities for statistical analyses, but MLSEM software tools have still not added some key functionality which is available in software packages for multilevel regression analyses. One such feature is the ability to calculate group deviations, and associated significance values, on endogenous dependent variables from the overarching grand mean. This type of post-hoc analysis allows the researcher to identify the specific groups in a multilevel analysis which significantly differ on a dependent variable, which allows for a fuller more nuanced picture. This research describes novel methods for calculating such group deviational effects in a MLSEM context using the popular Mplus software package (L. Muthén, Muthén, Asparouhov, & Nguyen, 2011). Given that these methods are new and not provided as a built-in option in the software, a multilevel regression model in Mplus will be compared with SAS, which provides such estimates as an option. This comparison provides validation for this technique which allows researchers to extend such techniques to a multilevel SEM context within Mplus.

ESTIMATION

Recent advancements in software functionality have allowed for the estimation of multilevel SEM models. The software allows the user to specify two-level models, as in traditional multilevel regression models, while also allowing for concurrent measurement and structural model estimation, as in SEM. This allows the user to identify random intercepts between groups, random slopes, the impact of group level covariates, cross-level interactions, etc. This has provided a much needed step forward in the researcher's methodological toolkit and provides for a much richer analysis on these types of datasets.

Traditional multilevel regression techniques have also allowed for the estimation of post-hoc tests of individual group deviations from the overall grand mean of all groups on the dependent variable. For example, when looking at students within schools using a dependent variable of math achievement, these post-hoc analyses allow us to see beyond the fact that schools may differ on average student math achievement, but also to see how each specific school differs in their average student math score from the overall grand mean of math scores. Furthermore, software packages allow the researcher to identify which of these differing schools significantly deviate from this grand mean, providing a method for identifying those schools that may be significantly lower on average student math score. This potentially allows for greater analysis as to why the school is significantly lower on average math achievement and ideally devise a plan for increasing the scores of the students in this school. Additionally, those schools whose students score significantly higher on math achievement can also be further analyzed to help identify helpful tips for other schools.

Some multiple regression software packages offer the ability to perform the above post-hoc tests. One popular package, SAS, provides the solution in a section titled "Solution for Random Effects." This solution provides the beta estimation of the group difference for each group, as well as the associated standard error, t-statistic, and p-value. To date, MLSEM software does not offer an option for specifying that these group differences and associated t-tests be calculated. Also, no method has yet been devised to utilize these MLSEM software packages to calculate these deviations of group means from the grand mean with associated significance values.[1]

This research devises a method for estimating group differences and associated significance values in a MLSEM context using the Mplus software package. Since this is the first attempt at such calculations, the methods used to calculate such differences need to be verified before they can be trusted fully. The SAS PROC MIXED procedure has been used extensively for multilevel regression models by researchers, and this method offers the ability to evaluate group differences in this multilevel regression context. Mplus is a software package which is capable of estimating MLSEMs, but no research to date has utilized this software to find group mean differences and associated significance values. One advantage to Mplus is that it is also capable of estimating multilevel regression models using the same basic syntactical approaches as it uses to estimate its multilevel structural equation models. Therefore, to help verify that Mplus is correctly estimating group differences in a MLSEM using our proposed method, its results in a multilevel regression model can be compared to that of SAS. If these numbers align, this can offer some verification that the same methods can be extended to an MLSEM context.

Model Description

To test the two software packages, the same dataset was utilized for both packages. This data consists of 309 high school students nested within 40 high schools. The research utilizes Lent's Social Cognitive Career Theory (SCCT) (Lent, 2005) to aid in the prediction of a student's choice to major in IT. For estimation of the multilevel regression model (to allow SAS's PROC MIXED procedure to verify the results from Mplus) the items for each of the student-level independent variables were averaged to create a single variable. School size was also utilized as a school-level covariate and the dependent variable of choice to major consists of a single item. The model and proposed hypotheses are displayed in Figure 1. Since this research is concerned with methodological issues, we will not fully detail the SCCT model or its underlying relationships. The reader is referred to (Lent, 2005; Lent, Brown, & Hackett, 1994) for a review of the SCCT model.

[1] As an example, if SAS does not include a specific option for some calculation, users have been known to develop their own solutions using SAS macros to estimate the item of interest. In regard to calculating group-level differences and corresponding significance tests, Mplus does not have a specific option for calculating these estimates as does SAS PROC MIXED, and furthermore, no one has yet devised a "homebrewed" method, as with a SAS macro, for filling in this functionality.

