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Economics of Electronic Micro-Payment Systems

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ABSTRACT

Despite the potential of micro-payment systems very few systems have been successful. Micro-payment markets exhibit two-sided network externalities and the business models for these markets are not very well understood. By using a parsimonious game theoretic model, this paper studies factors affecting the existence of a market for micro-payment systems, how the users and merchants choose their acceptance levels, and how a profit maximizing system provider sets the prices under the presence of two-sided network effects. We find that there is a ‘survival mass’ of merchants and users for the market to exist and a ‘critical mass’ for the acceptance levels to take off and remain stable. There is also a lower bound for the user and merchant demands. The conditions for the existence of the market are derived. We find the non-intuitive result that lowering the user-side adoption cost will actually hurt the chances for the micro-payment market to exist. Anecdotal evidence supports this. We also find that to achieve full acceptance levels, the system provider needs to subsidize both users and merchants, which is not feasible in practice.

Keywords

Network Externalities, Two-sided Market, Micro-payment Systems, Smart Card Technology, Electronic Cash, Game Theory

1. INTRODUCTION

The idea of having a cashless world has long been around. The costs of handling cash are high compared to that of electronic money. Printing, distributing and controlling cash are estimated to cost a developed economy 0.75% of annual GDP and an emerging economy 1% to 2% (Banker Middle East, 2003). Social savings of using electronic micro-payment means over cash are substantial.

Given the huge potential savings electronic micro-payment can bring about, there is much room for profits. Electronic micro-payment is also essential for all kinds of electronic and mobile commerce. This further enhances the incentives for firms to enter this market. Consequently, major credit card operators and financial institutions had been trying to capitalize on this business throughout the 90s. Initiatives like Mondex and Visa Cash (Westland, 1998; Westland, Kwok, Shu, Kwok, and Ho, 1997) got little success. In an early pilot test (Hove, 2000), the acceptance levels of both Mondex and Visa Cash were disappointing.

The Octopus card was originally a fare-payment smart card for the Hong Kong passenger transportation system. A joint venture firm called Creative Star Limited was formed by the five major public transportation operators to develop the system. It was introduced to the public in 1997, targeting a public transportation market with 10 million passenger journeys per day and total daily transactions valuing over 2.5 million dollars (Poon and Chau, 2001). A critical mass was quickly gained and the Octopus card system is now growing to support non-transportation micro-payment transactions too. With over 7 million cards been issued, it is now the closest thing to an electronic-cash system anywhere in the world (Yoon, 2001).

Success of the Octopus card has attracted a lot of attention. There are many similar applications in other parts of the world – Singapore, Belgium, Germany, Denmark, Sweden and Austria are just a few examples (CPSS Survey, 2001). The Octopus card is simply the most successful among them.

The overwhelming success of the Octopus card is surprising. Other options (Mondex, Visa Cash) are equally or even better supported but have failed to attain a comparable level of acceptance. Technically, the Octopus card is not more secure (Hong
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Kong Economic Times, 2000). It also does not have a gigantic client base compared with Visa and Mondex, which are supported by Visa Card and Master Card respectively.

The micro-payment market is a two-sided market: involving users and merchants, two very different parties. This unique market structure has two important implications. First, the demand for a micro-payment system must come jointly from users and merchants. Second, system providers face the well-known chicken and egg dilemma: users want merchants on board and vice versa. The business models of these two-sided markets are not very well understood and this may lead to the mixed results of various micro-payment initiatives.

To investigate the business model implications of such a two-sided market structure, we study factors affecting the existence of a market for micro-payment systems, how the users and merchants choose their acceptance levels, and how the profit maximizing system provider sets the prices, by examining network externalities that arise from the usage of such systems. Normally, network externalities derive directly from the number of users in the network. Telecommunication networks are a common example. Network externalities of electronic payment systems are different - the value of the system to users depends on the number of merchants adopting the system and vice versa. We study the following research questions by developing an analytical model: What determines the user and merchant acceptance levels? What are the factors affecting the existence of a market for micro-payment system? What are the pricing guidelines for system providers?

