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ABSTRACT

Based on the work of Krishna and Rosenthal (1996) about combinatorial auctions bidding equilibrium analysis and Che(1993)'s research about one-unit multi-attributes auctions, we construct a multi-attributes procurement simultaneous auction for 2 objects, through a first-score(price), sealed-bid format. There are two kinds of bidders: *simple bidders* and a *diverse bidder* considered in this model. Each *simple bidder* is interested in only one of the objects, while the *diverse bidder* is interested in both. With some further assumptions, we finally obtain the equilibrium bidding strategies for the both two kinds of bidders.

Keywords: Bidding Strategies, Procurement, Combinatorial Auctions

1. INTRODUCTION

Many auctions involve the sale (or purchase) of a variety of distinct assets. Examples are auctions for airport time slots, delivery routes and FCC radio spectrum rights. Because of complementarities (or substitution) between the auctioned assets, bidders have preferences not just for particular items but for sets or bundles of items. For this reason, economic efficiency is enhanced if bidders are allowed to bid on bundles or combinations of different assets. Possibly because of FCC combinational auction, auctions where bidders submit bids on combinations have recently received much attention [Rothkopf et. al.(1998), Sandholm (1999)]. Actually, such auctions were proposed as early as 1976, by Jackson, for radio spectrum rights. Increases in computing power have made them more attractive to implement. The value of assets for a particular bidder, may depend strongly on which other assets she wins. Because of the possibility of such synergy, the auction designers require attentions to allowing bids not just for individual asset, but also bids for the combinations of assets, which is called combinational auctions (CAs). When an auctioneer procure items by auctions, she may allow the potential suppliers to bid for various products bundles, which is called Procurement Combinatorial Auctions. It will make variety multi-units procurement much fairer, more feasible and more efficient.

One of the most challenging problems in Procurement CAs is Winners Determination Problem (WDP). The general allocation problem in combinational auctions can be commonly formulated as an integer programming problem which is computationally intractable. Literature about CAs mechanism design is quite rich [e.g., Krishna and Rosenthal, 2000; Parkes, 2001,]. Cantillon and Pesendorfer (2003) consider the problem of identification and estimation in the first price multi-unit auction. The topics on one-unit auctions' bidding strategies and winning probability are abundantly addressed. However, discussions about CAs bidding strategies are inadequate. Krishna and Rosenthal (1996) considered situations where multiple objects are auctioned simultaneously by means of a second-price sealed-bid auction, and derived the bidding equilibrium strategies for the synergy bidder. Rosenthal and Wang (1996) studied a simultaneous-auction model with synergies, common values, and constructed strategies in which bidders of different types randomize over different bid intervals, also provided necessary and sufficient conditions for such equilibria strategies. Che (1993) studies design competition in procurement by developing a model of two-dimensional (price and quality) auctions, and corresponding bids are evaluated by a scoring rule designed by auctioneer. In this paper, we will discuss bidding strategies in a specific multi-attributes procurement CA setting.

2. MODEL AND ITS ASSUMPTIONS

In this model, there are 2 objects to be procured by an auction simultaneously through a first-price, sealed-bid format. And two kinds of bidders, i.e. *simple bidder* and *diverse bidder* are considered. Each *simple bidder* is interested in only one of the objects, while the *diverse bidder* is interested in two objects. For each of the 2 objects, there are n interested *simple bidders*. Each *simple bidder* has a privately known supply cost function for the object.

Other assumptions and their mathematical expressions are below:

(1) Bidding vector is $\{p,q\} \in R_+^{\ 2}$, p and q denote bidding price and supply quality separately. Especially, quality, in our model, is a unified number to measure all non-price factors of the objects. The scoring function of the bidding vectors has quasi-linear form

$$S(q,p) = s(q) - p \tag{1}$$

where $s(.)$ is strictly increasing with respect to q.

(2) Number of *simple bidders* for each object is n, only one *diverse bidder* in the basic model.