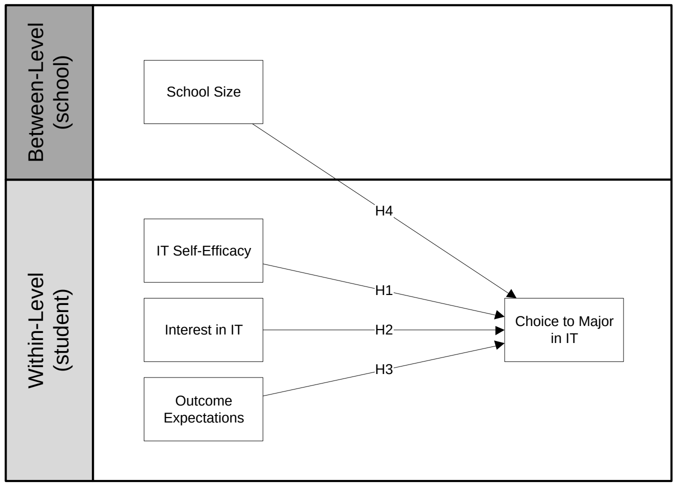

Figure 1. Hypothesized research model

Syntax Setup for SAS and Mplus

In order to compare model output between SAS and Mplus, we first need to estimate each model. This section will describe each of the syntax files used to estimate the SCCT model.

Figure 2 shows the SAS syntax for running a multilevel SCCT model.

```
DATA mydata;
       INFILE "C:\fileName.dat";
       INPUT  Interest3_1 Interest5_1 Interest7_1 Interest9_1 Interest11_1
              ITSE1_1 ITSE2_1 ITSE3_1 ITSE4_1 ITSE5_1 ITSE6_1 ITSE7_1 ITSE8_1 ITSE9_1
              Intent3_1
              Career1_1 Career2_1 Career3_1 Career4_1
              school_number school_size;
RUN;

DATA mydata1;
    SET mydata;
       ITSE = MEAN(ITSE1_1,ITSE2_1,ITSE3_1,ITSE4_1,ITSE5_1,ITSE6_1,ITSE7_1,ITSE8_1,ITSE9_1);
       Interest = MEAN(Interest3_1,Interest5_1,Interest7_1,Interest9_1,Interest11_1);
       Career = MEAN(Career1_1,Career2_1,Career3_1,Career4_1);
RUN;

PROC SQL;
    CREATE TABLE grpmeanctr AS
    SELECT school_number, ITSE - MEAN(ITSE) AS grpcITSE, Interest - MEAN(Interest) AS
       grpcInterest, Career - MEAN(Career) AS grpcCareer, Intent3_1, school_size
    FROM mydata1
    GROUP BY school_number;

    CREATE TABLE grpANDgrandmeanctr AS
    SELECT *, school_size - MEAN(school_size) AS grdcschool_size
    FROM grpmeanctr;
QUIT;

PROC MIXED DATA=grpANDgrandmeanctr METHOD=ML COVTEST;
    CLASS school_number;
    MODEL Intent3_1 = grpcITSE grpcInterest grpcCareer grdcschool_size /SOLUTION DDFM=BW;
    RANDOM INTERCEPT / SOLUTION SUBJECT=school_number TYPE=UN;
RUN;
```

Figure 2. SAS syntax for multilevel regression model of SCCT

The first DATA statement above is used to bring in the data from the associated file which contains the data needed for the analysis. This method uses a fixed ASCII file (the same exact file will be used as input to Mplus), but the user can use other methods such as a SAS file or importing an SPSS file. The second DATA statement is used to compute each of the student-level independent variables. Given that this is a regression analysis, we must compute one observed variable by taking the average of each of the items which will be used to compute each of the three independent variables. By taking the average, this also allows for centering of variables, which is required when running this type of multilevel analysis. The PROC SQL statement is used to first center each of the student-level independent variables within their associated school group (also referred to as centering within context) using the first CREATE TABLE statement. The second CREATE TABLE statement then uses the table created by the first CREATE TABLE statement and centers the school-level variable of school_size based on the grand mean of all school sizes. An in-depth discussion of SQL is beyond the scope of this manuscript, but this or other methods should be used to center the student-level variables within school and the school-level variables across schools before performing the multilevel analysis.

The PROC MIXED statement is used to run the actual multilevel regression analysis. The DATA statement tells SAS which dataset to use, while the METHOD informs SAS to use a maximum likelihood (ML) estimation method and COVTEST tells the program to run a significance test of the student-level and school-level covariance estimates. Restricted maximum likelihood (REML) is typically used as the default estimation method, but ML must be used to compare the SAS output with Mplus as Mplus does not offer a REML option. Also, ML is the default method used for full SEM models, so this will facilitate the move from regression-based multilevel modeling to SEM-based multilevel modeling in the future.