Our findings have important managerial implications. We find that there is a “survival mass” of merchants and users for the market to exist and a “critical mass” for the acceptance levels to take off and remain stable. There is also a lower bound for the user and merchant demands. The conditions for the existence of the market are derived. We find the non-intuitive result that lowering the user-side adoption cost will actually hurt the chances for the micro-payment market to exist. An early pilot test on Visa Cash and Mondex (Hove, 2000) supports this result. Also we find that to achieve full acceptance levels, the system provider needs to subsidize both users and merchants.

The remaining parts of this paper proceed as follows: Section 2 briefly reviews previous studies related to electronic payment systems, network externalities and multiproduct pricing. Section 3 describes the model used. Section 4 analyses the interaction between users and merchants. Section 5 lays out the analysis of the pricing decisions in a monopoly market. Section 6 concludes and discusses the results, and identifies future research directions.

2. LITERATURE REVIEW

When the value of a product depends on the number of users, the product exhibits network externalities. When the value increases with the number of users, there are positive network externalities. Consider a telephone network. The network is more valuable when there are more users with whom one can communicate. Network externalities could be negative too. When there are too many cars, i.e. users, on the road, undesirable traffic jams occur.

Network externalities form a broad stream of economic literature. Economides (1996) provides an excellent survey. Researches have shown that network externalities have significant impacts on firm strategies and consumer behaviors. The failure of Dvorak Simplified Keyboard is a classic example (David, 1985). Studies in the spreadsheet market demonstrate how network effects cause higher prices and set common standards (Brynjolfsson and Kemerer, 1996; Gandal, 1994). In studies of Electronic Data Interchange (EDI) networks, Wang and Seidmann (1995) show the presence of both positive and negative network externalities. Riggins, Kriebel, and Mukhopadhyay (1994) investigate how a buyer can attract suppliers to its EDI network while Barua and Lee (1997) study how subsidizing suppliers can increase the adoption of an EDI system.

Hove (2000) illustrated that network externality and communication channel are the two most important factors governing the adoption decision for an electronic payment system of users and merchants. Firms can use communication channels to disseminate information in their favor so that users and merchants are more willing to adopt by having a better understanding of the benefits of using the electronic payment system.

However, when it comes to network externality, things are complex. There is a joint demand requirement and a chicken-and-egg dilemma. Merchants will not adopt the system unless there is sufficient number of users. At the same time, users will not consider the system until there are enough adopted merchants. Figure 1 depicts this market structure.
Thus, the value of an electronic payment system, in the eyes of users, increases as more merchants join in. On the other hand, the value of the system in the eyes of merchants increases as more users adopt. This dilemma makes the market dynamics complicated and there are no guidelines for system providers to set their pricing strategies. The determinants of acceptance level in such markets are also not very well understood.

In a previous research, these types of interdependent network effects are called as two-sided network externalities (Yoo, Choudhary and Mukhopadhyay, 2002). We adopt this term. The research of two-sided network externalities studied B2B marketplace. The context of electronic payment systems differs from this in three important ways. First, users and merchants are two clearly separate entities while a participant of a B2B marketplace could act as both a buyer and a seller. Second, we have no negative network effects for the market of electronic micro-payment systems. B2B marketplace could exhibit significant negative network effects due to competitions for businesses among participants. Finally, when there is zero acceptance on any side of the market, an electronic payment system has no value while a B2B marketplace can still generate values by providing information.

Rochet and Tirole (2001) investigate the price allocation and welfare implications in two-sided market competition. Their work linked up network economics and multiproduct pricing together by analyzing the price allocation question in the presence of two-sided network externalities. Their model is of very general nature and provides valuable starting points in analyzing two-sided markets. However, issues important to our context, like the asymmetry nature of merchant side and user side, are not considered.

We apply game theory to examine the non-cooperative outcomes of a monopoly market in which the system provider, merchants and users each act in a way to maximize their own payoffs. With two-sided network externalities in effect, the strategic interaction among the three players is our focus in this analysis. Game theory is the right tool for such multi-agent decision problems.

3. MODEL SPECIFICATIONS

Consider a market with three parties: a system provider, a number of merchants, and a number of users. Users and merchants trade with each other. These trades are settled with transactions of small amount. Each small amount trade is one transaction in our model. The Octopus card system is an example of a micro-payment based market – the joint venture firm is the system provider, the different public transportation operators are the merchants and the general public are the users. A transaction takes place when a user pays a public transportation operator with either cash or the Octopus card.