(3) The cost functions of *simple bidders* are

$$c(q,\theta_i)=\begin{cases} c_1(q,\theta_i), & if \quad bid \quad item1 \\ c_2(q,\theta_i), & if \quad bid \quad item2 \end{cases},$$

where θ_i denotes type of *simple bidder* i, which is private information of bidder i, and θ_i is drawn from interval $[\underline{\theta},\overline{\theta}]$ with identical distribution $F(.)$ which are common knowledge. $c_1(.,.)$ and $c_2(.,.)$ are, increasing with respect to q and θ_i. Type parameter of the *diverse bidder* is denoted as θ which is drawn from interval $[\underline{\theta},\overline{\theta}]$ with the identical distribution $F(.)$, θ is also private information, the cost functions of the *diverse bidder* are $c_1(q,\theta)$ for object 1 and $c_2(q,\theta)$ for object 2.

(4) Profit of *simple bidder* i, with bid $\{p,q\}\in R_+^{\ 2}$, is denoted as

$$\pi_i(q,p)=p-c_r(q,\theta_i), r=1,2 \quad (2)$$

Profit of the *diverse bidder*, with bid $\{p,q\}\in R_+^{\ 2}$, is

$$\begin{cases} p_1-c_1(q,\theta), get \quad item1 \\ p_2-c_2(q,\theta), get \quad item2 \\ p_1-c_1(q,\theta)+p_2-c_2(q,\theta)+\alpha, get \quad both \end{cases} \quad (3)$$

where α is a non-negative constant which is diverse bidder's private information.

3. BIDDING STRATEGY ANALYSIS

Let $S_0(\theta)\equiv \max[s(q)-c(q,\theta)], for \quad all\,\theta\in[\underline{\theta},\overline{\theta}]$, by using envelope theorem, we know $S_0(.)$ is a strictly decreasing function, therefore its inverse function exits. Let $v\equiv S_0(\theta)$, v can be regarded as scoring capability of supplier. Let $H(v)\equiv 1-F(S_0^{-1}(v))$, then distribution $F(.)$ of θ is transformed into distribution $H(.)$ of v, which also imply that the one-to-one mapping from v to θ.

Let $b\equiv S(q_s(\theta),p)$, where $q_s(\theta)=\arg\max[s(q)-c(q,\theta)]$, and b is the score of the bidding by supplier with type parameter θ. Let $B(.)$ denote the scoring function with the independent variable v, and $B(.)$ is strictly increasing with v. Note that $p-c(q_s(\theta),\theta)=v-b$, then we have the expected revenue of any *simple bidder* with type θ_i and quality bidding $q_s(\theta_i)$:

$$\begin{aligned}\pi(q(\theta_i),p|\theta_i)&=[p-c(q(\theta_i),\theta_i)] \\ &\quad\times prob\{win|S(q(\theta_i),p)\} \quad (4) \\ &=[v-b]\{H(B^{-1}(b))\}^n\end{aligned}$$

where $prob\{win|S(q(\theta_i),p)\}$ represents the winning probability of bidder whose bidding is $\{q(\theta_i),p\}$.

Therefore, Any *simple bidder* with type θ_i has the following bidding equilibria strategy [Riley and Samuelson, 1981; Che, 1993,]:

$$\begin{cases} q(\theta_i)=\arg\max[s(q)-c_r(q,\theta_i)], \quad r=1 \quad or \quad 2 \\ p(\theta_i)=c(q,\theta_i)+\int_{\theta_i}^{\overline{\theta}} c(q(t),t)[\frac{1-F(t)}{1-F(\theta_i)}]^n dt \end{cases} \quad (5)$$

It is convenient to define

$$L(b)=\{H(B^{-1}(b))\}^n \quad (6)$$

Intuitively, $L(b)$ is the probability of *diverse bidder* to win item 1(or item 2) with score *b*, that is, the probability of all the *simple bidders* who are interested in item1 (or item 2) get score less than *b*.

We've already known the *simple bidder* will use the bidding strategy in (5). Therefore, the expected revenue of *diverse bidder* with type θ is

$$\begin{aligned}&\pi(S_1(q_1(\theta),p_1),S_2(q_2(\theta),p_2)|\theta) \\ &=L(S_1)L(S_2)[(p_1-c_1)+(p_2-c_2)+\alpha] \\ &\quad+L(S_1)(1-L(S_2))(p_1-c_1) \\ &\quad+L(S_2)(1-L(S_1))(p_2-c_2)\end{aligned} \quad (7)$$

In the above equation, first term is the expected revenue of winning both items, while the last two terms separately corresponds to expected revenue of winning only item 1 or only item 2.

