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DYNAMICS OF PLATFORM COMPETITION: EXPLORING THE ROLE OF INSTALLED BASE, PLATFORM QUALITY AND CONSUMER EXPECTATIONS

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Abstract

Researchers debate the role of installed base, platform quality and consumer expectations in driving the success of platforms. We analyze a dynamic model in which a new entrant with superior quality competes with an incumbent platform, and examine the long-run market outcomes. We find that the driver of market dynamics depends critically on the strength of indirect network effects and the consumer discount factor of future applications. We empirically examine the competition between the Xbox and PlayStation 2 consoles. We find that Xbox has a small quality advantage over PlayStation 2, and that the strength of indirect network effects and the discount factor in this market are within the range in which the market dynamics are quality driven. Counterfactual experiments suggest that PlayStation 2 could drive Xbox out of the market if the strength of indirect network effects more than doubles or the consumer discount factor increases by fifty percent.

Keywords: platform competition, indirect network effects, platform quality, two-sided markets, video game industry
Introduction

Many high-technology markets are mediated by platforms. These markets are often viewed as two-sided, since platform providers must get both consumers and developers of complementary applications on board in order to succeed (e.g., Evans 2003; Parker and Alstyne 2005). Indirect network effects are prevalent in these markets (e.g., Eisenmann et al. 2006). They arise because of an interdependence between demands for platforms and demands for their associated applications: having more applications on a platform leads to greater demand for that platform; at the same time, a larger installed base of consumers leads to a larger supply of applications.

Researchers debate the role of installed base, platform quality and consumer expectations in driving the success of a platform. Some scholars argue that the success of a platform is driven by its installed-base advantage. As new users value the size of an existing network, a platform that has a small lead on both sides of the market is likely to take over the entire market even if its quality is inferior to its rivals (e.g., Arthur 1989, 1994; Shapiro and Varian 1999). The QWERTY keyboard and the VHS video-recording format are examples of allegedly inferior standards that won the battles over the Dvorak keyboard and the Betamax format because of indirect network effects (David 1985; Cusumano et al. 1992). In the Microsoft trial, the government claimed that indirect network effects materially strengthened Microsoft’s monopoly power (Bresnahan 2002).

Some scholars argue that consumer expectations are the most critical factor in determining market domination. This expectation-driven view is largely supported by static models of indirect network effects. In these models, consumers often form rational, or self-fulfilled, expectations regarding the market size of each platform. Expectations are important as each consumer prefers to adopt the platform that will be adopted by the majority of other consumers to maximize his benefit from application provision. When two platforms compete with each other, there often exist “monopoly equilibria” in which all consumers and application developers adopt one platform, and an “oligopolistic equilibrium” in which the two platforms share the market on each side. The monopoly outcome occurs when consumers and developers hold favorable expectations of one platform—they believe everyone else will adopt the same platform. As entrants lack installed bases, consumers tend to hold favorable expectations of the established platforms. Therefore this view also suggests first-mover advantages and predicts that incumbents are likely to dominate the markets.

Other scholars challenge these views. Rangan and Adner (2001) point out that there is no guarantee that the benefit from network effects will go to the first mover and emphasize the importance of being the best. Liebowitz (2002) argues that even in cases where the winner takes all, having the highest quality matters more than being first to market in these markets. Indeed, many studies have shown in various contexts that quality has a significant positive influence on market share (e.g., Jacobson and Aaker 1985; Sethi 2000) and stock market returns (e.g., Aaker and Jacobson 1994; Johnson and Tellis forthcoming).

Understanding the drivers of platform success has profound implications. Policy makers could use this information to determine the likelihood that a society is locked into inferior technologies and decide whether to intervene. Platform providers could make more informed decisions regarding entry strategies. The three views outlined above would lead to distinct implications. The installed-base driven view emphasizes the importance of early leads and suggests that platform providers should rush to the market even if their products are premature. The expectation-driven view emphasizes the importance of expectation management. Both the installed-base driven view and the expectation-driven view suggest that a potential entrant, even with a superior technology, may not succeed in the market given that indirect network effects protect the incumbent. Thus the market outcome could be socially inefficient and government intervention may be necessary. On the other hand, the quality-driven view suggests that the market is potentially efficient. According to this view, platform providers should invest heavily in R&D to improve their quality and only enter the market when their products are superior.

In this paper, we analyze the role of each of these three factors in driving the success of platforms. We first build a dynamic model in which a new entrant with superior quality competes with an incumbent platform, and examine the long-run market outcomes. We find that the driver of market dynamics depends critically on two parameters: the strength of indirect network effects and the consumer discount factor of future applications. There are three cases. First, when the two parameters are below certain thresholds, the market dynamics are quality driven: both indirect

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1 A platform is a system with well-defined access points and rules on which other parties can build applications or services (Iansiti and Levien 2004; Eisenmann et al. 2006).
network effects and forward-looking consumer behavior enhance quality advantage and lead to an efficient outcome whereby the superior platform has a larger market share. Interestingly, in this case the long-run market structure is independent of the initial installed bases. Second, when the strength of indirect network effects is large and the discount factor is small, the market dynamics are driven by installed-base advantage and inefficient outcomes may appear with the incumbent becoming the monopoly. Finally, when the discount factor is close to 1, consumer expectations drive market dynamics. In this case, the platform with favorable consumer expectations will dominate the market. Our results provide a positive reconciliation of the mixed views: each of the three factors could drive market dynamics under different values of the strength of indirect network effects and the consumer discount factor in this market. The results also suggest that one cannot make an \textit{a priori} statement for the market outcome and empirical analysis is essential to understand market dynamics.