A two-stage game is used to model the market. The system provider, users and merchants are the players. Each player knows everyone’s payoff. In stage one, the system provider offers a pair of prices $P_m$ and $P_u$ to the merchants and users
respectively to use the system. In stage two, both users and merchants observe the offered prices and then move (decide whether to adopt) simultaneously. The resulting proportion of adopted users and merchants are $D_u$ and $D_m$ respectively. Hence, we have $0 \leq D_u, D_m \leq 1$.

### 3.1 User preference

Users are heterogeneous in the number of transactions made with the merchants. A user type corresponds to the number of transactions he/she makes. Two users are of the same type if they make the same number of transactions. A user $i$ does a certain number of transactions $\theta_i q_i$. The type of a user $i$ is denoted by $\theta_i$ and we assume that $\theta_i$ distribute uniformly over $[0,1]$. The transaction frequency of a particular user market is captured by $q_i$. For instance, a transportation market has a much higher transaction frequency (higher $q_i$) than a retail one.

There are several benefits an adopted user can get each time he/she uses the micro-payment system vis-à-vis cash. For example, the user can decrease the time needed for the payment process. The process of payment is also simplified in most cases. For instance, when paying for transportation fares in Hong Kong, an Octopus user can decrease the time for payment and avoid the troubles to take cash or some stored value cards out of the wallet.

Time and convenience constitutes a major benefit of adopting and using an electronic payment system. This comes from the contactless nature of the card in most cases. Technology plays a dominant role in realizing the system benefits. All benefits per transaction are summarized and represented by $b^U$.

Researches suggested that the nature of transactions is important to user preference (Hove, 2000). Some types of transactions can create higher values for users. Unattended point-of-sales applications are one example. These are uses for which cash is truly inconvenient (Clemons, Croson, and Weber, 1996; Weaver 1998). Under this model, these considerations can be easily handled by using different values for $b^U$.

By observing prices $P_u$ and $P_m$, users form an expectation on merchant acceptance. At equilibrium, this expectation equals the resulting merchant acceptance $D_m$ (Katz & Shapiro, 1985). The more the number of merchants adopting the system, the more different places an adopted user can use the system. In short, users benefits from having more merchants on board and prefer a higher $D_m$. In particular, users cannot use the system with no merchant adopting it. Consequently, a user who makes $\theta_i q_i$ transactions gets a gross benefit $D_m \theta_i q_i b^U$.

The gross benefit $D_m \theta_i q_i b^U$ is totally dependent on the network effect of the merchant market to a user $i$. Therefore, the intensity of the network effect is determined by the type of the user. In other words, the user’s frequency of using the micro-payment system defines his/her intensity of the network effect.

If a user decides to adopt the electronic payment system, he/she needs to learn how to use the micro-payment system. Denotes this learning cost by $\phi^U$, which is assumed the same for all users. System design features will be one important determinant for the learning cost. For example, using a contactless smart card is considered to be simpler and easier to learn than using a magnetic card.

Summarizing all the above, the net surplus of a particular user $i$ is:

$$U_i = D_m \theta_i q_i b^U - P_u - \phi^U$$

if he/she decides to adopt the electronic payment system. Otherwise, the user gets the reservation utility. Without loss of generality, we assume a zero reservation utility. Therefore, we have $U_i = 0$ if the user decides not to adopt the electronic payment system. We further denote the overall user-side system benefits $q_i b^U$, which partly due to the transaction frequency $q_i$, and partly due to the technology $b^U$, by $B^U$. Obviously, a user will adopt the electronic payment system if and only if

$$U_i = D_m \theta_i B^U - P_u - \phi^U \geq 0$$
3.2 Merchant preference

Similarly, merchants are heterogeneous in number of transactions makes with users and the number of transactions a merchant makes defines its type. A merchant \( j \) makes \( \theta_j n \) transactions. Hence the type of a merchant \( j \) is represented by \( \theta_j \). Assume that \( \theta_j \) is distributed uniformly over \([0,1]\). The transaction frequency of the merchant market is captured by \( n \).

The merchant expects a user demand \( D_u \).