Lemma 1. The *diverse bidder* with type θ follows the bidding strategy whose quality-bid $(q_{1s}(\theta),q_{2s}(\theta))$ respectively is

$$q_{1s}(\theta)=\arg\max[s(q)-c_1(q,\theta)]$$
$$q_{2s}(\theta)=\arg\max[s(q)-c_2(q,\theta)] \quad (8)$$

Proof. Assume there exists another equilibrium strategy $\{(q_1,p_1),(q_2,p_2)\}$, and $q_1\neq q_{1s}, q_2\neq q_{2s}$,

Then construct a bid $\{(q_1',p_1'),(q_2',p_2')\}$, where

$$\begin{aligned} q_1'&=q_{1s}, \\ q_2'&=q_{2s}, \\ p_1'&=p_1+s(q_{1s})-s(q_1), \\ p_2'&=p_2+s(q_{2s})-s(q_2) \end{aligned} \quad (9)$$

Note

$$S(q_1', p_1') = S(q_1, p_1) = S_1, \quad S(q_2', p_2') = S(q_2, p_2) = S_2 \tag{10}$$

Then, with bid $\{(q_1', p_1'), (q_2', p_2')\}$, the expected revenue of diverse bidder is below,

$$\begin{aligned} &\pi((q_1'(\theta), p_1'), (q_2'(\theta), p_2') | \theta) \\ &= L(S_1)L(S_2)\{(p_1 - c_1(q_1) + [s(q_{1s}) - c_1(q_{1s}) \\ &\quad - (s(q_1) - c_1(q_1))] + (p_2 - c_2(q_2) \\ &\quad + [s(q_{2s}) - c_2(q_{2s}) - (s(q_2) - c_2(q_2))] + \alpha\} \\ &\quad + L(S_1)(1 - L(S_2)\{(p_1 - c_1(q_1) \\ &\quad + [s(q_{1s}) - c_1(q_{1s}) - (s(q_1) - c_1(q_1))]\} \\ &\quad + L(S_2)(1 - L(S_1))\{(p_2 - c_2(q_2) \\ &\quad + [s(q_{2s}) - c_2(q_{2s}) - (s(q_2) - c_2(q_2))]\} \end{aligned} \tag{11}$$

Note

$q_{1s}(\theta) = \arg\max s(q) - c_1(q, \theta)$

and $q_{2s}(\theta) = \arg\max s(q) - c_2(q, \theta)$,

therefore,

$$\begin{aligned} &\pi((q_1'(\theta), p_1'), (q_2'(\theta), p_2') | \theta) \\ &> L(S_1)L(S_2)[(p_1 - c_1(q_1)) \\ &\quad + (p_2 - c_2(q_2)) + \alpha] + L(S_1)(1 - L(S_2)(p_1 - c_1(q_1)) \\ &\quad + L(S_2)(1 - L(S_1))(p_2 - c_2(q_2)) \end{aligned} \tag{12}$$

Thus, we have

$$\begin{aligned} &\pi((q_1'(\theta), p_1'), (q_2'(\theta), p_2') | \theta) \\ &> \pi((q_1(\theta), p_1), (q_2(\theta), p_2) | \theta) \end{aligned} \tag{13}$$

This completes the proof.□

From LEMMA 1 and (7), we know the optimization problem of diverse bidder with type θ can be expressed as follows:

$$\begin{aligned} &\max_{p_1 p_2} \pi((q_{1s}(\theta), p_1), (q_{2s}(\theta), p_2) | \theta) \\ &= \max_{p_1 p_2} \{L(s(q_{1s}(\theta)) - p_1)L(s(q_{2s}(\theta)) - p_2) \\ &\quad [(p_1 - c_1(q_{1s}(\theta))) + (p_2 - c_2(q_{2s}(\theta))) + \alpha] \\ &\quad + L(s(q_{1s}(\theta)) - p_1)(p_1 - c_1) \\ &\quad \times (1 - L(s(q_{2s}(\theta)) - p_2)) \\ &\quad + L(s(q_{2s}(\theta)) - p_2)(p_2 - c_2)\} \\ &\quad \times (1 - L(s(q_{1s}(\theta)) - p_1)) \end{aligned} \tag{14}$$