We empirically examine the competition between the Xbox and PlayStation 2 consoles. We find that Xbox has a small quality advantage over PlayStation 2, and that the strength of indirect network effects and the discount factor in this market are within the range in which the market dynamics are quality driven. Counterfactual experiments suggest that PlayStation 2 could drive Xbox out of the market if the strength of indirect network effects more than doubles or the consumer discount factor increases by fifty percent. These results help explain the successful entry of Xbox into the home video game market and provide support for our theoretical model.

\textbf{Literature Review}

Our work is related to two strands of literature. First, as platform-based markets are two-sided, our work extends theoretical literature on network effects and two-sided markets. Network effects and two-sided markets have been studied extensively, mostly in the context of static models (e.g. Farrell and Saloner 1985, 1986; Katz and Shapiro 1985; Caillaud and Jullien 2003; Rochet and Tirole 2003; Evans et al. 2005; Parker and Alstyne 2005; Armstrong 2007, among others). These static models often lead to multiple equilibria as a result of the increasing return to demand in these markets. In many cases, the market can either be monopolistic or oligopolistic in equilibrium. As industries characterized by network effects are among the most dynamic industries, there is a need to develop dynamic models to address the equilibrium selection problem.

There have been only a handful of dynamic models on network effects. These models often focus on direct network effects, in which consumers benefit directly from the existence of other consumers, rather than indirect network effects. In addition, consumers are often assumed to be myopic (e.g. Kandori and Rob 1998; Auriol and Benaïm 2000; Drinea et al. 2002; Zhu and Mitzenmacher 2005), or only live for a single period (e.g. Skrzypacz and Mitchell 2006; Economides et al. 2005). As most network effects arise in indirect manner (Rochet and Tirole 2003), we examine indirect network effects, and allow both consumers and application developers to forward-look and live for many periods.

Our work also contributes to the empirical literature on indirect network effects. Researchers have examined indirect network effects in the context of digital television, DVD and Divx players, yellow pages, personal digital assistants and home video games (e.g. Gupta et al. 1999; Gandal et al. 2000; Dranove and Gandal 2003; Rysman 2004; Nair et al. 2004; Bayus and Shankar 2003; Park 2004; Clements and Ohashi 2005; Stremersch et al. 2007). All of these studies, with the exception of Gandal et al. (2000) and Park (2004), reply on static frameworks. An implicit assumption of these static approaches is that both consumers and application developers act myopically. Gandal et al. (2000) analyze dynamic demand for a market with a homogeneous product (DVD players), while we study a market with two competing products. Park (2004) analyzes the competition between VHS and Betamax. As Park does not have data on the number of movie titles available for each technology, he essentially models indirect network effects as if they were direct: consumer utility is a function of the installed base of consumers rather than movie variety. Our empirical study builds on the approaches in Gandal et al. (2000) and Park (2004), and considers forward-looking behavior in a market with differentiated products.

\textbf{The Model}

We consider two competing platforms, a new entrant $E$ and an incumbent $I$. Each platform is associated with a group of consumers and application developers on each side of the market. The two platform technologies are incompatible with each other: applications developed for one platform cannot be used on the other platform. Each
application developer supplies one application. Consumers are assumed to single-home: they adopt one platform only.

The timing is as follows. In each period, (1) a group of new consumers choose platforms and purchase available applications, (2) a group of new application developers choose the platforms, incur fixed costs and sell their applications to the installed base of consumers. The two actions occur simultaneously. When we go to the next period, the same set of actions is repeated. We assume, for simplicity, that each consumer allocates a fixed budget, $y$, to purchase applications in each period.

We also assume that the two platforms are priced at the same level. This assumption allows us to focus on the interaction of installed base, platform quality and consumer expectations. Such intentional simplification is a time-honored approach and is used frequently in laboratory experiments and simulations. In addition, this assumption is valid for many markets where the platforms are based on non-proprietary technologies or sponsored by advertisers.

**Consumer Adoption**

We use a representative consumer approach and model the consumer preferences of the applications using a modified CES (Constant Elasticity of Substitution) utility function. The representative consumer approach provides an aggregate description of an underlying population characterized by discrete choices at the individual level—while an individual consumer often purchases only a few of the available applications, the representative consumer typically buys some of every application. This approach may abstract away some dynamics of application demand, but it retains the fundamental interdependence between platform demand and application supply.

We first examine the myopic case in which a consumer’s utility from platform $j$ is derived from the number of applications associated with platform $j$ at the time of adoption, and relax this assumption in Section 5. We use $b_{jt}$ and $d_{jt}$ to denote the installed base of consumers and the total number of applications (equivalently, the total number of developers) associated with platform $j \in \{E,I\}$ at the beginning of period $t$. The utility from adopting platform $j$ in period $t$ is: $u_{jt} = \ln (Q_j z_{jt})$, where $z_{jt} = (\sum_{k=1}^{d_{jt}} x_{kit})^\beta$ and $x_{kit}$ is the amount of applications a consumer purchases from developer $k$ at time $t$. $\beta \geq 1$ is a constant. Thus $z_{jt}$ captures indirect network effects resulting from consumer preference for application variety. $Q_j$ is the platform quality. Our functional form of $u_{jt}$ allows both quality effect and indirect network effects to occur simultaneously. We use logarithm transformation so that the utility function is concave in the application variety and platform quality. We assume that $Q_j$ is constant over the life-cycle of the platforms.

