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Aussadavut Dumrongsiri

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Aussadavut Dumrongsiri, Sirindhorn International Institute of Technology, Thammasat University,
aussadavut@siit.tu.ac.th

Abstract

This short version of the paper offers a model and analysis of the rental process at a subscription-based business like Netflix. We analyze a priority scheduling scheme, currently in use, that gives priority to light renters over heavy renters and show that, in some situations, it may be more profitable for the firm to give priority to heavy renters which is opposite to the Netflix's current practice. The conditions determining the optimal initial inventory are proved analytically. Extensions are briefly discussed.

Keywords: Rental business, Inventory, Consumer Model, Two classes of customers

1. Introduction

Online subscription-based rental services have recently gained prominence. The customer pays a fixed fees per period for the right to rent goods from the rental firm. This business model is popular in DVD rental industry (Netflix or Blockbuster) but is also in use in many other products such as handbags (http://www.bagborroworsteal.com) and books (www.bookswim.com). In this paper, our model is mainly motivated by DVD rental-by-mail industry. Netflix, the largest online DVD rental, constantly gains the subscribers. The company's published subscriber count increased from one million in the fourth quarter of 2002 to around 7.5 million at the end of the fourth quarter of 2007 [1]. Netflix's growth has been fueled by the fast spread of DVD players in households; as of 2004, nearly two-thirds of U.S. homes had a DVD player. Online offerings, not brick-and-mortar stores, will spearhead the rental market by 2010, according to a new study. The report, by The Convergence Consulting Group Limited in Toronto, said 44% of movie rentals will originate from stores in 2010, compared to 71% in 2007. Online rental, including Netflix and Blockbuster Online, will generate 37% of revenue, up from 25% in 2007 [3]. The recent advertising blitz by Blockbuster for its own rental-by-mail service, and the spreading of this model to many other countries (for example, Glowria in France and Lovefilm in UK) suggest that the industry will continue to thrive in near future.

The rental contract limits the number of DVDs a customer may have at any point in time but allows the customer to keep a DVD as long she likes. As soon as a customer returns a DVD, the rental firm sends her the next DVD on her list. From the rental firm's perspective, this leads to an interesting classification of customers. Customers who are heavy renters return and get a new DVD more times in a month than do the customers who are, comparatively speaking, light renters. Since the rental firm pays for the postage, and since all customers in a subscription class pay the same monthly fees, the rental firm may think of heavy renters as less profitable customers than the light renters. While there may be other benefits of having heavy renters as customers, this observation about profitability may be driving Netflix's policy of reserving the right to delay the rental deliveries to heavy renters [5]. Such differentiation between customers is more visible to customers in situations where demand is high, such as, a new blockbuster release. After weathering much criticism at Web based customer forums for discriminating against certain customers without explicitly saying so, Netflix has finally added a statement describing the policy on its Web site.

We develop a model to consider if such a policy to delay heavy renters is indeed useful for the rental firm. We introduce two considerations not mentioned above. First, heavy use also means that the DVD is quickly back into circulation and can be used to satisfy next demand and second, such a delay policy creates disutility for heavy rental customers. In this paper, using rational expectation framework, customers see the backlog or the accumulated number of shortage of rental items as the indicators of service quality. Our results show that, contrary to current practice, the rental firm, in some situations, may find it beneficial to delay the light users rather than the heavy users.

This short version of the paper is limited to addressing the above issue but our model has been extended to several new features that are part of this setting. One example is the list of movie titles a customer is required to keep. The rental firm uses this list to decide the next title that must be sent to a customer every time she returns a title. As a result, with the release of a new title, the rental firm has advance information about the demand, based on the number of people who have the new title on their list. Our model extension proposes ways to use this information and thus quantify its value.

The need to differentiate between customer classes based on operational policy is tied to the fact that the more common tool for differentiation, price, is not available due to subscription-based nature of the service. Our model explores the possibility of creating a price-based differentiation scheme.

Inventory models of rental firms are typically difficult to analyze due to the complexity introduced by the return process which is incorporated in our model. In addition, the model also captures a decreasing demand pattern (over time) as [4] shows from historical rental data. Also our paper incorporates more than one customer segment with different rental behaviors. No papers in the literature we are aware of has included all these factors in one model. A recent paper with some of these features is [2] but their model has stationary demand pattern and one class of customer and also the focus of the paper is different from ours. The following section presents our basic model and some initial results.

2. The Model

We model a rental firm with customers who pay monthly fee for unlimited rentals of DVD movies sent by postal service. When a customer wants to watch a movie, she includes it in her list. When the customer returns her currently occupied movie, the firm sends her the next movie on the list. We model the rental process of one newly released movie title. The firm acquires an initial inventory of *M* copies of the movie. When a customer who has this title on the top of her list returns a movie to the firm, that returns constitutes the rental demand for one unit. The firm may chose to delay satisfying this demand. It is possible that the firm sends this customer another movie on her list while keeping her waiting for her first choice in the list. If the firm delays the customer's demand, it is considered backlogged because the movie title still remains first on the customer's list. The backlog will clear when the firm has an available copy and sends it to the customer.

