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A GRP-basedHesitant Fuzzy Multiple Attribute Decision MakingMethod and Its Application to E-Commerce Risk Assessment

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Abstract. With respect to multiple attribute decision making (MADM) problems in which the attribute values take the form of hesitant fuzzy elements, the traditional grey relational projection (GRP) method is extended to solve multiple attribute decision making problems under hesitant fuzzy environment. Based on the hesitant fuzzy decision matrix provided by decision makers, all feasible alternatives are ranked according to the descending order of relative grey relational projections, and the most desirable alternative(s) should have the largest grey relational projection on positive ideal solution and the smallest grey relational projection on negative ideal solution. Finally, a numerical example of e-commerce risk assessment is given to illustrate the application of the proposed method.

Keywords: multiple attribute group decision making (MAGDM),hesitant fuzzy set,grey relational projection

1. INTRODUCTION

As an important part of decision science, multiple attribute decision making (MADM) is one process to select the most desirable alternative(s) from a discrete set of feasible alternatives with respect to a finite set of attributes. At present, many MADM methodshave been proposed, such as TOPSIS (technique for order performance by similarity to ideal solution)[1], AHP (analytic hierarchy process)[2], VIKOR (Serbian: vlsekriterijumskaoptimizacija i kompromisnoresenje)[3, 4]and so on. The grey relational projection (GRP) method, a well-known classical MADM method, was firstly proposed by Lü and Cui[5]. The basic idea of the GRP method is to calculate the grey relational projection of the decision alternative on the ideal solution. Since its appearance, the GRP method has received a great deal of attentions from researchers[6-8]. For example, Zheng et al.[9] used an improved grey relational projection method to evaluate the sustainable building envelope performance; Zhang et al.[10] extended the grey relational projection method to solve the multiple attribute decision making problems with intuitionistic trapezoidal fuzzy numbers.

Due to the time pressure, knowledge limitation and lack of data, the difficulty of determining the membership degree of an elementmay not because we have a margin of error or some possibility distribution on the possible values, but because we have several possible values. Therefore, it is necessary to adopt the form of hesitant fuzzy set to describe the preference information of decision makers.Hesitant fuzzy set (HFS), a generalization of fuzzy set, was proposed by Torra and Narukawa[11] and Torra[12]which permits the membership degree of an element to a given set havingseveral possible values. It can reflect the human's hesitance more objectively than the other classical extensions of fuzzy set[13], such as intuitionistic fuzzy sets (IFSs)[14-16], type-2 fuzzy sets[17-19], fuzzy multisets[20,21] and so on.Xia and Xu[22] proposed some aggregation operators for hesitant fuzzy information and applied them to solve hesitant fuzzy multiple attribute decision making problems. Zhu et al.[23] and Zhang[24] developed hesitant fuzzy geometric Bonferroni means and hesitant fuzzy power aggregation operators, respectively, by considering the interrelationships among attributes. Wei[25]introduced the hesitant fuzzy prioritizedaggregation operators to solve the hesitant fuzzy multiple attribute decision making problems

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where the attributes are in different priority level. Chen et al.[26]proposed some correlation coefficient formulas for HFS and applied them to clustering analysis. Peng et al.[27] proposed the generalized hesitant fuzzy synergetic weighted distance measures and gave their applications to MADM problem. Xu and Xia[28-30] investigated some measures involving distance, correlation, similarity and entropy for hesitant fuzzy set. Xu and Zhang[31] developed a novel method based on TOPSIS and the maximizing deviation method for solving hesitant fuzzy multiple attribute decision making problems with incompleteweight information. Liao and Xu[32] and Zhang and Wei[33] extended the VIKOR method to solve the MADM problems with hesitant fuzzy information. From the above analysis, it can be found that the existingGRP methods can solve information taking the forms of real numbers and intuitionistic trapezoidal fuzzy numbers, and yet they fail in dealing with the hesitant fuzzy information. Therefore, the aim of this paper is to extend the traditional GRP method to solveMADM problems under hesitant fuzzy environment.