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2Even though some consumers may own multiple platforms (i.e., multi-homing), at a given point in time, they are usually using one platform.
3It is possible that the fixed costs are incurred at a different time or over a long period. In such cases, we can evaluate the total fixed costs at the time of entry.
4Anderson et al. (1992) show that CES models can be constructed as being representative of a population of consumers making discrete choices.
5Many studies have used CES utility functions to model product variety (see, for example, Spence 1976; Chou and Shy 1990; Church and Gandal 1992, 1993; Nair et al. 2004).
6Consumers may also take application quality into consideration. We do not use a separate measure for application quality for two reasons. First, platform quality is often more important. The purchase experience on an auction site depends critically on the design and security protection of the site. Second, platform quality is often highly correlated with application quality. For example, applications written for a more powerful platform tend to run faster. Therefore, we use a single measure $Q_j$ to capture the overall quality.
7Similar transformations have been used in other research. For example, Nair et al. (2004) use $g(z_j) = z_j^{\frac{1}{\alpha}}$, $(\alpha \geq 1)$, to ensure concavity. We choose the log-transformation to derive a more analytically tractable form.
8Essentially, when a platform is upgraded, we consider it as a new platform. Major platforms are often upgraded infrequently. For example, major upgrades to operating systems and Web browsers happen every few years, and new generations of video game consoles are released every six years on average. Therefore, we assume $Q_j$ does not change over time and leave the competitive dynamics over multiple generations for future research.
A new consumer maximizes his utility subject to the budget constraint \( \sum_{k=1}^{d_j} \rho_{kjt} x_{kjt} \leq y \), where \( \rho_{kjt} \) is the price of the application sold by developer \( k \). The consumer’s optimal demand for each application can be derived as:

\[
x_{kjt}^* = \frac{y \cdot \phi_{jt}^{1/(\beta-1)}}{\rho_{kjt}^{\beta/(\beta-1)}}
\]

where \( \phi_{jt} = \left( \sum_{k=1}^{d_j} \rho_{kjt} \right)^{1-\beta} \) is often referred to as the price index for the applications.

A developer maximizes his total profit. Assume the marginal cost of each application developed for platform \( j \), \( mc_j \), is the same. There exists a symmetric equilibrium in which the prices of applications for each platform are the same, i.e., \( \rho_{kjt} = \rho_j = \beta mc_j \). While the marginal cost of producing many digital goods is close to zero, platform providers often charge developers for applications sold. For instance, game console providers charge royalty fees for each game sold for their consoles. Auction houses also charge each seller a listing fee. In addition, the marginal cost of each application does not have to be the payment to the console providers. For example, in the software industry, the marginal cost could include the cost of making a CD and providing service support to end users.

The demand for each application thus becomes \( x_{kjt}^* = \frac{y}{d_j \rho_j} \). Therefore, the demand for each application increases with the amount of budget each consumer allocates, and decreases with the total number of applications and application price. Substituting this demand expression into the utility function, we obtain the indirect utility function of the consumer:

\[
V_{jt} = \ln y + \ln \frac{Q_j}{\rho_j} + e \ln d_j,
\]

where \( e = \beta - 1 > 0 \) and \( \frac{Q_j}{\rho_j} \) is the platform quality adjusted by the application price.

We use the standard logit model to capture heterogeneity in consumer tastes in platforms. In particular, we assume that the unobserved random portion of a consumer’s utility independently and identically follows a Type-I extreme-value (Gumbel) distribution. Hence, the percentage of consumers choosing platform \( j \) in period \( t \) is (McFadden 1973):

\[
s_{Et} = \frac{Q \cdot d_{Et}^e}{Q \cdot d_{Et}^e + d_{It}^e} \quad \text{and} \quad s_{It} = \frac{d_{It}^e}{Q \cdot d_{Et}^e + d_{It}^e},
\]

where \( Q = \frac{Q_j}{\rho_j} \). We refer to \( Q \) as the price-adjusted quality ratio of the two platforms on the consumer side. It measures the quality advantage of the entrant over the incumbent. From equation (2), we have:

\[
\frac{s_{Et}}{s_{It}} = Q \left( \frac{d_{Et}}{d_{It}} \right)^e.
\]

We use \( e \) as our measure for the strength of indirect network effects as it measures how much platform demand at time \( t \) responds to the change in the ratio between the number of associated applications.

**Developer Entry**

Each existing consumer purchases \( x_{kjt}^* = \frac{y}{\rho_{kjt}^{\beta/(\beta-1)}} \) amount of applications from each new developer in period \( t \). Therefore, each new developer in period \( t \) pays a fixed cost of \( F_j \) and earns \( \pi_j = b_j \cdot x_{kjt}^* \cdot \left( \rho_{jt} - mc_j \right) \).