Generally, the pairwise point (p_1, p_2) which satisfies the first order condition and the second order condition of (14), is the solution to the maximal expected revenue. Now restrict attention to equal-bid for two identical objects, that is, item 1 and item 2 are the same. Naturally, we can let

$q_{1s}(\theta) = q_{2s}(\theta) = q_s(\theta)$,

and $c_1(q_s(\theta), \theta) = c_2(q_s(\theta), \theta)$,

By LEMMA 1, the quality-bid, $q_s(\theta)$, in equilibrium strategy is uniquely determined by θ, then the corresponding cost $c(q_s(\theta), \theta)$ is also determined by θ uniquely. Now, define optimal cost of bidder with type θ as

$$c^*(\theta) = c_1(q_s(\theta), \theta) = c_2(q_s(\theta), \theta)$$

Thus rewrite (14) as

$$\begin{aligned} &\max_p \pi((q_s(\theta), p), (q_s(\theta), p) | \theta) \\ &= \max_p \{(L^2(s(q_s(\theta)) - p) \times (2(p - c^*(\theta)) + \alpha)) \\ &\quad + 2L(s(q_s(\theta)) - p)(p - c^*(\theta))\} \\ &\quad \times (1 - L(s(q_s(\theta)) - p)) \end{aligned} \tag{15}$$

Without loss of generality, let $s(q_s(\theta))$ equal 1, and assume price p is drawn from [0,1]. Define $G(.) \equiv H(B^{-1}(.))$, and rewrite (15) as

$$\begin{aligned} &\max_p \pi((q_s(\theta), p), (q_s(\theta), p) | \theta) \\ &= \max_p \{(G^{2n}(1 - p) \times (2(p - c^*(\theta)) + \alpha)) \\ &\quad + 2G^n(1 - p)(1 - G^n(1 - p))(p - c^*(\theta))\} \end{aligned} \tag{16}$$

By first derivative of (16) with respect to *p*, we have

$$\frac{d\pi}{dp} = -\alpha G^n(1 - p) - p + \frac{1}{n} G(1 - p) + c^*(\theta) = 0 \tag{17}$$

Now considering the relationship between the optimal cost of *diverse bidder* with type θ, i.e., $c^*(\theta)$, and her price bid *p*, from (17) we have

$$c^*(\theta) = \alpha G^n(1 - p) + p - \frac{1}{n} G(1 - p) \tag{18}$$

Actually, (18) is the correspondence from optimal cost $c^*(\theta)$ to optimal price bid *p*, and we can abstractly express (18) as $c^*(\theta) = c_p(p)$. The only special feature used in the argument is that function $c_p(.)$ is convex. Generally, convexity of $c_p(.)$ is not easy to characterize in terms of the primitive assumptions of our model. However, if $G(.)$ is uniform distribution, i.e., the distribution of production capability *v* is uniform, then the function $c_p(.)$ is indeed convex. If $c_p(.)$ is not convex, the situation becomes more complicated and not worth pursuing here. Therefore, there exit at most 2 roots of equation (18).

Corresponding to different optimal cost $c^*(\theta)$, denote $p(c^*(\theta))$ as the solutions of (17). Define $p^*(c^*(\theta))$ as the unique root or greater one of the two roots, i.e.,

$$p^*(c^*(\theta)) = \begin{cases} p(c^*(\theta)), & \text{for unique solution} \\ \max\{p(c^*(\theta))\}, & \text{for other case} \end{cases} \tag{19}$$

By convexity of $c_p(.)$, it's easy to know $d\pi/dp = -\alpha G^n(1-p) - p + G(1-p)/n + c^*(\theta)$ is concave. Therefore, the derivative of $d\pi/dp$ is non-positive at the point of the greater root. That means the second derivative of expected revenue with respect to *p*, at the greater root, is non-positive, i.e., $\pi''(p^*(c^*(\theta))) < 0$. It is the second-order sufficient condition for local maximizer. Consequently, we are sure that $p^*(c^*(\theta))$ is the local maximizer, while the smaller root is not.

