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ABSTRACT

The supplier and buyers, with different objectives and self-interest, are separate economic entities acting independently and opportunistically to maximize their individual profits. In this paper, a GBA model in the B2B market is studied, where one supplier faces 2 different retailers, who cooperate in the order decision making. Firstly, the optimal ordering decision of the retailers was analyzed. Then, from the perspective of the supplier, the optimal pricing strategy of the supplier is also studied. Finally, it is concluded that the group buying online auction is a useful and efficient pricing mechanism in the B2B e-market, under which, all members of the supply chain will improve their payoffs.

Keywords: Group-buying online auction, supply chain, newsvendor model, B2B market

1. INTRODUCTION

The supplier and buyers, with different objectives and self-interest, are separate economic entities acting independently and opportunistically to maximize (minimize) their individual profits (costs). An effective supply chain network requires a cooperative relationship between the vendor and the buyer. Based on mutual trust, the cooperation includes the sharing of information, resources and profit. One of the most common strategies is to setup a pricing policy acceptable to both the vendor and the buyer. (Yang, 2004).

The price discount policy is a subject of significant interest. Starting from Crowther (1964), Monahan (1984) was one of the early authors who analyzed a vendor-oriented optimal quantity discount policy that maximized the vendor's gain; it is done at no additional cost to the buyer. Lee and Rosenblatt (1985) generalized Monahan's model and developed an algorithm to solve the vendor's ordering and discount-pricing policy. Buscher and Lindner (2004) further extend Lee and Rosenblatt's model.

However, their objective function is restricted to maximizing the supplier's profits only. Zahir and Sarker (1991) consider a price-dependent demand function for multiple regional wholesalers who are served by a single manufacturer. Meantime, in recent years, some studies have paid the attention on the perishable product with the policy of price discount. Wee (1999) introduces a deterministic inventory model with quantity discount, pricing and partial backordering when the product in stock deteriorates with time. Papachristos and Skouri (2003) generalize the work of Wee and consider a model where the demand rate is a convex decreasing function of the selling price and the backlogging rate is a time-dependent function, which ensures that the rate of backlogged demand increases as the waiting time to the following replenishment point decreases.

In our paper, we also analyze the two-echelon supply chain consisting of one vendor and multiple different buyers, in which, the products is perishable. The main difference between our work and theirs is that a particular pricing mechanism, the Group buying online auction, is employed in our study.

Group-buying auction is also a kind of quantity discount form. However, the GBA, unlike the traditional discount, does not lead to price discrimination among different buyers and every buyer will be charged the same price. Each buyer in the GBA will only buy a piece of product which means the price declines monotonically in total purchase quantities of all buyers, and not just an individual buyer's purchase quantities. These are much different from the traditional quantity discount policy.

Moreover, the group-buying auction (GBA), a homogeneous multi-unit auction, has many users on sites such as LetsBuyIt.com and Yabuy.com. Different from traditional auctions, where bidders compete against one another to be the "winner" with the highest price, the GBA enables bidders to aggregate to offer a lower price at which they all "win" (Horn et al. 2000). When using the GBA as the pricing mechanism, it is helpful to refer to traditional auction models. By considering the seller's expected revenue, Vickrey (1961) proves the well-known Revenue Equivalent Principle and Myerson (1979) proves the Revelation Principle. The details about traditional auction studies can be found in Klemperer's (1999) survey. However, the traditional auction theory does not consider inventory because it is not an important issue for those products. Internet expands online auction markets, where almost all products can be auctioned. Hence, our study tries to bridge the inventory management and auction process.

Although the GBA is widely used in the E-market, it lacks a theoretical framework and the study on the GBA is rare. Chen et al. (2002) build a dynamic game model for the GBA, based on which they study the bidders' optimal strategy. They prove that the mechanism is incentive compatible for bidders under the IPV (independent private values) assumption. Kauffman and Wang (2002a) conduct an experimental study of the GBA. They analyze the changes in the number of bids for Mobshop-listed products over various periods. Based on the empirical study of twelve GBA websites, Kauffman and Wang (2002b) point out that because the GBA is still a new concept, the websites must educate consumers to achieve a critical mass. Anand and Aron (2003) devise an analytical model to study the GBA. They compare the posted pricing mechanism with the GBA in different scenarios, e.g., demand uncertainty and scale economies of production. On the other hand, the inventory literature has paid little notice to the marketing trends (Arcelus et al., 2003). Chen et al. (2004b) consider a GBA model for a monopolistic manufacturer selling novel products in the uncertain market and link this new pricing mechanism to the newsboy model.