Next the actual model to be estimated is specified. The CLASS statement tells the program which variable will be used to group the observations. Given that this is school data, the associated school_number will be used to group student-level variables. Next, the model statement tells the program to regress the dependent variable Intent3_1 (Intent to Major in IT) on the student-level independent variables of grpcITSE (group-centered IT Self-Efficacy), grpcInterest (group-centered Interest in IT), and grpcCareer (group-centered Outcome Expectations), as well as the school-level covariate of grdcschool_size (grand mean-centered school size). The /SOLUTION option tells the program to give estimates and associated significance values for each of the independent variables in the model and DDFM=BW tells SAS to use the between/within method for computing the denominator degrees of freedom for fixed effect hypothesis tests. Next, the RANDOM statement tells the model to estimate random intercepts (for each school) and also to give estimates and associated significance values for each of these random intercepts. By adding this SOLUTION statement, SAS will provide the solution for random effects that we need to see the difference between schools on average student Intent to Major in IT. Finally, the SUBJECT parameter tells SAS which variable will identify the subjects in this analysis (given this is a multilevel analysis, the school will actually be considered the subject level) while the TYPE parameter tells SAS to assume a completely generalized covariance where no restrictions are assumed.

Figure 3 shows the Mplus syntax for running the same multilevel regression SCCT model.

```
TITLE:
    SCCT MLM;

DATA:
    FILE IS filename.dat;

VARIABLE:
    NAMES ARE ITSE1 ITSE2 ITSE3 ITSE4 ITSE5 ITSE6 ITSE7 ITSE8 ITSE9
        Int1 Int2 Int3 Int4 Int5
        Career1 Career2 Career3 Career4
        Intent3
        sch_num sch_size;

    USEVARIABLES = Intent3 sch_size ITSE Interest Career;

    WITHIN = ITSE Interest Career;
    BETWEEN = sch_size;
    CLUSTER = sch_num;
    CENTERING = GROUPMEAN(ITSE Interest Career)
        GRANDMEAN(sch_size);

ANALYSIS:
    TYPE = TWOLEVEL;
    ESTIMATOR = ML;

DEFINE:
    ITSE = MEAN(ITSE1 ITSE2 ITSE3 ITSE4 ITSE5 ITSE6 ITSE7 ITSE8 ITSE9);
    Interest = MEAN(Int1 Int2 Int3 Int4 Int5);
    Career = MEAN(Career1 Career2 Career3 Career4);
```

```
MODEL:
    %WITHIN%
    Intent3 ON ITSE Interest Career;

    %BETWEEN%
    Intent3 ON sch_size;

    f BY;
    Intent3 ON f@1;
    Intent3@0;

OUTPUT:
    SAMP STANDARDIZED;

SAVEDATA:
    FILE = mlm_output.txt;
    SAVE = FS;
```

Figure 3. Mplus syntax for multilevel regression model of SCCT

Mplus syntax is broken up into separate sections. The TITLE section provides a place for providing a descriptive title of the analysis being performed. The DATA section allows you to specify the file to be used employing the FILE IS statement. The file used here is a fixed format ASCII file without any column identifiers in the file itself (i.e. only numbers). Next, the VARIABLE section allows you to identify the variables in both the file and those to be used in the analysis. The NAMES ARE statement provides a listing of the various data columns present in the data file. For this analysis, only the variables used in the analysis were included in the data file, but if the data file includes more variables than are used in the analysis, the USEVARIABLES statement allows for the specification of which variables from the file will actually be used in the analysis. Notice that since we will be combining individual variables into their respective constructs, we only specify the combined variables as those which will be used in the analysis. The WITHIN and BETWEEN statements identify which variables used in the analysis are at the within level (i.e. student level) and which are at the between level (i.e. school level), while the CLUSTERING statement identifies which variable will be used to identify groups (i.e. schools) within the data. The CENTERING option allows the identification of which variables should be grand-mean centered and which should be group-mean centered, which is an improvement over the necessary SQL steps needed in SAS. The ANALYSIS section allows for the identification of the types of analysis to be used. Here we are using a TWOLEVEL analysis with ML as the estimation technique. The DEFINE section allows for the computation of other variables to be used in the model. Given that this is a regression model, we have defined combined measures for each student-level independent variable using the average of the constituent items for each.

The MODEL section is used to define the model which will be estimated by Mplus. When estimating a multilevel model, the MODEL section contains two subsections, the %WITHIN% (i.e. student-level) subsection and the %BETWEEN% (i.e. school-level) subsection. The within subsection of our model is used to specify the primary regression model. For this research we regress Intent to Major in IT on IT Self-Efficacy, Interest in IT, and Outcome Expectations. The between subsection specifies the added regression of Intent to Major on school size.