The remainder of the paper is organized as follows: In Section 2, some basic definitions related to hesitant fuzzy setand projection are introduced briefly. In Section 3,the traditional GRP methodis extended to solve MADM problems under hesitant fuzzy environment. In Section 4, a numerical example of e-commerce risk assessment is given to illustrate the application of the proposed method. The paper is concluded in Section 5.

2. PRELIMINARIES

In this section, some basicdefinitions related to hesitant fuzzy set and the projection method are briefly introduced to facilitate the following discussion.

Hesitant fuzzy set (HFS) which permits the membership degree of an element to a given set to be represented as several possible values between 0 and 1 was originally proposed by Torra[12] and Torra and Narukawa[11]. It is a useful tool to deal with the situation where experts have hesitancy in providing their preferences on objects in a practical decision making process.

Definition 1[11,12]. Let *X*be a fixed set, a HFS on *X* is in terms of a function that when applied to *X* returns a subset of [0,1], which can be represented as the following mathematical symbol:

$$E = \{< x, h_E(x) > | \, x \in X\}, \qquad (1)$$

where$h_E(x)$ is a set of values in [0,1], denoting the possible membership degrees of the element $x \in X$ to the set *E*. For convenience, Xia and Xu[22] called $h=h_E(x)$ a hesitant fuzzy element (HFE) and *H* the set of all hesitant fuzzy elements (HFEs).

Definition 2[22]. Let h, h_1 and h_2 be any three HFEsand $\beta > 0$, then the operational laws of HFEs are defined as follows:

(1) $h^\beta = \bigcup_{r\in h}\{r^\beta\}$; (2)

(2) $\beta h = \bigcup_{r\in h}\{1-(1-r)^\beta\}$; (3)

(3) $h_1 \oplus h_2 = \bigcup_{r_1\in h_1, r_2\in h_2}\{r_1 + r_2 - r_1 r_2\}$; (4)

(4) $h_1 \otimes h_2 = \bigcup_{r_1\in h_1, r_2\in h_2}\{r_1 r_2\}$.(5)

Definition 3[31]. Let h_1 and h_2 be any two HFEs, $S(h_i) = \frac{1}{l(h_i)}\sum_{r_i\in h_i} r_i$ and $V(h_i) = \frac{1}{l(h_i)}\sqrt{\sum_{r_i,r_j\in h_i}(r_i - r_j)^2}$ be the score function and variance function of $h_i(i=1,2)$, respectively, where$l(h_i)$ is the number of elements in h_i (i=1,2), then

If $S(h_1) < S(h_2)$, then h_1 is smaller than h_2, denoted by $h_1 < h_2$.

If $S(h_1) = S(h_2)$, then

(1) If $V(h_1) < V(h_2)$, then h_1 is larger than h_2, denoted by $h_1 > h_2$;

(2) If $V(h_1) = V(h_2)$, then h_1 is equal to h_2, denoted by $h_1 = h_2$.

Definition 4[32]. Let h_1 and h_2 be any two HFEs, then the normalized Hamming distance between h_1 and h_2 is defined as follows:

$$d(h_1,h_2)=\frac{1}{l}\sum_{j=1}^{l}\left|h_{1\sigma(j)}-h_{2\sigma(j)}\right|, \quad (6)$$

where $h_{1\sigma(j)}$ and $h_{2\sigma(j)}$ are the $j$thsmallest element in h_1 and h_2, respectively; $l(h)$ is the number of elements in h.

In most cases, $l(h_1)\neq l(h_2)$, and for convenience, let $l=\max\{l(h_1),l(h_2)\}$. To operate correctly, the shorter one should be extended until both of them have the same length when we compare them. The best extending way is to add the same value several times in it. In fact, we can add any value in the shorter one to extend it. The selection of this value mainly depends on the decision makers' risk preference. The optimistic decision makers may add the maximum value to extend the shorter one, while the pessimists who expect unfavorable outcomes may add the minimum value.Although the results may be different if we extend the shorter one by adding different values, this is reasonable since the decision makers' risk preference can directly influence the final decisionresults[34-36]. In this paper, it is assumed that all decision makers are optimistic (other situations can be studied similarly).