Developers choose application prices, \( \rho_{jt} \), to maximize their profits. Given the concavity of the profit function, the first-order condition requires that each developer set the price such that his marginal revenue equals his marginal cost. Following the literature (e.g. Nair et al. 2004), we assume that, given the large number of applications available in the market, the effect of a single application’s price on the aggregate price index, \( \phi_{jt} \), is negligible and can be
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ignored. The price elasticity of game demand, $\eta_j$, can then be obtained as $\beta(\beta-1)$. Therefore, the marginal revenue of the developer is $\rho_j(1-\eta_j)/\eta_j = \rho_j/\beta$. We solve for the optimal price by equating the marginal revenue to the marginal cost and have $\rho_j = \beta mc_j$. Hence the maximal profit is $\pi_j = \frac{y \cdot b_j \cdot (\beta - 1)}{\Delta d_{jt} \beta}$, where $\Delta d_{jt}$ is the equilibrium number of new developers supporting platform $j$ at time $t$. Thereafter, new consumers purchase applications from each developer, and similarly, $\pi_{jt} = \frac{y_{jt} \cdot b_j \cdot (\beta - 1)}{\Delta d_{jt} \beta}$ for $s > t$, where $\Delta b_{jt}$ is the number of new consumers adopting platform $j$ at time $s$.

We follow the approach due to Gandal et al. (2000) to analyze the dynamics on the developer side. The total profit the developer entering in period $t$ earns is $-F_{jt} + \pi_{jt} + \varphi_d \pi_{j,t+1} + \varphi_d' \pi_{j,t+2} + \cdots$, where $\varphi_d$ is the discount factor of future earnings for the developers.

If the developer enters in period $t+1$, he earns a discounted profit evaluated at period $t$ of $-\varphi_d F_{j,t+1} + \varphi_d \pi'_{j,t+1} + \varphi_d' \pi'_{j,t+2} + \cdots$, where $\pi'_{j,t+1} = \frac{y_{jt} \cdot b_j \cdot (\beta - 1)}{\Delta d_{jt} \beta}$ and $\pi'_{j,t+i} = \pi_{j,t+i}$ for $i \geq 2$. As the cost of developing applications usually drops over time in reality, we assume $F_{j,t+1} < F_{jt}$. A free-entry condition requires that developers are indifferent between the two options. This implies

$$(-F_{jt} + \pi_{jt} + \varphi_d \pi_{j,t+1} + \varphi_d' \pi_{j,t+2} + \cdots) = (-\varphi_d F_{j,t+1} + \varphi_d \pi'_{j,t+1} + \varphi_d' \pi'_{j,t+2} + \cdots),$$

(4)

Given the large installed base of consumers and applications, and relatively small number of new consumers and new applications in each period, we have $\frac{b_{jt+1}}{\Delta d_{jt+1}} \gg \frac{\Delta b_{jt+1}}{\Delta d_{jt+1}}$ and $\pi_{jt} \approx \pi'_{j,t+1}$. Therefore, we can simplify equation (4) as:

$$F_{jt} - \varphi_d F_{j,t+1} = \frac{(1-\varphi_d) \cdot y \cdot (\beta - 1)}{\Delta d_{jt} \beta} \cdot b_{jt},$$

(5)

The left-hand side represents the gain from waiting, and the right-hand side represents the cost of waiting. Assume the development cost drops at the same rate for both platforms: $F_{j,t+1} = (1-\gamma')F_{jt}$, where $0 < \gamma' < 1$, so that the fixed cost drops at a decreasing rate over time and does not converge to zero when $t$ becomes very large. We have:

$$\Delta d_{jt} = \frac{\alpha_t}{F_{jt}} \cdot b_{jt},$$

(6)

where $\alpha_t = \frac{(1-\varphi_d) \cdot y \cdot (\beta - 1)}{\beta(1-\varphi_d'(1-\gamma'))}$. The equation suggests that exogenous reductions in fixed cost, $F_{jt}$, and increases in the installed base of consumers, $b_{jt}$, could induce more application developers to enter the market. Let $F = F_{jt}/F_{Et}$ be the ratio of the fixed costs for developers to support each platform. As $F_{kt}$ and $F_{Et}$ drop at the same rate, $F$ does not vary over time. $F$ measures the quality advantage of platform $E$ over platform $I$ on the developer side. In equilibrium, the percentage of new developers supporting platform $E$ is $\frac{\Delta b_{kt}}{\Delta d_{kt} + \Delta b_{kt}} = \frac{F_{kt}}{F_{Et} - F_{kt}}$, and the percentage for platform $I$ is $\frac{b_{kt}}{F_{kt} + b_{kt}}$. Therefore, a platform is more likely to be supported if the cost of application development is lower, or its installed base is larger.
**Long-Run Equilibrium Analysis**

We now extend the one-period model into multiple periods. When we consider multiple periods, it is important to take into consideration the decay in the installed base of consumers and applications. Consumers may cease to use platforms or switch to other ones. Application popularity tends to decrease over time and consumers may not value outdated ones.

We use \( \delta_b \in [0,1] \) and \( \delta_d \in [0,1] \) to denote the “rate of decay” (or “death rate”) of the installed base and associated applications. Let \( M_t \) be the total number of new consumers at time \( t \). We can express the change in the installed base of platform \( E \) as

\[
\dot{b}_E = M_t \cdot \frac{Q \cdot d^E_t}{Q \cdot d^E_t + d^E_h} - \delta_b^E b_E - \delta_h^E b_h.
\]

The equation is intuitive: the change of the installed base of platform \( E \) is the number of new consumers adopting platform \( E \) less the number of existing consumers who exit the installed base in a given period. By incorporating \( \delta_b \), we essentially allow consumers to re-enter the potential market and re-consider their platform choices. We expect \( \delta_b \) to decrease with the switching cost: the more costly it is to switch, the lower the rate of decay of the installed base.