Since the domain of expected revenue (16) is closed set, in order to find global maximizer, what we should do is to compare the boundary solutions with interior solution. Next, define a $\hat{c}(\theta)$ as follows,

$$\hat{c}(\theta) = \max\{c^*(\theta) : \pi(0|c^*(\theta)) - \pi(p^*(c^*(\theta))|c^*(\theta)) \geq 0\} \tag{20}$$

and

$$\frac{d\{\pi(0|c^*(\theta)) - \pi(p^*(c^*(\theta))|c^*(\theta))\}}{dc^*(\theta)} = -2 + 2L(1-p) \leq 0 \tag{21}$$

It follows that if there exits $\hat{c}(\theta)$, such that $\pi(0|\hat{c}(\theta)) - \pi(p^*(c^*(\theta))|\hat{c}(\theta)) = 0$, then for all $c^*(\theta) : 0 < c^*(\theta) < \hat{c}(\theta)$, we have

$$\pi(0|c^*(\theta)) > \pi(p^*(c^*(\theta))|c^*(\theta)) \tag{22}$$

Essentially, in order to compare boundary solutions with interior solutions, we need to define $\hat{c}(\theta)$. In other words, $\hat{c}(\theta)$ is the *separating point* in which boundary solution is better than interior solution, and the following result is immediate.

Theorem 1. *The following constitutes an equilibrium of our Multi-attributes procurement auction model.*
The simple bidders with type θ_i *follow the strategy*:

$$\begin{cases} q(\theta_i) = \arg\max[s(q) - c_r(q,\theta_i)], r = 1 \ \ or \ \ 2 \\ \mathrm{p}(\theta_i) = c(q,\theta_i) + \int_{\theta_i}^{\bar{\theta}} c(q(t),t)[\frac{1-F(t)}{1-F(\theta_i)}]^n dt \end{cases} \tag{23}$$

The diverse bidder with type θ *follows the strategy*:

Case 1: *For any given* $c^*(\theta)$, *there exits a unique solution to* (17)

$$\begin{cases} q_s(\theta) = \arg\max[s(q) - c(q,\theta)] \\ \mathrm{p}(\theta) = p(c^*(\theta)) \end{cases} \tag{24}$$

Case 2: *For any given* $c^*(\theta)$, *there exit two solutions to* (17)

$$\begin{cases} q_s(\theta) = \arg\max[s(q) - c(q,\theta)] \\ \mathrm{p}(\theta) = \begin{cases} p^*(c^*(\theta)) & if \ \ c^*(\theta) > \hat{c}(\theta) \\ 0 & if \ \ 0 < c^*(\theta) < \hat{c}(\theta) \end{cases} \end{cases} \tag{25}$$

4. CONCLUSION REMARKS

In this paper, we construct a multi-attributes procurement simultaneous auction for 2 objects, through a first-score (price), sealed-bid format. There are two kinds of bidders, *simple bidder* and *diverse bidder*, are considered in our model. Each *simple bidder* is interested in only one of the objects, while the *diverse bidder* is interested in both. With some assumptions, we obtain the equilibrium bidding strategies for the both two kinds of bidders. Basically, the difference between ours work and past correlated research (Krishna and Rosenthal (1996)) are: multi-attributes vs. single attribute (price), first-score(price) vs. second-price, and, most important, procurement auction vs. selling auction. Also, this piece of research can not be realized without Che (1993)'s work about one-unit multi-attributes procurement auction. This work could be regarded as a first step of constructing equilibria and designing mechanism for multi-attributes procurement combinatorial auctions. Future work includes extensions to multiple *diverse bidders* in primal model, and heterogeneous items assumption. It may also be interesting to study collaboration and competition relationship between bidders in multi-attributes procurement combinatorial auctions, in more general or practical environment, which could be undertaken on the basis of this work.

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