The final portion of the between section of the model is a novel method we developed as a way of estimating the group differences that mirror the solution for random effects estimated by SAS. Mplus does not have a method for easily estimating these parameters, but our developed statements provide a method for doing so. First, we create a latent variable f which has no indicators. We set the variance of this latent variable to 1 and then regress Intent to Major on this latent variable. Finally, we set the variance of Intent to Major at 0. What this does is move the residual variance term of Intent to Major to the latent f variable, which will be manifested as the estimate ascribed to the latent variable. By having a separate variable f to hold the residual of the dependent variable, our analysis will be able to provide an estimate and associated standard error for this variable across groups. We then use these estimates and associated standard errors to derive an estimation of a t-statistic for each group residual on Intent to Major as well as an associated significance value using the output from the SAVEDATA command (described below). The OUTPUT subsection specifies that we would like to get sample and standardized statistics related to the fixed effects in the model.

The SAVEDATA command allows for a separate data file apart from the standard Mplus output to be saved along with the analysis. The user can choose where the file should be saved as well as what items should be saved in the file beyond the standard variable values. We have specified that factor scores (FS) should be saved as these will provide the deviational estimates for endogenous variables from the grand mean on that endogenous variable and associated standard errors. The combination of the added f commands in the model statement as well as the FS option in the SAVEDATA section provides the ability to estimate group differences and significance values described below.

Model Statistics Output Comparison between SAS and Mplus

The model above was estimated using both SAS PROC MIXED and Mplus. Before discussing the group differences, we will first compare standard output from SAS and Mplus to help verify that these two software packages are arriving at the same results. First, looking at the basic descriptors about the models, we see that both software packages are showing 309 observations at the student level and 40 observations at the group level (referred to as subjects and clusters in SAS and Mplus, respectively). Also notice that both models have the same number of independent variables (SAS counts the intercept in its "Columns in X," so it is one greater than Mplus).

Dimensions	
Covariance Parameters	2
Columns in X	5
Columns in Z Per Subject	1
Subjects	40
Max Obs Per Subject	24

Number of Observations	
Number of Observations Read	309
Number of Observations Used	309
Number of Observations Not Used	0

Table 1. SAS basic model descriptors.

```
Number of groups                                                 1
Number of observations                                         309

Number of dependent variables                                    1
Number of independent variables                                  4
Number of continuous latent variables                            1

.....

SUMMARY OF DATA

     Number of clusters                         40

     Average cluster size                    7.725
```

Table 2. Mplus basic model descriptors.

SAS offers very few fit statistics, but it does offer some focused on the information criteria method.

Fit Statistics	
-2 Log Likelihood	1137.6
AIC (smaller is better)	1151.6
AICC (smaller is better)	1151.9
BIC (smaller is better)	1163.4

Table 3. SAS model fit statistics.

By comparison, Mplus offers a much larger number and variety and of fit statistics. This will become important when Mplus is used in a MLSEM context, but here we will just focus on those fit statistics that align with SAS.

```
TESTS OF MODEL FIT

Chi-Square Test of Model Fit

          Value                              0.000
```

```
          Degrees of Freedom                     0
          P-Value                           1.0000

Chi-Square Test of Model Fit for the Baseline Model

          Value                             76.652
          Degrees of Freedom                     4
          P-Value                           0.0000

CFI/TLI

          CFI                                1.000
          TLI                                1.000

Loglikelihood

          H0 Value                        -568.777
          H1 Value                        -568.777

Information Criteria

          Number of Free Parameters              7
          Akaike (AIC)                    1151.554
          Bayesian (BIC)                  1177.687
          Sample-Size Adjusted BIC        1155.486
            (n* = (n + 2) / 24)

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.000

SRMR (Standardized Root Mean Square Residual)

          Value for Within                   0.000
          Value for Between                  0.000
```

Table 4. Mplus model fit statistics.

First, as you can see, the AIC measure aligns perfectly with SAS at 1151.6. Mplus provides a Log Likelihood value (-568.777) whereas SAS offers a -2 Log Likelihood value (1137.6), but if you take the value provided by Mplus times -2, this value matches up with SAS perfectly (568.777 * 2 = 1137.6). The only other value displayed by both SAS and Mplus is the BIC value. As you can see, the values are similar, but not the same (1163.4 vs. 1177.687). This is a notable difference which we will discuss further in the Discussion section.

Next, we look at the statistical output for both programs including both covariance parameters and fixed effects.

Covariance Parameter Estimates					
Cov Parm	**Subject**	**Estimate**	**Standard Error**	**Z Value**	**Pr > Z**
UN(1,1)	school_number	0.3036	0.1391	2.18	0.0146
Residual		2.1354	0.1814	11.77	<.0001

Table 5. SAS covariance parameter estimates.