They, however, all do not study the GBA mechanism in the B2B market, which is a more prosper and profitable segment. Kauffman and Wang (2002b) suggest that the GBA websites oriented toward the B2B market should be better positioned for future growth. In the business practice, Mobshop changed its strategic direction from the B2C to the B2B market (Clark 2001) which is more profitable. According to Anand and Aron (2003), in the B2B market, the GBA has been employed to sell diverse goods and services, such as, furniture, commercial print, computer hardware, software, telecommunications & connectivity, electricity and gas, computer equipment, office supplies and vehicles, vegetables and company travel. To the best of our knowledge, our paper is the first in academia to study the GBA in the B2B e-market. Our effort is to regard the GBA as a useful and resultful coordination strategy in a two echelon supply chain in order to make a more effective and efficient decision in the B2B e-Market environment.

The rest of this paper is organized as follows. In section 2, we build the model with one supplier and 2 retailers. Section 3 deduces the optimal order strategy for the retailers. In Section 4, we study the supplier's optimal discount strategy. In section 5, we summarize the paper and describe the future research directions.

2. MODEL DESCRIPTION

Suppose two retailers, which will cooperate during the ordering decision process, order products from the same supplier and each retailer faces independent markets. First, the supplier will set the price curve (p_1, p_2, l), where $p_1 > p_2$. Then the retailers will bid their order quantity. If the sum of the order quantity is equal to or greater than l, then all the retailers pay unit price p_2 to get their order. Otherwise, the unit price is p_1. Suppose retailer i faces the ith markets with unit sale price S_i, and the demand is a random variable with PDF $F_i(\cdot)$. At the end of the auction, the unsold products are perished with salvage 0. All information is common knowledge and the retailers make their decision simultaneously. i.e., they face a statistic game.

3. THE ORDER STRATEGY FOR THE RETAILERS

When no quantity discount is offered, the supplier's unit selling price is p. Then each retailer faces the situation as the newsboy model and the payoff for retailer i is $\int_0^{n_i} S_i x dF_i(x) + \int_{n_i}^{+\infty} S_i n_i dF_i(x) - n_i p$. The optimal order quantity is $n_i^* = F_i^{-1}(\frac{S_i - p}{S_i})$ and the payoff is $\int_0^{n_i^*} S_i x dF_i(x)$. When using the GBA, the wholesale price is related to the sum of the retailers' order quantity, i.e., the wholesale price

$$p = H(n_1 + n_2 - l)p_2 + (1 - H(n_1 + n_2 - l))p_1,$$

where $$H(x) = \begin{cases} 1, & if \quad x \geq 0 \\ 0, & if \quad x < 0 \end{cases}.$$

Hence the payoff for retailer i is

$$\int_0^{n_i} S_i x dF_i(x) + \int_{n_i}^{+\infty} S_i n_i dF_i(x) - n_i (H(n_1 + n_2 - l)p_2 + (1 - H(n_1 + n_2 - l))p_1)$$

When the two retailers cooperate, their optimal strategy is to maximize the whole profit of both retailers, i.e.,

$$\max_{n_1, n_2} (\int_0^{n_1} S_1 x dF_1(x) + \int_{n_1}^{+\infty} S_1 n_1 dF_1(x) + \int_0^{n_2} S_2 x dF_2(x) + \int_{n_2}^{+\infty} S_2 n_2 dF_2(x)) - (n_1 + n_2)(H(n_1 + n_2 - l)p_2 + (1 - H(n_1 + n_2 - l))p_1)$$

This problem can be solved in two steps:

[Step I]