We apply the same approach to the developer side and obtain a system of four equations:

\[
\begin{align*}
\dot{d}_E &= \frac{\alpha_E}{F_E} - \delta_d^E d_E, \\
\dot{d}_h &= \frac{\alpha_h}{F_h} - \delta_d^h d_h,
\end{align*}
\]

**Proposition 1.** The ultimate market structure depends on the strength of indirect network effects, \( e \):

1. When \( e > 1 \), the market evolves towards a monopoly. That is, one platform eventually dominates the market.
2. When \( e < 1 \), the market evolves towards an oligopoly. That is, both platforms co-exist in the long run. In the long-run equilibrium, the ratio between the number of consumers in the two platforms is \( Q \frac{e}{F} \), and the ratio between the number of developers is \( (QF)^{\frac{e}{F}} \).

**Proof.** See the Appendix.

Proposition 1 suggests that the market may evolve toward one of the two regimes: a monopoly regime and an oligopoly regime. The market structure transits from one regime to the other at \( e = 1 \). The intuition is that for a monopoly platform to emerge, indirect network effects have to be strong enough so that the market share advantage increases over time. When the indirect network effects are not sufficiently strong, an initial installed-base advantage of the incumbent will actually diminish over time and eventually the market reaches a steady state.

We are particularly interested in the relative impact of installed-base advantage and quality advantage on the dynamics and efficiency of the market when a later platform enters with superior quality. Therefore, in the following discussion, we assume \( Q \geq 1 \) and \( F \geq 1 \), and \( b_{E,0} < b_{I,0} \) and \( d_{E,0} < d_{I,0} \). That is, the new entrant, \( E \), has quality advantage but the incumbent, \( I \), has installed-base advantage. Based on Proposition 1, we have the following corollary:

**Corollary 1.** When \( e < 1 \), as \( e \) increases, the market share of platform \( E \) in the long-run equilibrium increases. The equilibrium market shares on both sides are independent of the initial installed bases.

Contrary to the popular belief that indirect network effects always protect the incumbent, in this case indirect network effects actually enhance quality advantage of the entrant. When \( e \) approaches 1, the entrant’s market share approaches 100% on each side even if its quality advantage is small. The intuition is that since indirect network effects are not sufficiently strong, the installed-base advantage of the incumbent diminishes. Because of the presence
of indirect network effects, however, more consumers will take the same action and buy the higher quality platform. Therefore, the market shares of the entrant become larger due to indirect network effects on both sides. Furthermore, Corollary 1 indicates that initial installed bases have no effects on the market outcome in the long run. This result is striking as it contrasts with predictions of static models of indirect network effects. It suggests that in the oligopoly regime, the installed-base advantage may not present barriers to entry, and that platform success is entirely driven by quality advantage.

While we have derived a closed-form solution for the long-run market share in the oligopoly regime, we do not have a closed-form solution to determine which platform will become the monopoly in the monopoly regime. We use numerical simulations to obtain insights. Simulation results indicate the following:

**Proposition 2.** There exists a threshold \( e^* \) such that when \( 1 < e < e^* \), platform \( E \) becomes the monopoly; when \( e > e^* \), platform \( I \) becomes the monopoly. \( e^* \) increases with the quality advantage of platform \( E \).

Hence when \( e < e^* \), indirect network effects enhance quality advantage. The market outcome is efficient as the platform with superior quality will have larger market share on each side. When \( e > e^* \), the platform with inferior quality may dominate the market.

**Forward-looking Consumers**

In our previous analysis, consumers are myopic—they make their decisions based on current state variables and do not consider utility from future available applications. Such an assumption is frequently used in dynamic models on technology adoptions to yield analytically tractable solutions (see, for example, Auriol and Benaïm 2000; Casadesus-Masanell and Ghemawat 2006). In markets where consumers need to incur large fixed costs for access to a platform, they may take into account not only the current utility but also the expected future utilities from new applications. Consumer expectations thus could play an important role in shaping the market dynamics. We extend our model to a setting in which consumers are forward-looking.

We assume that consumers form rational expectations of the number of new applications in future periods. Let \( T \) be the life-expectancy of the platform. In this case, we have \( u_{jt} = \ln Q_j + \sum_{s=t}^{T} \ln z_{js} \), where \( z_{jt} \) captures utility from current associated applications and \( z_{js} \) captures utility from new applications released in period \( s \). A consumer’s indirect utility function is, therefore,

\[
V_{js} = \ln y_s - \ln \rho_j + e \ln \Delta d_{js},
\]

for \( s = t+1, \cdots, T \). The total utility a consumer derives from platform \( j \) can be obtained as:

\[
V_{j\text{Total}} = \beta_y + \ln Q_j / \rho_j + e \ln N_{jt},
\]

where \( \beta_y = \ln y_t + \sum_{s=t+1}^{T} \phi^{s-t} (\ln y_s - \ln \rho_j) \) and \( N_{jt} = \exp \left( \ln d_{jt} + \sum_{s=t+1}^{T} \phi^{s-t} \ln \Delta d_{js} \right) \) is the discount factor of future utility for consumers. When \( \phi = 0 \), consumers place no value on future applications and we are back to the myopic case. As \( \phi \) approaches 1, consumers are patient and consider future applications to be as important as those currently available.