Solution for Fixed Effects					
Effect	**Estimate**	**Standard Error**	**DF**	**t Value**	**Pr > |t|**
Intercept	4.9953	0.1314	38	38.02	<.0001
grpcITSE	0.09415	0.07760	266	1.21	0.2261
grpcInterest	0.5176	0.09836	266	5.26	<.0001
grpcCareer	0.3268	0.1089	266	3.00	0.0030
grdcschool_size	0.000641	0.000257	38	2.50	0.0169

Table 6. SAS model fixed effects.

```
MODEL RESULTS

                                                        Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Within Level

 INTENT3    ON
    ITSE               0.094      0.078      1.213      0.225
    INTEREST           0.518      0.098      5.262      0.000
    CAREER             0.327      0.109      3.001      0.003

 Residual Variances
    INTENT3            2.135      0.181     11.773      0.000

Between Level

 INTENT3    ON
    F                  1.000      0.000    999.000    999.000

 INTENT3    ON
    SCH_SIZE           0.001      0.000      2.489      0.013

 Intercepts
    INTENT3            4.995      0.132     37.769      0.000

 Variances
    F                  0.304      0.139      2.181      0.029

 Residual Variances
    INTENT3            0.000      0.000    999.000    999.000
```

Table 7. Mplus model statistics (including covariance parameters and fixed effects).

The first thing that becomes apparent is the difference in placement of several of the output measures between the two programs, specifically the covariance parameters. The group-level covariance parameter (also referred to as τ_{11}) in the SAS output is located in Mplus under the Variances section for F (τ_{11} = 0.304) while the individual-level covariance parameter (also referred to as σ^2) is located in Mplus under the Residual Variances section for INTENT3 (σ^2 = 2.135). Conversely, the fixed effects are fairly easy to identify in the output between the two models with the most notable difference being that Mplus provides a separate Between Level section for group-level variables. There are two differences to note before we proceed. First, is the inclusion in the Mplus Between Level effects of the variable F. This is used to estimate the post-hoc group differences in Mplus. Given that Mplus does not offer a specific option for estimating these between group differences, we had to add separate syntax to allow for calculation of these group-level differences (described above). Secondly, the t-values and associated significance values slightly differ between the two programs for those values analyzed at the between level in the Mplus output (i.e. τ_{11}, Intercept, and SCH_SIZE) while the estimates match up exactly. This slight difference is due to the default maximum likelihood estimation algorithms and the associated method for analyzing group differences with regard to variances for each. This will be discussed in more detail in the Discussion section.

One additional piece of information offered by Mplus is the R^2 value for the dependent variable of choice to major in IT.

```
R-SQUARE

Within Level

    Observed                                            Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value

    INTENT3            0.207      0.041      5.017      0.000
```

Table 8. Mplus R-squared output

While you can use either the Raudenbush and Bryk method (1992) or the Snijders and Bosker method (1999) for estimating R2 values for a multilevel model, Mplus offers this as a standard portion of the output.

After analyzing the above model using both SAS and Mplus we find that both software products estimate the multilevel regression model producing the same output in each. Figure 4 shows a graphical display of the model findings with the associated estimates and significance values which were found in both SAS and Mplus.

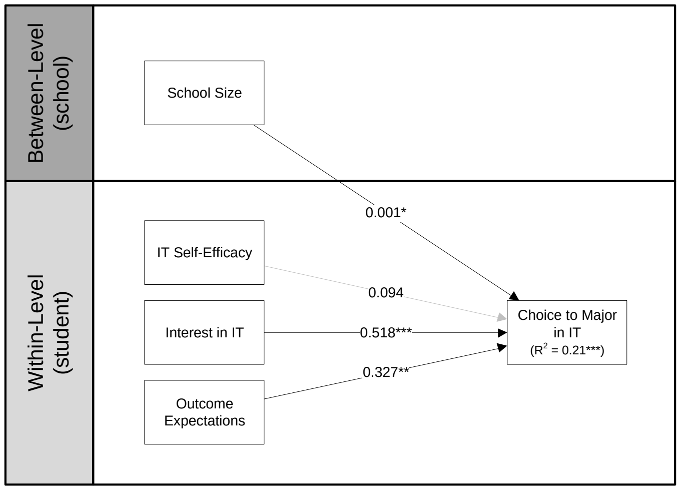

Figure 4. Multilevel regression model (hierarchical linear model) with estimates (*p < 0.05, **p < 0.01, ***p < 0.001).