Solve two nonlinear programming problems

Programming I

$$\max_{n_1, n_2} \int_0^{n_1} S_1 x dF_1(x) + \int_{n_1}^{+\infty} S_1 n_1 dF_1(x) + \int_0^{n_2} S_2 x dF_2(x) + \int_{n_2}^{+\infty} S_2 n_2 dF_2(x) - (n_1 + n_2) p_2 \tag{1}$$

$$\text{s.t.} \begin{cases} n_1 + n_2 \geq l \\ n_1 \geq 0 \\ n_2 \geq 0 \end{cases}$$

Programming II

$$\max_{n_1,n_2} \int_0^{n_1} S_1 x dF_1(x) + \int_{n_1}^{+\infty} S_1 n_1 dF_1(x) + \int_0^{n_2} S_2 x dF_2(x) + \int_{n_2}^{+\infty} S_2 n_2 dF_2(x) - (n_1 + n_2) p_1 \quad (2)$$

$$\text{S.T.} \begin{cases} n_1 + n_2 < l \\ n_1 \geq 0 \\ n_2 \geq 0 \end{cases}$$

[Step II]
Comparing the value of the objective function (1) and (2), the larger one is the optimal solution.

Using this method and based on the following theorems, we can get the retailers' optimal order strategy.

Theorem 1 If $n_1^* + n_2^* \geq l$, the optimal order quantity for retailer 1 is $N_1^* = n_1^*$; and that for retailer 2 is $N_2^* = n_2^*$.

Proof: Because

$$\begin{aligned}
&(\int_0^{n_1} S_1 x dF_1(x) + \int_{n_1}^{+\infty} S_1 n_1 dF_1(x) + \int_0^{n_2} S_2 x dF_2(x) \\
&+ \int_{n_2}^{+\infty} S_2 n_2 dF_2(x)) - (n_1 + n_2)(H(n_1 + n_2 - l) p_2 \\
&+ (1 - H(n_1 + n_2 - l)) p_1) \\
&\leq (\int_0^{n_1} S_1 x dF_1(x) + \int_{n_1}^{+\infty} S_1 n_1 dF_1(x) + \int_0^{n_2} S_2 x dF_2(x) \\
&+ \int_{n_2}^{+\infty} S_2 n_2 dF_2(x)) - (n_1 + n_2) p_2
\end{aligned}$$

we can get that

$$\begin{aligned}
&\max_{n_1,n_2}(\int_0^{n_1} S_1 x dF_1(x) + \int_{n_1}^{+\infty} S_1 n_1 dF_1(x) + \int_0^{n_2} S_2 x dF_2(x) \\
&+ \int_{n_2}^{+\infty} S_2 n_2 dF_2(x) - (n_1 + n_2)(H(n_1 + n_2 - l) p_2 \\
&+ (1 - H(n_1 + n_2 - l)) p_1)) \\
&\leq \max_{n_1,n_2}(\int_0^{n_1} S_1 x dF_1(x) + \int_{n_1}^{+\infty} S_1 n_1 dF_1(x) + \int_0^{n_2} S_2 x dF_2(x) \\
&+ \int_{n_2}^{+\infty} S_2 n_2 dF_2(x) - (n_1 + n_2) p_2) \\
&= \int_0^{n_1^*} S_1 x dF_1(x) + \int_{n_1^*}^{+\infty} S_1 n_1^* dF_1(x) + \int_0^{n_2^*} S_2 x dF_2(x) \\
&+ \int_{n_2^*}^{+\infty} S_2 n_2^* dF_2(x) - (n_1^* + n_2^*) p_2)
\end{aligned}$$

since $n_1^* + n_2^* \geq l$, hence

$$\begin{aligned}
&\int_0^{n_1^*} S_1 x dF_1(x) + \int_{n_1^*}^{+\infty} S_1 n_1^* dF_1(x) + \int_0^{n_2^*} S_2 x dF_2(x) \\
&+ \int_{n_2^*}^{+\infty} S_2 n_2^* dF_2(x) - (n_1^* + n_2^*) p_2) \\
&= \int_0^{n_1^*} S_1 x dF_1(x) + \int_{n_1^*}^{+\infty} S_1 n_1^* dF_1(x) + \int_0^{n_2^*} S_2 x dF_2(x) \\
&+ \int_{n_2^*}^{+\infty} S_2 n_2^* dF_2(x) - (n_1^* + n_2^*)(H(n_1^* + n_2^* - l) p_2 \\
&+ (1 - H(n_1^* + n_2^* - l)) p_1))
\end{aligned}$$