For each transaction, a merchant can get \( b^M \) benefits. These mainly come from the savings generated by reducing the needs to handle cash and decreasing frauds. For the passenger transportation sector in Hong Kong, processing of coin payments could cost up to 4% of the fares collected (Poon and Chau, 2001). The performance of the underlying technology will be a major factor on how much savings can be generated.

To adopt and use the micro-payment system, merchants need to install readers and terminals. Their staffs are trained on processing payments using the new system. All these one-time costs are represented by \( \phi^M \). These costs are partly determined by the nature of the technology, which defines equipment requirements and user interface effectiveness.

Hence the net surplus an adopted merchant can get is:

\[
V_j = D_u \theta_j n b^M - P_m - \phi^M
\]

If a merchant does not adopt the electronic payment system, it will get the reservation utility. Without loss of generality, we assume a zero reservation utility. Here we also denote the overall system benefits as \( B^M = nb^M \). Therefore, a merchant will adopt the electronic payment system if and only if

\[
V_j = D_u \theta_j B^M - P_m - \phi^M \geq 0
\]

4. USER AND MERCHANT INTERACTION

Suppose the marginal user and merchant (i.e. the user and merchant who is indifferent between adopting the system and staying without it) are of type \( \hat{\theta} \) and \( \hat{\vartheta} \). By uniform distributions of \( \theta_j \) and \( \vartheta_j \):

\[
D_u = 1 - \hat{\theta} \quad \text{and} \quad D_m = 1 - \hat{\vartheta}
\]

By simple computation, zero utilities for the marginal user and the marginal merchant will imply:

\[
\begin{align*}
D_u &= 1 - \frac{P_u + \phi^U}{B^U D_m} \\
D_m &= 1 - \frac{P_m + \phi^M}{B^M D_u}
\end{align*}
\]

The two equations are the best response functions of the two markets (users and merchants). Denote the user-side normalized cost of adoption \( \frac{P_u + \phi^U}{B^U} \) by \( r_u \), and the merchant-side normalized cost of adoption \( \frac{P_m + \phi^M}{B^M} \) by \( r_m \). Solving the above two equation gives:

\[
\begin{align*}
D_m &= \frac{(1-r_m + r_u) \pm \sqrt{(1-r_u + r_m)^2 - 4r_u}}{2} \\
D_u &= \frac{(1-r_u + r_m) \pm \sqrt{(1-r_u + r_m)^2 - 4r_m}}{2}
\end{align*}
\]
In general, there are two solutions for $D_u$ and $D_m$. Graphically, the two best response functions will intersect as below:

From Figure 2 we can see that there is a minimum level of user (merchant) acceptance required to have any positive merchant (user) acceptance. We call these levels the survival masses. It is interesting to see that the merchant-side (user-side) survival masses (i.e. the minimum required level of user (merchant) acceptance) is determined by the normalized cost of adoption of the merchant (user) market while the equilibrium merchant and user demands are determined by the normalized costs of adoption of both sides of the market.

The survival masses are the minimum user and merchant demands needed for a market of the micro-payment to exist. Figure 3 summarized this phenomenon.

However, simply attaining the survival masses cannot guarantee a stable market. There are two intersection points of the two best response functions. Both are possible equilibrium points. One corresponds to low acceptance levels while other corresponds to high acceptance levels.

When taking a closer look at the solution, we can see that only the high acceptance level point is a stable equilibrium. The market will automatically go back to the high equilibrium point given any small deviations due to some shocks.

The low equilibrium point may be interpreted as the point of critical mass. A small deviation below this point will cause the acceptance levels to go to zero. If the deviation is above, acceptance levels automatically go to the high equilibrium point due to market forces. Any network size below the critical mass will have negative expectations dominant and the acceptance level tends to go to zero, while any network size above will ignite positive expectations and the acceptance level goes to a very high level without much difficulty – exactly the process described by Shapiro and Varian (1999).

In a nutshell, a level of survival mass is required for a market to exist while a level of critical mass has to be exceeded to have a stable market.

The lower bound of equilibrium acceptance levels can also be established.
Proposition 1: The optimal user and merchant acceptance levels are both above 50%:

\[ D_u^* > \frac{1}{2}, D_m^* > \frac{1}{2} \]

Proof: Please refer to appendix for proof details.

To have micro-payment system based market (in other words, at any Nash equilibrium), the optimal acceptance levels for both sides of the market (user and merchant) has to be greater than 50%. This is the critical acceptance level that has to be exceeded for the market to be stable.