We assume that there are two segments of customers: light renters (denoted by subscript l) and heavy renters (denoted by subscript h). The light renters request a fewer number of movies per month than the heavy renters while both of them are charged the same monthly fee. We assume that the demand rate $\lambda_i(t)$ for segment i when i = 1 and 2 are known and decreasing in time and then $\lambda_i^{'}(t) < 0$. At any point in time, the number of copies of the movie occupied by segment i customers is $k_i(t)$. A proportion p_i of segment i customers occupying the movie return the movie to the firm. Then the return rate of segment i customers is $p_i k_i(t)$. We assume that the light renters segment has a lower value for this proportion p_l than the heavy renters segment has for its proportion p_h; that is, $p_l < p_h$. This assumption helps us to differentiate between the segments based on their return behavior. For example, if both segments occupy the same number of copies, we would expect the number of returns per time-unit to be greater for the heavy renters segment because of the fact that the customers in this segment keep their movies for shorter periods.

We model a priority policy at the rental firm. The customers fall into two classes: class 1 with high priority and class 2 with low priority. This allows us the flexibility to assign high priority to either light renters segment or heavy renters segment. In the basic model presented here, we assume that the company will satisfy both classes of demand but when the demand exceeds the available copies, class 1 demand will have priority over class 2 demand. It is further assumed that the initial inventory M is large enough to satisfy all class 1 demands at all times; however to make the problem non-trivial we also assume that the total demand is larger than number of copies available at the start or there is shortage of copies of DVD. This is a reasonable assumption because shortage of DVD copies is observed in practice initially and also the demand of DVD peaks at the beginning and starts to decrease over time [4]. Therefore, it is more realistic to investigate the case with initial shortage. However, this assumption is relaxed in extensions to this short paper.

The number of copies occupied by class 1 customers is governed by:
$k_1(t+\Delta t) = (\lambda_1(t) - p_1 k_1(t))\Delta t + k_1(t)$

Let Δt go to zero and we get: $\frac{dk_1}{dt} = \lambda_1(t) - p_1 k_1(t)$

Solving differential equations, the number of copies occupied by class 1 customers, is: $k_1(t) = e^{-p_1 t}[\int_0^t \lambda_1(x)e^{p_1 x}dx + K_1]$ where K_1 is the initial copies class 1 customers occupy at time 0.

During the time when there is not enough copies to satisfy both demands, $k_2(t) = M - k_1(t)$ and the return rate is then determined by $p_1 k_1(t) + p_2 k_2(t) = p_1 k_1(t) + p_2(M - k_1(t))$. Because the demand rates are decreasing over time, at some t_1, the return rate is enough to satisfy both demand rates for the first time. We find t_1 by using the following identity:

$$p_1 k_1(t_1) + p_2(M - k_1(t_1)) = \lambda_1(t_1) + \lambda_2(t_1) \quad (1)$$

Note that t_1 is a function of *M*. The total number of units backlogged is:
$B(M) = \int_0^{t_1(M)} \lambda_1(t) + \lambda_2(t) - \left(p_1 k_1(t) + p_2\left(M - k_1(t)\right)\right)dt$ or, equivalently,

$$B(M) = \int_0^{t_1(M)} \lambda_1(t) + \lambda_2(t) - p_2 M + \left(p_2 - p_1\right)k_1(t)dt\,. \quad (2)$$

In this paper, only the sketches of the proofs are given or the proofs are omitted but all the results are proved analytically. By using (1) and the fact that t_1 is the first time the backlog goes to zero we can show the result in Lemma 1.

LEMMA 1. $\frac{dt_1}{dM} = \frac{p_2}{\frac{d}{dt}\left[\lambda_1(t) + \lambda_2(t) - \left(p_1 k_1(t) + p_2 k_2(t)\right)\right]_{t=t_1}} < 0$

This means as *M* increases, the duration over which backlog accumulates decreases. Using (2) and Lemma 1, we arrive at Lemma 2.

LEMMA 2. $\frac{dB(M)}{dM} = -1 - p_2 t_1(M) < 0$ and $\frac{d^2B}{dM^2} = -p_2\frac{dt_1(M)}{dM} > 0$

Lemma 2 states that as M increases the total backlog decreases and that $B(M)$ is strictly convex in *M*.

We are now ready to develop a profit function for the rental firm. First, let N_i denote the total number of class i demand. We do not present a detailed customer utility model here but, briefly, in a rational expectation framework, the backlog B is seen as an indicator of quality of service by class 2 customers and influences their decision to join the subscription service. The consumer model leads to $\frac{dN_2(B)}{dB} < 0$ and $\frac{dN_2(B)}{dB}$ is not a function of *M*.