Therefore, it can be deduced that (n_1^*, n_2^*)

$$\begin{aligned}
&= \arg\max_{n_1,n_2}(\int_0^{n_1} S_1 x dF_1(x) + \int_{n_1}^{+\infty} S_1 n_1 dF_1(x) + \int_0^{n_2} S_2 x dF_2(x) \\
&+ \int_{n_2}^{+\infty} S_2 n_2 dF_2(x) - (n_1 + n_2)(H(n_1 + n_2 - l) p_2 \\
&+ (1 - H(n_1 + n_2 - l)) p_1))
\end{aligned}$$

Q.E.D

Lemma 1 The equation group

$$\begin{cases} n_1 + n_2 = l \\ S_1(1 - F_1(n_1)) = S_2(1 - F_2(n_2)) \end{cases}$$

has one and only one solution (z_1, z_2).

Proof: The existence of the solution is clear. Now, we will prove the uniqueness of the solution. Suppose there are two pairs of solutions (z_1, z_2), (z_1', z_2'). Because $z_1 + z_2 = z_1' + z_2' = l$, $z_1 - z_1' = -(z_2 - z_2')$, hence $(F_1(z_1) - F_1(z_1'))(F_2(z_2) - F_2(z_2')) < 0$.
Since

$$S_1(1 - F_1(z_1)) = S_2(1 - F_2(z_2)),$$
$$S_1(1 - F_1(z_1')) = S_2(1 - F_2(z_2')),$$

We can deduce that

$$S_1(F_1(z_1) - F_1(z_1')) = S_2(F_2(z_2) - F_2(z_2'))$$

which contrasts to

$(F_1(z_1) - F_1(z_1'))(F_2(z_2) - F_2(z_2')) < 0$. **Q.E.D**.

Suppose

$$\begin{aligned}
\pi_1 &= (\int_0^{z_1} S_1 x dF_1(x) + \int_{z_1}^{+\infty} S_1 z_1 dF_1(x) + \int_0^{z_2} S_2 x dF_2(x) \\
&+ \int_{z_2}^{+\infty} S_2 z_2 dF_2(x)) - l p_2 \\
\pi_2 &= (\int_0^{n_1^*} S_1 x dF_1(x) + \int_{n_1^*}^{+\infty} S_1 n_1^* dF_1(x) + \int_0^{n_2^*} S_2 x dF_2(x) \\
&+ \int_{n_2^*}^{+\infty} S_2 n_2^* dF_2(x)) - (n_1^* + n_2^*) p_1
\end{aligned}$$

Theorem 2 If $n_1^* + n_2^* < l$ and $\pi_1 > \pi_2$, the optimal order quantity for tow retailers is $N_1^* = z_1$; $N_2^* = z_2$ respectively. Otherwise, the optimal solution is

$N_1^* = n_1^*;\ \ N_2^* = n_2^*$

Proof: If $n_1^* + n_2^* < l$ .Using K-T condition, the optimal solution for Programming I is (z_1, z_2) and the objective value is π_1. The optimal solution for Programming II is (n_1^*, n_2^*)) and the objective value is π_2**.** Therefore, we can deduce the result directly.

Q.E.D

Based on the above theorems, the optimal order quantity for retailer i, N_i^*, can be calculated with the following algorithm.

[Step I]

Comparing $n_1^* + n_2^*$ with l, if $n_1^* + n_2^* \geq l$, the optimal retailer 1 is $N_1^* = n_1^*$; and that for retailer 2 is $N_2^* = n_2^*$. Otherwise, we will begin the step II.

[Step II.]