5. PRICING DECISION IN A MONOPOLY MARKET

In stage one, the system provider offers prices to users and merchants. The system provider selects prices to maximize profit. Assume that there are only fixed costs and we drop the fixed cost notation. Hence the profit function is:

\[ \pi = P_u D_u + P_m D_m \]

It is widely observed that merchants have a greater incentive to adopt the micro-payment system, since the cost savings are great:

Assumption 1: Merchants, if adopting the micro-payment system, have more benefits than users. That is, \( B^M > B^U \).

It usually costs much more for merchants to adopt the system, since it involves installing the necessary equipments and training the staff to use it. The users on the other hand only need to get the card and learn how to use it:

Assumption 2: It costs more for merchants than users to adopt the micro-payment system. That is, \( \phi^M > \phi^U \).

The two assumptions simply represent market facts and thus have face validity.
5.1 Interior Solution

**Proposition 2:** Given the conditions below:

(i) \( B^U > \frac{B^M}{2} \);

(ii) \( \frac{B^M}{\phi^M} > \frac{1}{2} \frac{B^U}{\phi^U} \)

(iii) \( B^U > 4\phi^M \),

there exists a unique sub-game perfect Nash equilibrium. The optimal price and acceptance levels for the user and merchant side of the market are given by:

\[
D_u^* = \alpha
\]

\[
D_m^* = -\frac{1}{2} \left( \frac{B^U \alpha^2 - B^U \alpha - B^M \alpha + \phi^M}{B^M \alpha} \right)
\]

\[
P_u^* = -\frac{1}{2} \left( \frac{B^U (1-\alpha)(B^U \alpha^2 - B^U \alpha - B^M \alpha + \phi^M)}{B^M \alpha} \right) - \phi^U
\]

\[
P_m^* = (1 + \frac{1}{2} \left( \frac{B^U \alpha^2 - B^U \alpha - B^M \alpha + \phi^M}{B^M \alpha} \right))B^M \alpha - \phi^M
\]

where \( \alpha \) is the second largest root of the equation:

\[
f(x) = 3(B^U)^2 x^4 - 4B^U (B^U + B^M) x^3 + ((B^U)^2 + (B^M)^2 + 2B^M B^U + 2\phi^M B^U - 4\phi^U B^M) x^2 - (\phi^M)^2 = 0
\]

**Proof:** For proof details, please refer to appendix.

The interpretation of conditions i-iii is as follows. There is a market for the micro-payment system if the user-side system benefit is large enough relative to the merchant-side (condition (i)), the merchant side market has a high enough benefit-to-cost ratio relative to the user side (condition (ii)), and the user-side benefit, relative to the merchant-side adoption cost, is high enough (condition (iii)).

Achieving the three conditions through adjusting market parameters, on the other hand, is not that simple. For instance, surprisingly, lowering the user-side adoption cost, instead of decreasing the lower bound condition requirement, will increase the lower bound requirement of condition (ii). Thus contrary to expectations, lowering the user-side adoption cost alone will not help enable the market. Table 1 details the various effects of market parameters on equilibrium conditions.

<table>
<thead>
<tr>
<th>Conditions for Equilibrium</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>User-side System Benefit ( B^U )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Merchant-side System Benefit ( B^M )</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>User-side Adoption Cost ( \phi^U )</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Merchant-side Adoption Cost ( \phi^M )</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 1:** Impact of Market Parameters on the Conditions for Equilibrium
5.2 Boundary Solution

**Proposition 3**: To achieve full acceptance level for either side of the market, subsidization is required.

**Proof**: By the inverse demand functions,

\[ P_u^* = -\phi^U < 0 \Leftrightarrow D_u^* = 1 \quad \text{and} \quad P_m^* = -\phi^M < 0 \Leftrightarrow D_m^* = 1. \] [QED]

To achieve full acceptance level for either side of the market, subsidization is required. Hence, it is not feasible for the system provider to have full acceptance levels for both sides of the market.

6. DISCUSSION, CONCLUSION AND IMPLICATION

Using a parsimonious model of two-sided network externalities, we analyse the interaction of users and merchants and the strategy of a system provider in the context of micro-payment. Our results have important managerial insights for micro-payment system markets, and help managers to better understand business models in the digital economy that show two sided network externalities.