Definition 5[37, 38]. Let $\alpha=(\alpha_1,\alpha_2,...,\alpha_n)$ and $\beta=(\beta_1,\beta_2,...,\beta_n)$ be two vectors, then the projection of vector α on β is defined as:

$$P_\beta(\alpha)=\|\alpha\|\cdot\cos(\alpha,\beta)=\sqrt{\sum_{j=1}^{n}\alpha_j^2}\times\frac{\sum_{j=1}^{n}(\alpha_j\beta_j)}{\sqrt{\sum_{j=1}^{n}\alpha_j^2}\times\sqrt{\sum_{j=1}^{n}\beta_j^2}}=\frac{\sum_{j=1}^{n}(\alpha_j\beta_j)}{\sqrt{\sum_{j=1}^{n}\beta_j^2}}. \quad (7)$$

The projection can be illustrated in Fig.1. In general, the larger the projection value $P_\beta(\alpha)$ is, the larger the similarity degree between vector α and β is.

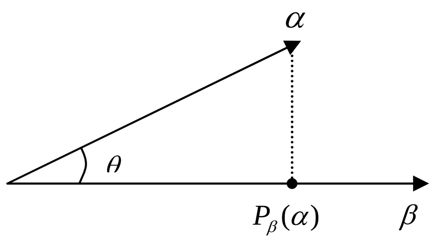

Figure 1. The projection of vector α on β.

3. A GRP-BASED HESITANT FUZZY MULTIPLE ATTRIBUTE DECISION MAKING METHOD

For a hesitant fuzzy multiple attribute decision making problem, let $A=\{A_1,A_2,...,A_m\}$ be a discrete set of m alternatives, $C=\{C_1,C_2,...,C_n\}$ bea set of n attributes, and $w=\{w_1,w_2,...,w_n\}$ be theweight vector of attributes, with $w_j\in[0,1]$, j=1,2,…,n, and $\sum_{j=1}^{n}w_j=1$. Suppose that the preference information provided by decision makers is described by the form of hesitant fuzzy decision matrix $H=(h_{ij})_{m\times n}$, where $h_{ij}=\bigcup_{r_{ij}\in h_{ij}}\{r_{ij}\}$, a hesitant fuzzy element, is the assessment value of the alternative A_i(i=1,2,…,m) with respect to the attribute C_j(j=1,2,…,n). Therefore, a novel hesitant fuzzy multiple attribute decision making method is proposed based on GRP, which involves the following steps:

Step 1. Normalize the hesitant fuzzy decision matrix $H=(h_{ij})_{m\times n}$. To eliminate the effect from different physical dimensions to decision results, the original decision matrices should be normalized firstly. We use the normalized method proposed by Xu and Hu[39]to normalize the hesitant fuzzy decision matrix $H=(h_{ij})_{m\times n}$.The normalized hesitant fuzzy decision matrix $\bar{H}=(\bar{h}_{ij})_{m\times n}$ is constructed as follows:

$$\bar{h}_{ij}=\begin{cases}\bigcup_{r_{ij}\in h_{ij}}\{r_{ij}\}, & j\in\Omega_B\\ \bigcup_{r_{ij}\in h_{ij}}\{1-r_{ij}\}, & j\in\Omega_C\end{cases}, i=1,2,....,m,\ j=1,2,...,n, \quad (8)$$

where Ω_B and Ω_C are the sets of benefit attributes and cost attributes, respectively.

Step 2. Determine the hesitant fuzzy positive ideal solution (HF-PIS) and the hesitant fuzzy negative ideal solution (HF-NIS) by Eqs. (9) and (10), respectively.

$$\bar{h}^+ = (\bar{h}_1^+, \bar{h}_2^+, ..., \bar{h}_n^+), \tag{9}$$

$$\bar{h}^- = (\bar{h}_1^-, \bar{h}_2^-, ..., \bar{h}_n^-), \tag{10}$$

where $\bar{h}_j^+ = \max_{1 \le i \le m} \bar{h}_{ij}$, $\bar{h}_j^- = \min_{1 \le i \le m} \bar{h}_{ij}$, j=1,2,…,n.