The firm acquires each copy of the movie at a cost of c dollars. By having a copy of movie available for a class i customer, the rental transaction generates the net profit f_i which is computed by dividing (monthly fees – expected postage cost) by expected value of number of rentals per month. It is obvious that the profit from a light renter, f_l, is greater than the profit from a heavy renter, f_h because both have the same monthly fees but heavy renters have higher number of rentals in a month. The profit function for this particular movie title is determined as:

$$\pi(M) = \sum_{i=1}^{2} N_i(M) f_i - cM\,.$$

$$\frac{d\pi(M)}{dM} = f_2 \frac{dN_2(M_2)}{dM} - c = f_2 \frac{dN_2}{dB}\frac{dB}{dM} - c$$

Apply first order condition for maximum point and Lemma 2, we yield:

$$\frac{d\pi(M)}{dM} = f_2 \frac{dN_2}{dB}\frac{dB}{dM}\left(-1 - p_2 t_1(M)\right) - c = 0$$

$$t_1^* = \frac{1}{p_2}\left(\frac{c}{(f_2 - c_p)\frac{dN_2}{dB}} - 1\right)$$

That is, the optimal *M* is chosen in such a way that the backlog stop at t_1^*.

LEMMA 3. *The profit function is maximized at*:

$$t_1^* = \frac{1}{p_2}\left(\frac{c}{(f_2 - c_p)\frac{dN_2}{dB}} - 1\right).$$

Next we use Lemma 3 to find optimal initial inventory *M*. Because (1) holds for any value of *M*, M^* satisfies identity (1):

$$\lambda_1(t_1^*) + \lambda_2(t_1^*) - \left(p_1 k_1(t_1^*) + p_2 k_2(t_1^*)\right) = 0$$

$$\lambda_1(t_1^*) + \lambda_2(t_1^*) - p_1 k_1(t_1^*) - p_2\left(M - k_1(t_1^*)\right) = 0$$

This yields,

$$M^* = \frac{1}{p_2}\left[\lambda_1(t_1^*) + \lambda_2(t_1^*) + (p_2 - p_1)k_1(t_1^*)\right]$$

Also, apply the second condition, we get.

$$\frac{d^2\pi(M)}{dM^2} = f_2 \frac{d^2 N_2(M)}{dM^2} = f_2 \frac{dN_2}{dB}\frac{d^2 B}{dM^2}$$

Using Lemma 2, $\frac{d^2\pi(M)}{dM^2} > 0$

$\pi(M)$ is a concave function of *M* and the optimal *M** is unique.
By using Lemma 2, Lemma 3 and the results above, we can conclude:

PROPOSITION 4. *The profit function is strictly concave in M and the optimal initial inventory M* is uniquely determined by:* $M^* = \frac{1}{p_2}\left[\lambda_1(t_1^*) + \lambda_2(t_1^*) + (p_2 - p_1)k_1(t_1^*)\right]$ and $t_1^* = \frac{1}{p_2}\left(\frac{c}{(f_2 - c_p)\frac{dN_2}{dB}} - 1\right)$.

Because of strict concavity of profit function, the optimal *M** could be easily searched using any line search algorithm such as Newton's search. Let $\overline{N_i}$ is the maximum number of class i customers with zero backlog where $i \in \{l, h\}$. Using the optimal *M**, we compare the profit functions using the policies giving priority to heavy and light users and arrive at the result in Proposition 5.

PROPOSITION 5. *Under the following condition, setting the heavy renters as class 1 yields better result than the* *opposite:* $f_l\left(\overline{N}_l - N_l\big|_{B=\overline{N}_l}\right) < f_h\left(\overline{N}_h - \left[N_h(M^*)\right]_{\text{priority to light renters}}\right)$

The left hand side of the condition is the maximum profit loss when setting the light renter as class 2 while the right hand side of the condition is the optimal profit loss when setting the heavy renter as class 2. When this condition holds, giving priority to heavy renters results in higher profit which is opposite of the current practice at Netflix. This is very interesting result because even with this stylized model, we prove that the policy used currently at Netflix is not always optimal and the further investigation is needed to find an optimal policy. When the heavy renter is sensitive to the backlog or when the total number of heavy renters is very large, the condition is more likely to hold. In such cases, even after the backorder-sensitive customers leave, the net number is still large compared to the light renters.

3. Conclusion and Extensions

We have built a model for rental firms that, unlike much of the existing literature, explicitly captures the return process and more than one customer class. The model offers insights into current scheduling practice at Netflix and offers an alternative. In extensions to this work, we explore the optimal scheduling policy and determine the value of the demand signal contained in customer's waiting lists. We are also working on comparing a price-based scheme to discriminate between the segments with the current scheduling-based scheme.

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