**Implications**

There is a “survival mass” of merchants and users for the market to exist and a “critical mass” for the acceptance levels to take off and remain stable. There is also a lower bound, which equals to 50%, for the user and merchant demands. This is the critical acceptance level that has to be achieved.

The Octopus card system attained acceptance levels of more than 50% on both sides of the market right from the beginning. The initial participation of the five major public transportation operators guaranteed the necessary merchant and user demands.

The relationship between market parameters and the existence of a market for the micro-payment system is complicated. Improving system benefits of either users or merchants has a mixed effect in achieving the conditions for existence of equilibrium. Improving the user side adoption costs may even prohibit a market to exist. Independently considering improving system benefits and/or adoption costs of either market side does not help. It is the complex relationship among market parameters that matters.

This sheds lights on why micro-payment projects, like Visa Cash and Mondex, that are supported by large firms did not take off. Thinking of market parameters independently kills them. For instance, in a large scale pilot project for Visa Cash and Mondex (Hove, 2000), the system provider tried to get the conditions for equilibrium by lowering user adoption costs. From Table 1, we see that such efforts will take the situation further away from the condition required for the existence of a micro-payment system based market.

Finally, to achieve full acceptance level for either side of the market, subsidization is required. Hence, it is not feasible for the system provider to have full acceptance levels for both sides of the market. In fact, no such situation is observed from real practice.

**Future Direction**

Our model does not consider the duopoly case. Future research may develop duopoly models to investigate competitive decisions of system providers offering different technologies. Researchers may also consider whether it is possible to have different system providers serving different market segments and if a first mover has any advantages over new entries. It will also be interesting to see if the divide-and-conquer strategy is a good one in different market structures.

Finally, researchers may further investigate the institutional question. What institutional settings will be optimal for micro-payment systems? In the credit card market, and also Mondex and Visa Cash, the payment system is owned by an independent firm. For the Octopus card, it is owned by the public transportation operators who are merchants themselves. This difference in institutional settings could be an explanation for success. In additions, the Octopus card – the most successful micro-payment system so far – operates locally in Hong Kong. The possibility of globalising it like the credit card will be an interesting question. The authors expect that globalisation may require different institutional settings than local operations.
REFERENCES

18. Westland, C. Mondex Electronic Cash in Hong Kong. (1998) Hong Kong University of Science and Technology Business Case Study, Hong Kong University of Science and Technology, Hong Kong.
APPENDIX

Proof of Proposition 1:

For the two best response functions to intersect:

\[
\frac{P_u + \phi^U}{B^U} < 1 - \frac{P_m + \phi^M}{B^M} \Rightarrow \frac{P_u + \phi^U}{B^U} + \frac{P_m + \phi^M}{B^M} < 1
\]

Optimal acceptance levels \( D_u^* \) and \( D_m^* \) are given by the larger root of the following two equations respectively:

\[
f(u) = 0 = u^2 - (1 - \frac{P_u + \phi^U}{B^U} + \frac{P_m + \phi^M}{B^M})u + \frac{P_m + \phi^M}{B^M} \\
g(v) = 0 = v^2 - (1 + \frac{P_u + \phi^U}{B^U} - \frac{P_m + \phi^M}{B^M})v + \frac{P_u + \phi^U}{B^U}
\]

Both \( f(u) \) and \( g(v) \) are convex.

By \( f\left(\frac{1}{2}\right) = g\left(\frac{1}{2}\right) = \frac{1}{2} \left( \frac{P_u + \phi^U}{B^U} + \frac{P_m + \phi^M}{B^M} - 1 \right) < 0 \) as \( \frac{P_u + \phi^U}{B^U} + \frac{P_m + \phi^M}{B^M} < 1 \),

\[D_u^* > \frac{1}{2}, D_m^* > \frac{1}{2}. \] [QED]

Proof of Proposition 2:

We prove the proposition in 3 steps:

Step 1: Restate the profit maximization problem using the inverse demand function instead of best response functions, as the two are mathematically equivalent and the inverse demand function representation is simpler to solve.