Step 3.Construct the grey relational coefficient matrices $\varepsilon^+ = (\varepsilon_{ij}^+)_{m\times n}$ and $\varepsilon^- = (\varepsilon_{ij}^-)_{m\times n}$ of each alternative from HF-PIS and HF-NIS by Eqs. (11) and (12), respectively.

$$\varepsilon_{ij}^+ = \frac{\min_{1\le i\le m}\min_{1\le j\le n} d(\bar{h}_{ij}, \bar{h}_j^+) + \rho \max_{1\le i\le m}\max_{1\le j\le n} d(\bar{h}_{ij}, \bar{h}_j^+)}{d(\bar{h}_{ij}, \bar{h}_j^+) + \rho \max_{1\le i\le m}\max_{1\le j\le n} d(\bar{h}_{ij}, \bar{h}_j^+)}, i = 1, 2, ..., m, \ j = 1, 2, ..., n, \tag{11}$$

$$\varepsilon_{ij}^- = \frac{\min_{1\le i\le m}\min_{1\le j\le n} d(\bar{h}_{ij}, \bar{h}_j^-) + \rho \max_{1\le i\le m}\max_{1\le j\le n} d(\bar{h}_{ij}, \bar{h}_j^-)}{d(\bar{h}_{ij}, \bar{h}_j^-) + \rho \max_{1\le i\le m}\max_{1\le j\le n} d(\bar{h}_{ij}, \bar{h}_j^-)}, i = 1, 2, ..., m, \ j = 1, 2, ..., n, \tag{12}$$

where $d(\bar{h}_{ij}, \bar{h}_j^+)$ is the normalized Hamming distance between $\bar{h}_{ij}$ and $\bar{h}_j^+$, $d(\bar{h}_{ij}, \bar{h}_j^-)$ is the normalized Hamming distance between $\bar{h}_{ij}$ and $\bar{h}_j^-$, and $\rho \in [0,1]$ is distinguishing coefficient. In general, $\rho = 0.5$.

According to Eq. (11), it is obvious that the grey relational coefficient vector between the HF-PIS $\bar{h}^+ = (\bar{h}_1^+, \bar{h}_2^+, ..., \bar{h}_n^+)$ and itself is $\varepsilon_0^+ = (1,1,...,1)$. Similarly, the grey relational coefficient vector between the HF-NIS $\bar{h}^- = (\bar{h}_1^-, \bar{h}_2^-, ..., \bar{h}_n^-)$ and itself is $\varepsilon_0^- = (1,1,...,1)$.

Step 4.Construct the weighted grey relational coefficient decision matrices $Y^+ = (y_{ij}^+)_{m\times n}$ and $Y^- = (y_{ij}^-)_{m\times n}$ of each alternative from HF-PIS and HF-NIS by Eqs. (13) and (14), respectively.

$$y_{ij}^+ = w_j \times \varepsilon_{ij}^+, i = 1, 2, ..., m, \ j = 1, 2, ..., n, \tag{13}$$

$$y_{ij}^- = w_j \times \varepsilon_{ij}^-, i = 1, 2, ..., m, \ j = 1, 2, ..., n, \tag{14}$$

where w_j is the weight of attribute C_j (j=1,2,…,n).

Therefore, the weighted grey relational coefficient vector between the HF-PIS $\bar{h}^+ = (\bar{h}_1^+, \bar{h}_2^+, ..., \bar{h}_n^+)$ and itself is $y_0^+ = (w_1, w_2, ..., w_n)$. Similarly, the weighted grey relational coefficient vector between the HF-NIS $\bar{h}^- = (\bar{h}_1^-, \bar{h}_2^-, ..., \bar{h}_n^-)$ and itself is $y_0^- = (w_1, w_2, ..., w_n)$.