Model Post-Hoc Group Differences Calculation and Comparison Between SAS and Mplus

The ability to estimate group-level deviations on a dependent variable from the overall group grand mean is of great usefulness for researchers in identifying those groups who may need further analysis and possible implementation of change programs. SAS offers this output in a table titled Solution for Random Effects which is provided by adding the SOLUTION parameter to the RANDOM portion of the PROC MIXED command. Table 9 shows the output from this command

Solution for Random Effects						
Effect	**school_number**	**Estimate**	**Std Err Pred**	**DF**	**t Value**	**Pr > |t|**
Intercept	19	0.03786	0.3589	304	0.11	0.9161
Intercept	20	0.3184	0.3116	304	1.02	0.3077
Intercept	22	-0.6024	0.4082	304	-1.48	0.1410
Intercept	24	-0.01115	0.4293	304	-0.03	0.9793
Intercept	26	-0.5339	0.3682	304	-1.45	0.1481
Intercept	29	0.9487	0.3474	304	2.73	0.0067
Intercept	30	0.7926	0.4432	304	1.79	0.0747
Intercept	33	0.4836	0.4643	304	1.04	0.2985
Intercept	36	0.1869	0.4268	304	0.44	0.6618
Intercept	37	0.1939	0.3864	304	0.50	0.6162
Intercept	38	-0.07913	0.3129	304	-0.25	0.8005
Intercept	40	-0.2152	0.3032	304	-0.71	0.4784
Intercept	44	-0.03580	0.4024	304	-0.09	0.9292
Intercept	46	0.3194	0.3732	304	0.86	0.3927
Intercept	56	-0.9565	0.3674	304	-2.60	0.0097
Intercept	65	0.1786	0.4878	304	0.37	0.7145
Intercept	94	0.2275	0.3031	304	0.75	0.4536
Intercept	96	-0.04197	0.4910	304	-0.09	0.9319
Intercept	97	-0.5456	0.3006	304	-1.82	0.0705
Intercept	100	0.07200	0.3633	304	0.20	0.8430
Intercept	101	-0.01016	0.3990	304	-0.03	0.9797
Intercept	104	0.04109	0.3515	304	0.12	0.9070
Intercept	107	0.1000	0.5159	304	0.19	0.8464
Intercept	109	-0.7405	0.3029	304	-2.44	0.0151
Intercept	111	-0.01218	0.4101	304	-0.03	0.9763

Intercept	114	-0.1920	0.4872	304	-0.39	0.6938
Intercept	115	0.04750	0.5168	304	0.09	0.9268
Intercept	116	-0.1384	0.4428	304	-0.31	0.7548
Intercept	117	-0.3646	0.4438	304	-0.82	0.4120
Intercept	118	0.1830	0.3978	304	0.46	0.6459
Intercept	140	-0.07716	0.4872	304	-0.16	0.8743
Intercept	141	0.1664	0.4875	304	0.34	0.7331
Intercept	142	0.3368	0.4261	304	0.79	0.4299
Intercept	144	0.2583	0.4446	304	0.58	0.5618
Intercept	152	0.1005	0.5159	304	0.19	0.8457
Intercept	157	-0.08113	0.5161	304	-0.16	0.8752
Intercept	158	0.1843	0.4879	304	0.38	0.7059
Intercept	163	-0.4648	0.3693	304	-1.26	0.2091
Intercept	166	-0.1575	0.4875	304	-0.32	0.7469
Intercept	171	0.08309	0.5161	304	0.16	0.8722

Table 9. SAS output for group differences from overall grand group mean.

The SAS output provides the group identifier (here school_number) the regression estimate for the group's deviation from the grand mean on the dependent variable of Intent to Major in IT, the associated standard error, degrees of freedom, as well as t-statistic and significance value for the estimate. This allows the researcher to identify those schools who are significantly higher (i.e. significant positive t-value) or lower (i.e. significantly negative t-value) on average student Intent to Major in the school.

The output above is very easy to retrieve in SAS by adding the single SOLUTION parameter. Conversely, Mplus does not offer an easy method to obtain the estimated school-level deviations on Intent to Major. As described in the syntax section above, a separate file must be saved in Mplus to provide the ability to estimate these values. When the SAVEDATA section is specified in Mplus, a separate file is saved without a very noticeable prompt to the user. The one portion of the standard Mplus output which indicates that a separate file is saved is at the bottom of the output entitled SAVEDATA INFORMATION (see Table 10). This section shows that a file was saved called mlm_output.txt and the columns of the file are in the order and format specified. Since Mplus does not save column names to the output file, this selected output becomes very important when using this file.

```
SAVEDATA INFORMATION

  Order and format of variables

    INTENT3        F10.3
    ITSE           F10.3
    INTEREST       F10.3
    CAREER         F10.3
    SCH_SIZE       F10.3
    F              F10.3
    B_INTENT3      F10.3
    B_INTENT3_SE   F10.3
    SCH_NUM        I4

  Save file
    mlm_output.txt
```

Table 10. Mplus output showing format of accompanying supplemental output file.