Step 2: Solve the problem using first order conditions and get the 4th order polynomial \( f(x) \). Two of the three second order conditions (the second partial derivatives with respect to acceptance levels) are clearly satisfied. In verifying the third second order condition, we first prove that it has positive values by showing that it is a concave function in \((D_u, D_m)\) and its boundary points on the \((D_u, D_m)\) plane all give positive functional values.

Step 3: Prove that only the second largest root of \( f(x) \) is the equilibrium solution. Here we use Rolle’s Theorem, which states that stationary points of a polynomial define the upper and/or lower bound of the value of the corresponding roots. We show that except the second largest root, all others are out of the range between 50% and 100% acceptances.

Here are the details:

Step 1: The inverse demand functions are:

\[
P_u = (1 - D_u) B^U D_m - \phi^U \\
P_m = (1 - D_m) B^M D_u - \phi^M
\]

By substituting them into the profit function of the system provider, the optimization becomes:

\[
\max_{D_u, D_m} (\pi = ((1 - D_u) B^U D_m - \phi^U) D_u + ((1 - D_m) B^M D_u - \phi^M) D_m)
\]

Step 2:

Using the first order conditions:
\[ D_u^* = \alpha \]
\[ D_m^* = -\frac{1}{2} \frac{B^U \alpha^2 - (B^U + B^M)\alpha + \phi^M}{B^M \alpha} \]

where \( \alpha \) is the root of the equation:

\[
f(x) = 3(B^U)^2 x^4 - 4B^U (B^U + B^M)x^3 + ((B^U)^2 + (B^M)^2 + 2B^M B^U + 2\phi^M B^U - 4\phi^U B^M)x^2 - (\phi^M)^2 = 0
\]

The second order conditions are:

\[
SOC1 = \frac{\partial^2 \pi}{\partial D_u^2} = -2 B^U D_m < 0
\]
\[
SOC2 = \frac{\partial^2 \pi}{\partial D_m^2} = -2 B^M D_u < 0
\]
\[
SOC3 = \frac{\partial^2 \pi}{\partial D_u^2} \frac{\partial^2 \pi}{\partial D_m^2} - \left( \frac{\partial^2 \pi}{\partial D_u \partial D_m} \right)^2
\]
\[
= 4 B^U B^M D_u D_m - ((1 - 2 D_u) B^U - (1 - 2 D_m) B^M)^2 > 0
\]

SOC1 and SOC2 are obviously true. For the last condition, note that

SOC3 is concave in \((D_u, D_m)\) as:

\[
\frac{\partial^2 SOC3}{\partial D_u^2} = -8(B^U)^2 < 0
\]
\[
\frac{\partial^2 SOC3}{\partial D_m^2} = -8(B^M)^2 < 0
\]
\[
\left( \frac{\partial^2 SOC3}{\partial D_u^2} \right) \left( \frac{\partial^2 SOC3}{\partial D_m^2} \right) - \left( \frac{\partial^2 SOC3}{\partial D_u \partial D_m} \right)^2 = 48(B^U B^M)^2 > 0
\]

We need to prove that \( SOC3 > 0 \) for possible optimal acceptance levels. The optimal levels lie between 1/2 and 1:

\[
SOC3 \bigg|_{D_u = D_m = 1/2} = B^U B^M > 0
\]
\[
SOC3 \bigg|_{D_u = D_m = 1} = 4B^U B^M - (B^U + B^M)^2 = (B^U - B^M)^2 > 0
\]
\[
SOC3 \bigg|_{D_u = 1, D_m = 1/2} = 2B^U B^M - B^U = (2B^M - B^U)B^U > 0 \text{ since } B^M > B^U
\]
\[
SOC3 \bigg|_{D_u = 1/2, D_m = 1} = 2B^U B^M - B^M = (2B^U - B^M)B^M > 0 \text{ as long as } B^U > \frac{B^M}{2}
\]
Hence, $D_u^*, D_m^*$ is a solution provided that $B^U > \frac{B^M}{2}$.

**Step 3:**

We proceed to prove that only one of four possible values for $\alpha$ is the solution. Let roots of $f(x) = 0$ be $\alpha_1 \geq \alpha_2 \geq \alpha_3 \geq \alpha_4$ and $\beta_1 > \beta_2 > \beta_3$ be roots of $f''(x) = 0$.