Following the same approach as in the myopic case, we derive a system of four equations:

\[
\dot{b}_{Et} = M_t \cdot \frac{Q \cdot N_{Et}^e}{Q \cdot N_{Et}^e + N_{Et}^e} - \delta_b b_{Et}, \quad \dot{b}_{It} = M_t \cdot \frac{N_{It}^e}{Q \cdot N_{Et}^e + N_{It}^e} - \delta_b b_{It},
\]

\[
\dot{d}_{Et} = \frac{\alpha_e}{F_{Et}} \cdot b_{Et} - \delta_d d_{Et}, \quad \dot{d}_{It} = \frac{\alpha_e}{F_{It}} \cdot b_{It} - \delta_d d_{It}.
\]
An important feature of this system is that the adoption behavior influences and at the same time is influenced by application provision in the future. As do other structural models of network effects (e.g. Ackerberg and Gowrisankaran forthcoming; Rysman 2004), we must confront the issue of multiple equilibria. Different equilibrium paths reflect the fact that in each period, new consumers could hold different expectations. While it is theoretically possible to compute all equilibria, this approach can take a prohibitive amount of time when the number of periods is large. To tackle this problem of dimensionality, researchers often impose additional constraints to limit the number of possible equilibria.

In our analysis, we assume that all consumers in all periods hold favorable expectations of one platform. As consumers hold favorable expectations of platform \( j \), when there are multiple solutions, we pick the solution that maximizes \( b_{jt} \) and \( d_{jt} \). This assumption is likely to hold when consumers coordinate with each other about their expectations. Equivalently, one may think that we are examining two polar cases. As we illustrate below, these two polar cases provide sufficient insights as to when consumer expectations matter.

The complexity of the model makes analytical solutions intractable. We solve the model numerically. Figure 1 provides simulation results for the case where \( T = 300 \), \( Q = F = 1.2 \) and \( b_{j,0} = d_{j,0} = 20 \). For different levels of \( e \), we plot the equilibrium market share of platform \( E \) on the consumer side as the discount factor, \( \phi \), increases from 0 to 1. The plots for the developer side are similar and thus omitted. When \( \phi = 0 \), we have the myopic outcome. Figure 1.A and 1.B show the results in which all consumers hold favorable expectations for the incumbent and the entrant respectively.

Comparing the two plots, we find that the market outcomes are different only when \( \phi \) is above a threshold for a given \( e \). The market tips the platform of which consumers hold favorable expectations. As \( e \) increases, the threshold decreases. We summarize these results in the next proposition:

**Proposition 3.** When \( \phi > \phi^* \), multiple equilibrium paths exist and the platform with favorable expectations will be the monopoly. \( \phi^* \) decreases with \( e \).

---

9 As the right hand side of each equation is a continuous function, it is immediate from Brouwer’s fixed point theorem that there must exist one fixed point (i.e., at least one solution for \( b_{jt} \) and \( d_{jt} \)).

10 For example, researchers often pose restrictions on the order of entry in empirical models related to entry (e.g. Berry 1992). In addition to imposing constraints, researchers often calibrate computational results with data from a real-world market. For example, Ackerberg and Gowrisankaran (forthcoming) and Rysman (2004) compute a limited set of equilibria and select one by matching them to the data. Ryan and Tucker (2007) use the two-step approach proposed in Bajari et al. (forthcoming) which recovers reduced-formed policy functions as a function of state variables in the first step and projects these functions into the dynamic model in the second step.
Therefore, consumer expectations affect market outcomes only when consumers are sufficiently patient. In this case, the success of a platform is entirely driven by expectations. Intuitively, when \( \varphi \) is large, consumers place large value on future applications. Hence their utilities become more similar to the utilities of future adopters, and they are more likely to take the same actions as future adopters. As a result, the market tips one platform.

We also find that when \( \varphi, \) ad \( e \) are low, consumers’ forward-looking behavior further enhances quality advantage: the market share of platform \( E \) increases as \( \varphi \) increases. Similar to the myopic case, simulation results show that equilibrium market shares are independent of the initial installed-base advantage.

**Proposition 4.** When \( \varphi < \varphi^{**} \) and \( e < e^* \), both indirect network effects and forward-looking behavior enhance quality advantage. In particular, when \( 0 < e < 1 \), the two platforms will co-exist in the long run; when \( 1 < e < e^* \), platform \( E \) will be the monopoly. The equilibrium market shares are independent of the initial installed-base advantage. \( \varphi^{**} \) decreases with \( e \).

Both \( e \) and \( \varphi \) determine the extent to which consumers value future applications. When consumers place a relatively small value on future applications, their beliefs about the evolution of the market have little impact on their adoption decisions and hence their pattern of adoption is close to that of the myopic case.