The saved mlm_output.txt file consists of a fixed ASCII file which contains columns for each of the variables specified in the SAVEDATA information section of the standard Mplus output. The file is composed of as many lines as there are subjects in the data (e.g. in this analysis we had 309 individuals). Each line has the associated values for that individual for each of the variables specified. For each student-level variable (i.e. INTENT3, ITSE, INTEREST, CAREER) the values will be specific to that individual. Conversely, for each school-level variable (i.e. SCH_SIZE, F, B_INTENT3, B_INTENT3_SE, SCH_NUM) the value will be the same for each individual from that specific school; therefore, the school-level variables will repeat for each individual from that specific school.

To use the values in this file, the values should be imported into some spreadsheet program (for this example we utilized Microsoft Excel for our analysis). Since we were only interested in group level effects, we erased all duplicate group records until we had one record for each school (i.e. 40 lines for the 40 groups in this analysis). The F variable was estimated by Mplus and represents the estimate of the group deviation from the grand group mean. Given that we specified the variance of INTENT3 in the model to be zero, the B_INTENT3_SE variable becomes the estimated standard error of the F variable as opposed to the B_INTENT3 variable. By dividing the estimate (i.e. F) by its associated standard error (i.e. B_INTENT3_SE) we are able to derive a t statistic for each of the group estimates in the file. Using this t value, we can use the two-tailed t distribution formula in Excel (T.DIST.2T) to derive a significance value for the derived t-statistic. The degrees of freedom for this formula are the number of students in the sample minus the number of variables used in the analysis, or 309 – 5 = 304. Also, the absolute value of the t-statistic should be used, as the significance formula cannot utilize negative values. Table 11 shows the calculated output.

INTENT3	ITSE	INTEREST	CAREER	SCH_SIZE	F	B_INTENT3	B_INTENT3_SE	SCH_NUM	t statistic (F/B_INTENT3_SE)	p-value (T.DIST.2T(ABS(t statistic),df)
4	0.365	-1.414	0.554	809.948	0.038	5.552	0.319	19	0.12	0.905
7	2.7	1.05	1.052	664.948	0.318	5.74	0.262	20	1.21	0.226
5	-0.032	1.2	0.5	776.948	-0.602	4.891	0.39	22	-1.54	0.124
7	-0.014	0.8	0.812	1404.948	-0.011	5.885	0.377	24	-0.03	0.977
5	0.833	1.24	0.95	-446.052	-0.534	4.176	0.354	26	-1.51	0.132
7	0.231	0.3	-1.042	-260.052	0.948	5.777	0.335	29	2.83	0.005
7	1.528	0.95	-0.562	-283.052	0.792	5.606	0.44	30	1.80	0.073
7	-0.111	-0.267	-1	-467.052	0.483	5.18	0.461	33	1.05	0.296
7	-0.178	0.24	-0.05	-401.052	0.187	4.925	0.421	36	0.44	0.657
5	0.056	-0.425	0.969	-366.052	0.194	4.955	0.377	37	0.51	0.607
4	0.066	-0.667	0.347	-341.052	-0.079	4.698	0.292	38	-0.27	0.787
4	2.456	0.926	0.184	133.948	-0.215	4.866	0.286	40	-0.75	0.453
5	1.349	0.971	-0.429	564.948	-0.036	5.322	0.39	44	-0.09	0.927
6	-0.679	0.822	0.222	-187.052	0.319	5.195	0.365	46	0.87	0.383
4	1.211	1.16	-1.025	-415.052	-0.956	3.773	0.354	56	-2.70	0.007
4	0.333	0.2	0.375	-471.052	0.179	4.872	0.486	65	0.37	0.713
7	-1.479	1.4	-0.05	-321.052	0.227	5.017	0.281	94	0.81	0.420
5	-0.222	-0.9	0.375	1082.948	-0.042	5.648	0.486	96	-0.09	0.931
5	-0.015	-0.182	-0.33	-438.052	-0.545	4.169	0.271	97	-2.01	0.045
3	0.444	0.86	0.875	-184.052	0.072	4.949	0.354	100	0.20	0.839
4	-0.454	0.073	-0.964	-420.052	-0.01	4.716	0.39	101	-0.03	0.980
6	0.437	-1.743	-0.411	690.948	0.041	5.479	0.319	104	0.13	0.898
6	0	0	0	313.948	0.1	5.297	0.516	107	0.19	0.846
3	0.044	-0.758	0	-134.052	-0.74	4.169	0.286	109	-2.59	0.010
4	0.204	-1.167	-1.708	-227.052	-0.012	4.838	0.405	111	-0.03	0.976
4	-0.5	-0.7	0.5	-200.052	-0.192	4.675	0.486	114	-0.40	0.693
6	0	0	0	971.948	0.047	5.666	0.516	115	0.09	0.927
6	0.611	-0.2	0.562	-177.052	-0.138	4.743	0.44	116	-0.31	0.754
7	0.745	0.165	1.188	-374.052	-0.364	4.391	0.44	117	-0.83	0.409
4	0.127	-0.343	-1.321	-342.052	0.183	4.959	0.39	118	0.47	0.639
2	0.389	-0.5	-0.25	-229.052	-0.077	4.771	0.486	140	-0.16	0.874
6	-0.444	-0.4	-0.375	-385.052	0.166	4.915	0.486	141	0.34	0.733
6	2.111	1.16	-0.1	-321.052	0.337	5.126	0.421	142	0.80	0.424
7	-0.944	-0.6	0.188	455.948	0.258	5.546	0.44	144	0.59	0.558
6	0	0	0	307.948	0.1	5.293	0.516	152	0.19	0.846
4	0	0	0	-536.052	-0.081	4.571	0.516	157	-0.16	0.875
7	1.681	0.9	0.25	-511.052	0.184	4.852	0.486	158	0.38	0.705
2	0.111	-0.211	-0.25	-483.052	-0.465	4.221	0.354	163	-1.31	0.190
6	0.278	0.5	0.375	336.948	-0.157	5.054	0.486	166	-0.32	0.747
6	0	0	0	525.948	0.083	5.416	0.516	171	0.16	0.872