Solving $f'(x) = 0$ gives $x = 0$ and:

$$g(x) = 6(B^U)^2 x^2 - 12B^U(B^U + B^M)x + 2((B^U)^2 + (B^M)^2 + 2B^MB^U + 2\phi^MB^U - 4\phi^U B^M)$$

$$= 0$$

$$g(0) = 2((B^U)^2 + (B^M)^2 + 2B^MB^U + 2\phi^MB^U - 4\phi^U B^M)$$

$$> 2((B^U)^2 + (B^M)^2 + 2B^MB^U + 2\phi^U B^U - 4\phi^U B^U) \text{ as } B^M > B^U, \phi^M > \phi^U$$

$$= 2((B^U)^2 + (B^M)^2 + 2(B^M - \phi^U)B^U) > 0 \text{ as } B^M > B^U > \phi^U$$

$$g\left(\frac{1}{2}\right) = 4[8(B^M)^2 - 10(B^U)^2 + 16\phi^M B^U - 8B^U B^M - 32\phi^U B^M]$$

$$= 4[8B^U B^M \left(\frac{B^M}{B^U} - 1 - 10 \frac{B^U}{B^M} + 16\phi^M \phi^U \left(\frac{B^U}{\phi^U} - 2 \frac{B^M}{\phi^M}\right)\right)]$$

$$< 0 \text{ as long as } B^U > \frac{B^M}{2}, \frac{B^M}{\phi^M} > \frac{1}{2} \frac{B^U}{\phi^U}$$

$$g''(x) = 12(B^U)^2 > 0, \text{ i.e., } g(x) \text{ is convex},$$

Therefore, $\beta_1 > \frac{1}{2} > \beta_2 > 0 = \beta_3$.

Solving $g'(x) = 12(B^U)x - 12B^U(B^U + B^M) = 0$ gives $x = \frac{B^U + B^M}{B^U} > 1 \Rightarrow \beta_1 > 1$.

By Rolle’s theorem, $\alpha_1 > \beta_1 > 1, \frac{1}{2} > \beta_2 > \alpha_3, 0 > \alpha_4$. Thus only $\alpha_2$ is a possible solution. It is the unique solution if it lies between $1/2$ and $1$.

To prove that, we first establish the following inequality:

$$B^M - B^U > 0 > 4(\phi^U - \phi^M) \text{ as } B^M > B^U, \phi^M > \phi^U$$

$$\Rightarrow B^M - 4\phi^U > B^U - 4\phi^M$$

$$\Rightarrow (B^M - 4\phi^U)^2 > (B^U - 4\phi^M)^2 \text{ as long as } B^M > 4\phi^U, B^U > 4\phi^M$$

By $B^M > B^U$ and $\phi^M > \phi^U, B^U > 4\phi^M \Rightarrow B^M > 4\phi^U$. 

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\[ f\left(\frac{1}{2}\right) = 16\left(4B^M\right)^2 - 16\phi^U B^M - (B^U)^2 - 16\left(\phi^M\right)^2 + 8\phi^M B^U \]
\[ = 16\left(4B^M (B^M - 4\phi^U) - (B^U - 4\phi^M)^2\right) \]
\[ > 16\left((B^M - 4\phi^U)^2 - (B^U - 4\phi^M)^2\right) \]
\[ > 0 \text{ since } (B^M - 4\phi^U)^2 > (B^U - 4\phi^M)^2 \text{ as long as } B^M > 4\phi^U, B^U > 4\phi^M \]

\[ f(1) = (B^M)^2 - 2B^U B^M - 4\phi^U B^M + 2\phi^M B^U - (\phi^M)^2 \]
\[ = B^M (B^M - 2B^U) - 2\phi^M \phi^U (2\frac{B^M}{\phi^M} - \frac{B^U}{\phi^U}) - (\phi^M)^2 \]
\[ < 0 \text{ as long as } B^U > \frac{B^M}{2} \text{ and } \frac{B^M}{\phi^M} > \frac{1}{2} \frac{B^U}{\phi^U} \]

Together with the fact that \( \beta_1 > 1 > \frac{1}{2} > \beta_2 \) and \( \beta_1 > \alpha_2 > \beta_2 \) by Rolle’s Theorem, we can conclude that \( 1 > \alpha_2 > \frac{1}{2} \) . [QED]