When \( e \) and \( \varphi \) become larger, we find that the market starts to tip the incumbent. In this case, future application provision becomes an important factor in consumers’ adoption decisions. When \( \varphi \) is not very large, consumers only value applications in the near future and are not patient enough to wait for the entrant to take over the leadership. Hence, the only self-consistent equilibrium path is the one in which the market tips the incumbent. When \( e \) increases, the installed-base advantage becomes more pronounced. Therefore, the range of \( \varphi \) for the market to tip the incumbent increases. We have:

**Proposition 5.** When \( e > e^* \) and \( \varphi^{**} < \varphi < \varphi^* \), platform \( I \) becomes the monopoly. \( \varphi^{**} \) decreases with \( e \), and \( \varphi^* \) increases with \( e \).

We summarize market outcomes for different values of \( e \) and \( \varphi \) in Figure 2.

---

**Figure 2. Summary of Theoretical Results**

Our results indicate that the success of a platform can be driven by different factors under different circumstances. When the values of \( e \) and \( \varphi \) lie in region A of Figure 2, the success is driven by quality advantage. In particular, when \( e > 1 \), the platform with superior quality will be the monopoly. In region B, platform success is driven by installed-base advantage, and the platform with such advantage will become the monopoly. In region C, the success is driven by consumer expectations. The platform with the favorable expectations will become the monopoly.

---

11 We use lines to segment the area for simplicity.
Our results also suggest that the long-run market structure can be either oligopolistic or monopolistic. The two platforms co-exist if and only if the market is quality-driven and \( e < 1 \) (the shaded region). Otherwise, we have the “winner-takes-all” outcome.

**Empirical Analysis**

We study the competition between Sony’s PlayStation 2 and Microsoft’s Xbox consoles. PlayStation 2 was introduced in October 2000 and is backward compatible with PlayStation 1. Xbox was introduced a year later. By the time Xbox was introduced, more than 4.5 million PlayStation 2 consoles had been sold in the United States and more than 1000 compatible game titles were available for PlayStation 2. While previous entrants to this market often came with next generation technology (e.g., Nintendo in 1986 and Sega in 1989), Xbox technology belongs to the same generation as PlayStation 2 (128 bits generation). Anecdotal evidence suggests that the quality difference between the two consoles is small.

Although many industry experts and scholars cast doubt on Xbox’s ability to seize a significant market share, Xbox made successful entry to this market. We compute the percentages by dividing the number for each console by the sum of the two consoles sold in each year. As the numbers indicate, Xbox has been very successful in growing its market shares on both sides. Its shares of installed base and associated games increased over years. In 2004, Xbox had more than 40% shares in both new console sales and new game releases. While the share of new game releases increased to 45% in 2005, the console sales slowed down in 2005, most likely due to the anticipated release of the next generation system, Xbox 360. We also compute the percentage of Xbox games supplied by Sony and the percentage of Xbox games supplied by Microsoft. The data suggest that console providers only produced a very small number of game titles (<10%) and thus third party game publishers are the major supplies of games in this market.

The competition between the two consoles provides an ideal setting for our empirical analysis for two reasons. First, as both consoles target adults between 18 and 34, they position them in direct competition with each other. While several other consoles were also available on the market during this period, they were either targeted at different demographics (such as Nintendo’s GameCube) or were obsolete (such as Nintendo’s N64 and Sega’s DreamCast). PlayStation 2 and Xbox together account for more than 80% of new console sales in 2005.

Second, our theoretical model assumes that platforms are priced at the same level. The pricing strategies of Microsoft and Sony fit this assumption well. The price differences are less than ten dollars in every month except March 2004 and April 2004. This pattern suggests that the two console providers quickly matched each other’s price over time. For example, in May 2002 Microsoft was forced to cut the Xbox price by $100 in response to a similar price reduction of PlayStation 2. As the consoles were offered at similar prices in each period, we expect that consumers made their purchase decisions based on the quality of the consoles and the variety of their associated games.

**Data**

Data on console sales and game sales are from the NPD Group, a leading market research firm that tracks this industry. NPD collects data from approximately 17 leading US retail chains that account for 80% of the U.S. market. From these data, NPD formulates estimates of sales figures for the entire U.S. market. We obtain monthly sales and price data for PlayStation 2 and Xbox consoles and their associated games up to October 2005. Game publishers continued to release new games for the two consoles after October 2005. We collect data on the number of new games released for each console in each month after October 2005 from GameSpot.com. According to Ranking.com, GameSpot.com is the 172th most visited site among all Web domains and is the most popular one on video games.

**Empirical Specifications**

---

1. In our dataset, the monthly sales data in units for games vary from 1 to 1.2 million and game sales decline at an average rate of 36%. As game players are likely to pay attention only to games with significant sales, we only count games that have more than 15,000 copies sold in each month. On average, these games account for more than 60% total game sales in each month for both consoles.

2. Xbox 360 was released in November 2005. Microsoft officially announced its release date in May 2005.
Our objective here is to measure the strength of indirect network effects, \( e \), the discount factor, \( \varphi \), and the two quality ratios, \( Q \) and \( F \). As \( Q \) and \( F \) are ratios, we could use console dummies in regressions to estimate them instead of developing metrics to explicitly measure them.