Table 11. Excel file showing estimate of each group (from the Mplus supplemental save file) and accompanying calculated t-statistic and p-value for each.

Upon inspection of both the SAS output for the solution for random effects in Table 9 as well as the Mplus output with additional calculations by Excel in Table 11, we find that schools 29, 56, and 109 are significantly different in their mean student Intent to Major in IT. Additionally, Mplus identifies an added school, 97, as significantly different. The estimates from SAS and Mplus match exactly while the standard errors and associated t-statistics are somewhat different. This difference will be examined in the discussion section. While slightly different, the results are highly consistent regarding the deviation of schools from the grand mean of Intent to Major in IT. Furthermore, the t-statistics provide directionality such that while school 29 is significantly higher in mean Intent to Major, schools 56, 97, and 109 are significantly lower.

DISCUSSION

The above analysis is meant as a tutorial and verification mechanism for a new method for finding group mean differences and associated significance in a multilevel model in Mplus. The estimation shows that standard output for both SAS and Mplus match up almost exactly. Furthermore, the newly devised method for estimation of group mean differences in Mplus is also highly consistent with the results from SAS. Given this verification mechanism, researchers can feel confident in extending these multilevel group mean difference analyses to a MLSEM context.

While this research shows extremely similar results between SAS and Mplus, there are some slight differences between the output of the two programs. These differences are not with the estimated betas in the model (which match exactly) but with the standard errors associated with these estimates, which are used in constructing t-statistics for significance tests. The primary reason for these differences is in the estimation algorithm used by both SAS and Mplus. While the above analyses have specified that both software products utilize ML estimation techniques, the algorithms utilized for the ML estimation differ. SAS utilizes the Newton-Raphson (NR) algorithm while Mplus uses the Expectation Maximization (EM) algorithm. While both methods offer robust mechanisms for estimating standard errors in multilevel models, the NR algorithm has been shown to provide better estimates of these standard errors by accounting for the variance in parameter estimates (Lindstrom & Bates, 1988). The effect of the variance estimates is generally not present when the number of groups is larger, which is why research has suggested that the number of groups in a multilevel analysis should exceed 50 to combat potential bias in standard error estimates (Maas & Hox, 2005). While only 40 groups were used above, only one group deviational estimate (school 97) was found to differ between the two methods, and this estimate was still quite close to the traditional cutoff significance value using both SAS and Mplus (0.07 and 0.05 respectively). Future research which utilizes less than 50 groups may want to use a more conservative significance value of 0.01.

The above novel technique of finding significant group differences within a MLSEM context is a much needed step forward in statistical estimation and adds to the arsenal of behavioral statisticians. This builds on previous multilevel regression techniques by allowing for the simultaneous estimation of measurement and structural models as well as the adding the ability to discover group differences. This type of analysis provides impetus for further research regarding the outlying groups to discover reasons for the nature of the differences in these groups. This can provide practical benefits by allowing for intervention programs for lower-than-average groups as well as analysis of above-average groups to aid in understanding how these groups can be used to help other groups succeed.

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