Step 5. Calculate the grey relational projections of each alternative A_i (i=1,2,…,m) on HF-PIS $\bar{h}^+ = (\bar{h}_1^+, \bar{h}_2^+, ..., \bar{h}_n^+)$ andHF-NIS $\bar{h}^- = (\bar{h}_1^-, \bar{h}_2^-, ..., \bar{h}_n^-)$, respectively.

Each line in the weighted grey relational coefficient decision matrix $Y^+ = (y_{ij}^+)_{m\times n}$ is considered as a row vector $y_i^+ = (y_{i1}^+, y_{i2}^+, ..., y_{in}^+)$, which corresponds to the alternative A_i (i=1,2,…,m). Therefore, the grey relational projection of the alternative A_i(i=1,2,…,m) on HF-PIS $\bar{h}^+ = (\bar{h}_1^+, \bar{h}_2^+, ..., \bar{h}_n^+)$ can be obtained by Eq. (15) as follows:

$$P_i^+ = \left\|y_i^+\right\| \times \cos(\theta_i^+) = \sqrt{\sum_{j=1}^n (w_j \varepsilon_{ij}^+)^2} \times \frac{\sum_{j=1}^n ((w_j \varepsilon_{ij}^+) \times w_j)}{\sqrt{\sum_{j=1}^n (w_j \varepsilon_{ij}^+)^2} \times \sqrt{\sum_{j=1}^n w_j^2}} = \frac{\sum_{j=1}^n (w_j^2 \varepsilon_{ij}^+)}{\sqrt{\sum_{j=1}^n w_j^2}}, i = 1,2,..., m. \tag{15}$$

Similarly, the grey relational projection of the alternative A_i (i=1,2,…,m) on HF-NIS $\bar{h}^- = (\bar{h}_1^-, \bar{h}_2^-, ..., \bar{h}_n^-)$ can be obtained by Eq. (16).

$$P_i^- = \frac{\sum_{j=1}^n (w_j^2 \varepsilon_{ij}^-)}{\sqrt{\sum_{j=1}^n w_j^2}}, i = 1,2,..., m. \tag{16}$$

Step 6. Calculate the relative grey relational projection of each alternative A_i (i=1,2,…,m) on HF-PIS.

For analternative, the larger the value of its grey relational projection on HF-PIS, the closer to HF-PIS it is, and the better the alternative is; On the other hand, the smaller the value of its grey relational projection on HF-NIS, the farther to HF-NIS it is, and the worse the alternative is. Therefore, considering both the grey relational projections on HF-PIS and HF-NIS simultaneously, we define the relative grey relational projection of alternative A_i (i=1,2,…,m) to HF-PISby Eq. (17).

$$P_i = \frac{P_i^+}{P_i^+ + P_i^-} = \frac{\sum_{j=1}^{n}(w_j^2 \varepsilon_{ij}^+)}{\sum_{j=1}^{n}(w_j^2 \varepsilon_{ij}^+) + \sum_{j=1}^{n}(w_j^2 \varepsilon_{ij}^-)}, \; i = 1,2,..., \; m. \tag{17}$$

Step 7. Rank the alternatives according to the descending order of corresponding relative grey relational projections P_i (i=1,2,…,m). That is, the larger the relative grey relational projection is, the better the alternative is.

Step 8. End.

4. ILLUSTRATIVE EXAMPLE

In this section, a numerical example ofe-commerce risk assessment is used to illustrate the application of hesitant fuzzy MAGDM method proposed in this paper. Suppose that an e-commerce enterprise intends to select a new investment of alternatives to maximize the expected profit. After preliminary screening, there are four possible alternatives A_i (i=1,2,3,4) to be selected under the following four attributes: (1) the profit of the hardware/software investment (C_1); (2) the contribution to the performance of the organization (C_2); (3) the effort to transfer from the current system (C_3); and (4) the reliability of the outsourcing software developer (C_4). The weight vector of attributes is w=(0.15,0.35,0.35,0.15)^T.The preference information of decision makers is given bya hesitant fuzzy decision matrix $H = (h_{ij})_{4\times4}$ shown in Table 1. Then, to determine the most desirable alternative(s), the proposed method is utilized, whichinvolves the following steps:

Table 1.Hesitant fuzzy decision matrix

	C_1	C_2	C_3	C_4
A_1	{0.5647,0.7911,0.9004, 0.9478,0.9750,0.9881}	{0.6476,0.7841,0.7851, 0.8992}	{0.6819,0.8779,0.9519, 0.9815}	{0.6969,0.9293,0.9758, 0.9991}
A_2	{0.6470,0.8529,0.8800, 0.9653}	{0.7890,0.9719,0.9818, 0.9989}	{0.6666,0.7958,0.9479}	{0.5674,0.7525,0.8674, 0.9241}
A_3	{0.3316,0.5178}	{0.4151,0.6095}	{0.9119,0.9956,0.9966, 0.9999,1.0000,1.0000}	{0.8243,0.9766,0.9955, 0.9994,0.9998,1.0000}
A_4	{0.1570,0.4006}	{0.8378,0.9561,0.9883, 0.9968}	{0.7544,0.9063,0.9646}	{0.8329,0.9765,0.9833, 0.9994}

Step 1.Utilize Eqs. (9) and (10) to determine the hesitant fuzzy positive ideal solution (HF-PIS) and the hesitant fuzzy negative ideal solution (HF-NIS), respectively.

$$\bar{h}^+ = (\{0.5647,0.7911,0.9004,0.9478,0.9750,0.9881\},\{0.8378,0.9561,0.9883,0.9968\},$$
$$\{0.9119,0.9956,0.9966,0.9999,1.0000,1.0000\},$$
$$\{0.8243,0.9766,0.9955,0.9994,0.9998,1.0000\});$$

$$\bar{h}^- = (\{0.1570,0.4006\},\{0.4151,0.6095\},\{0.6666,0.7958,0.9479\},$$
$$\{0.5674,0.7525,0.8674,0.9241\}).$$

Step 2.Utilize Eqs. (11) and (12) (Suppose $\rho = 0.5$) to calculate the grey relational coefficient decision matrices $\varepsilon^+ = (\varepsilon_{ij}^+)_{4\times4}$ and $\varepsilon^- = (\varepsilon_{ij}^-)_{4\times4}$ of each alternative from HF-PIS and HF-NIS, respectively, which are shown in Tables 2 and 3.

Table 2.The grey relational coefficient decision matrix of each alternative from HF-PIS

	C_1	C_2	C_3	C_4
A_1	1.0000	0.6020	0.7706	0.8846
A_2	0.8753	0.9319	0.6982	0.6427
A_3	0.4010	0.3950	1.0000	1.0000
A_4	0.3333	1.0000	0.7961	0.9855

Table 3.The grey relational coefficient decision matrix of each alternative from HF-NIS

	C_1	C_2	C_3	C_4
A_1	0.3333	0.5347	0.8812	0.6718
A_2	0.3354	0.4009	1.0000	1.0000
A_3	0.6320	1.0000	0.6982	0.6427
A_4	1.0000	0.3950	0.7775	0.5955

It is obvious that the grey relational coefficient vector between the HF-PIS $\bar{h}^+ = (\bar{h}_1^+, \bar{h}_2^+, ..., \bar{h}_n^+)$ and itself is $\varepsilon_0^+ = (1,1,...,1)$. Similarly, the grey relational coefficient vector between the HF-NIS $\bar{h}^- = (\bar{h}_1^-, \bar{h}_2^-, ..., \bar{h}_n^-)$ and itself is $\varepsilon_0^- = (1,1,...,1)$.

Step 3. Utilize Eqs. (13) and (14) to construct the weighted grey relational coefficient decision matrices $Y^+ = (y_{ij}^+)_{4\times4}$ and $Y^- = (y_{ij}^-)_{4\times4}$ of each alternative from HF-PIS and HF-NIS, respectively, which are shown in Tables 4 and 5.