Comparing π_1 with π_2**,** If $n_1^* + n_2^* < l$ and $\pi_1 > \pi_2$, the optimal solution is $N_1^* = z_1;\ N_2^* = z_2$. Otherwise, the optimal solution is $N_1^* = n_1^*;\ \ N_2^* = n_2^*$

4. THE SUPPLIER'S OPTIMAL DISCOUNT STRATEGY

We use a Stackelberg game framework to study the supplier's strategy. When no quantity discount is offered, the supplier's unit selling price is p. Then the supplier's profit is $(p-c)(n_1^* + n_2^*)$ .When using GBA, with price curve Q, suppose the retailer i's optimal order quantity according the former section is $N_i(Q)$, the payoff of the supplier is

$$(H(N_1(Q)+N_2(Q)-l)p_2 + (1-H(N_1(Q)+N_2(Q)-l))$$
$$p_1 - c)(N_1(Q)+N_2(Q))$$

The supplier will determine the discount price p_2 and discount quantity threshold l.

Assumption 1 Suppose if order N_1 and N_2 will bring the retailer the same expected profit and $N_1 < N_2$, the retailer will choose N_2 because more order quantity implies a higher consumer satisfaction.

The supplier will determine the discount price p_2 and discount quantity threshold l.

Let we define

$$\pi = \int_0^{n_1^*} S_1 x dF_1(x) + \int_0^{n_2^*} S_2 x dF_2(x)\,,$$

$n_1(l)$ and $n_2(l)$ s.t. $\begin{cases} n_1(l) + n_2(l) = l \\ S_1(1-F_1(n_1(l))) = S_2(1-F_2(n_2(l))) \end{cases}$,

for any given p_2, suppose the discount threshold is $l(p_2)$ s.t. $l(p_2) > n_1^* + n_2^*$ and

$$\pi = (\int_0^{n_1(l)} S_1 x dF_1(x) + \int_{n_1(l)}^{+\infty} S_1 n_1(l) dF_1(x) + \int_0^{n_2(l)} S_2 x dF_2(x)$$
$$+ \int_{n_2(l)}^{+\infty} S_2 n_2(l) dF_2(x)) - l(p_2)p_2 \,.$$

Lemma 2 For any given p_2, optimal $l^*(p_2) \geq l(p_2)$

Proof: If $l < l(p_2)$

Case 1: if $l \leq n_1^* + n_2^*$, the sold quantity is $n_1^* + n_2^*$ and the price is p_2. The supplier's profit is $(n_1^* p_2 + n_2^* p_2)(p_2 - c) < l^*(p_2)(p_2 - c)$.

Case 2: if $n_1^* + n_2^* < l < l(p_2)$, the sold quantity is l, and the supplier's profit is $l(p_2 - c) < l^*(p_2)(p_2 - c)$.

Hence set $l < l(p_2)$ is not optimal for the supplier.

Q.E.D

If $l > l(p_2)$, the retailer i's order quantity is n_i^* and the supplier's profit is $(p_1 - c)(n_1^* + n_2^*)$. When $l = l(p_2)$, the supplier's profit is $(p_2 - c)l = (p_2 - c)l(p_2)$ .Let $p_2^* = \arg\max_{p_2}(p_2 - c)l(p_2)$ .Then, the supplier's optimal profit is $(p_2^* - c)l(p_2^*)$.

Theorem 3 If $l = l(p_2)$ and $(p_2^* - c)l(p_2^*)) > n_1^* + n_2^*$, the optimal price curve is (p, p_2^*;l); else, trading price will keep p_1, i.e. the supplier should not use GBA, i.e., do not offer any discount.

Employing the GBA, all the supplier and retailers will get more profit, i.e., it is a Pareto improvement.

5. CONCLUSIONS AND FUTURE RESEARCH

In this paper, we employ the group-buying auction as a special pricing mechanism and study the supplier and retailers' discount strategy and order strategy in the B2B e-Market. After building up the model framework, we optimize the retailers' payoff and deduce the optimal order strategy. Then, from the perspective of the supplier, the optimal discount strategy is also studied. We conclude that with the GBA, all members of a supply chain will improve their profits.

In our model we assume that the retailers are cooperative with each other during the ordering decision. However, if they make the decision completely independently, it is another story. Hence, it is interesting and meaningful to study such a case.

Another important area for the future research is to extend the number of the retailers from 2 to k. It is more practical and helpful for the decision making of the supplier.

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