Table 4.The weighted grey relational coefficient decision matrix of each alternative from HF-PIS

	C_1	C_2	C_3	C_4
A_1	0.1500	0.2107	0.2697	0.1327
A_2	0.1313	0.3262	0.2444	0.0964
A_3	0.0602	0.1383	0.3500	0.1500
A_4	0.0500	0.3500	0.2786	0.1478

Table 5.The weighted grey relational coefficient decision matrix of each alternative from HF-NIS

	C_1	C_2	C_3	C_4
A_1	0.0500	0.1871	0.3084	0.1008
A_2	0.0503	0.1403	0.3500	0.1500
A_3	0.0948	0.3500	0.2444	0.0964
A_4	0.1500	0.1383	0.2721	0.0893

Obviously, the weighted grey relational coefficient vector between the HF-PIS $\bar{h}^+$ and itself is $y_0^+ = (0.1500, 0.3500, 0.3500, 0.1500)$. Similarly, the weighted grey relational coefficient vector between the HF-NIS $\bar{h}^-$ and itself is $y_0^- = (0.1500, 0.3500, 0.3500, 0.1500)$.

Step 4. Utilize Eqs. (15) and (16) to calculate the grey relational projections of each alternative A_i (i=1,2,3,4) on HF-PIS $\bar{h}^+$ and HF-NIS $\bar{h}^-$, respectively. For example, according to Eq. (15), the grey relational projection of the alternative A_1 on HF-PIS is calcualated as follows:

$$P_1^+ = \frac{1.0000\times0.1500^2 + 0.6020\times0.3500^2 + 0.7706\times0.3500^2 + 0.8846\times0.1500^2}{\sqrt{0.1500^2 + 0.3500^2 + 0.3500^2 + 0.1500^2}} = 0.3910.$$

Similarly, the other grey relational projections of alternatives A_i (i=1,2,3,4) on HF-PIS $\bar{h}^+$ and HF-NIS $\bar{h}^-$ can be obtained.

$$P_2^+ = 0.4343\ , P_3^+ = 0.3759\ , P_4^+ = 0.4637\ ;$$

$$P_1^- = 0.3641\ ,\ P_2^- = 0.3745\ ,\ P_3^- = 0.4396\ ,\ P_4^- = 0.3334\ .$$

Step 5. Utilize Eq. (17) to calculate the relative grey relational projections of alternativesA_i (i=1,2,3,4) on HF-PIS. The relative grey relational projections are shown as follows:

$$P_1 = 0.5178\ ,\ P_2 = 0.5370\ ,\ P_3 = 0.4609\ ,\ P_4 = 0.5817\ .$$

Step 6. According to the descending order of corresponding relative grey relational projections P_i (i=1,2,3,4),the ranking of all feasiblealternativesA_i(i=1,2,3,4) is obtained as follows:

$$A_4 \succ A_2 \succ A_1 \succ A_3.$$

Note that the symbol '$\succ$' means 'superior to'. Therefore, A_4 is the most desirable alternative.

5. CONLUSIONS

The traditional grey relation projection method is generally suitable for dealing with MADM problems in which the attribute values take the form of real numbers, and yet it failswhen dealing with hesitant fuzzy information. Therefore, in this paper, with respect to MADM problems in which the attribute values take the form of hesitant fuzzy elements, a GRP-based hesitant fuzzy multiple attribute decision making method is investigated. Based on the hesitant fuzzy decision matrix provided by decision makers, all feasible alternatives are ranked according to the descending order of relative grey relational projections, and the most desirable alternative(s) should have the largest grey relational projection on positive ideal solution and the smallest grey relational projection on negative ideal solution. Finally, a numerical example of e-commerce risk assessment is given to illustrate the application of the proposed method. In future research, we will focus on extending the application of the proposed method in various domains, such as investment, personnel evaluation and sharing economy and so on.

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