Equation (3) can be transformed to yield the following specification:

\[
\ln s_{t} = \ln s_{t-1} + \beta_{Q} + \eta \ln N_{t} + \ln N_{t-1} + \beta_{D0}D_{2005} + \xi
\]

where \( \beta_{Q} \) captures the quality advantage of Xbox over PlayStation 2. The quality ratio, \( Q \), can be obtained as \( \exp(\beta_{Q}) \). While the log-difference specification takes away time-specific effects that are common to both consoles, we include a dummy for year 2005 to control for potential cannibalization effects from the planned release of Xbox 360. \( N_{\cdot t} \) measures the game variety, which includes both available games at time \( t \) and discounted games from future releases. As PlayStation 2 consoles can be used to play PlayStation games, PlayStation games are also included as PlayStation 2 games when they have significant sales. Similar to much of the empirical work related to forward-looking consumers, we adopt the “errors-in-variables” approach (e.g., Wickens 1982). That is, as we assume consumer expectations are fulfilled, we could use the actual game release data in the future and express \( N_{\cdot t} \) as a function of \( \eta \).

We now consider the developer side. Based on equation (6), we obtain the following specification:

\[
\ln \Delta d_{j,t} = \beta_{0} + \ln b_{j,t} + \beta_{2}D_{2005} + \sum_{i=3}^{5} \beta_{i}D_{2000+i} + \beta_{3}D_{Holiday} + \xi
\]

where the dependent variable, \( \ln \Delta d_{j,t} \), is the logarithm of the number of new games released for console \( j \) at time \( t \), and \( b_{j,t} \) is the installed base of console \( j \) in period \( t \). The size of the installed base by console and by month is obtained from the cumulative console sales up to the current month, subject to a constant rate of decay. We experiment with different decay rates and compare our data on the console market share with survey results from other sources. We find that the annual decay rate of 10\%, which corresponds to a monthly decay rate of 0.87\%, provides the best match. According to our theoretical model, we expect \( \beta_{1} \) to be 1. The quality ratio, \( F \), can be obtained as \( \exp(\beta_{2}) \). As we do not observe \( y \) and \( F_{\cdot j,t} \) in each period, we include year dummies and a holiday dummy (which equals 1 when the month is November or December and 0 otherwise) to control for their variations over time.

**Results**

Table 1 presents our regression results. Panel A reports results for console adoption on the consumer side. In Model I, we estimate equation (8), assuming consumers to be myopic (i.e., \( \varphi = 0 \)). In Model II, we relax this myopia assumption and use non-linear least square (NLS) estimation. The sample period starts from the introduction of Xbox into the US market, November 2001, to October 2005. Our results indicate significant indirect network effects in this market. The estimated strength of indirect network effects is 0.69 and 0.62 in the two models, suggesting that myopic models may overestimate the strength. We also find that the discount factor is small (0.31). In addition, we find a small quality advantage of Xbox over PlayStation 2. The quality ratio \( Q \) is \( \exp(\beta_{Q}) \approx 1.3 \). Finally, the significant negative coefficients of the dummy for year 2005 suggests that the anticipated release of Xbox 360 significantly slowed down the adoption of Xbox.

We conduct several robustness checks. First, we include the price difference between the two consoles as an additional control variable to test our assumption that price difference does not have a significant impact on the relative console sales. The result from Model III suggests that price difference indeed does not affect consumer choices and is consistent with the observation that two console providers matched each other’s price quickly. We have fewer observations as Xbox entered the Japanese market in February 2002. We are also concerned that game players may only value games with high quality when making purchase decisions. We therefore collect professional ratings for games from GameSpot.com. We only count games with ratings greater than 7.0 on a 10.0 point scale.
Games with scores above 7.0 are considered good according to GameSpot’s rating system. We repeat our analysis and obtain very similar results (Model IV).

Panel B reports results for game supply on the developer side. We use equation (9) as the empirical specification and report the result in Model I. In Model II, we estimate equation (9) without the holiday dummy. In Model III, we also add an interaction variable between the dummy for Xbox and the dummy for year 2005 to control for the potential negative effect from the release of Xbox 360. In Model IV, we only count games with ratings greater than 7.0 on a 10.0 scale at GameSpot.com. We obtain similar results in all four models. We find that the coefficients of $\ln b_{jt}$, $\beta_1$, are above 0.80 in all models. T-tests cannot reject the hypothesis that $\beta_1$ is 1 in any of the models and thus support our theoretical model. We also find that the quality difference between the two consoles on the game developer side is statistically indistinguishable from zero. The negative coefficients of the year dummies and positive coefficients of the holiday dummy suggest that game players might allocate smaller budgets for purchasing games over time and larger budgets during holiday seasons. Finally, we do not detect significant negative impact from the release of Xbox 360 on game supply. This result is most likely due to the fact that Xbox 360 is backward compatible and can be used to play Xbox games.

### Table 1. Regression Results for Both Console Adoption and Game Supply

#### Panel A: Console Adoption

<table>
<thead>
<tr>
<th>Model</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{F}_n$</td>
<td>0.28*</td>
<td>0.30**</td>
<td>0.32</td>
<td>0.28**</td>
</tr>
<tr>
<td></td>
<td>[0.14]</td>
<td>[0.14]</td>
<td>[0.26]</td>
<td>[0.13]</td>
</tr>
<tr>
<td>$\ln N_{E1} - \ln N_{I1}$</td>
<td>0.62***</td>
<td>0.62***</td>
<td>0.64***</td>
<td>0.63***</td>
</tr>
<tr>
<td></td>
<td>[0.10]</td>
<td>[0.10]</td>
<td>[0.12]</td>
<td>[0.11]</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.31*</td>
<td>0.34*</td>
<td>0.36*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.18]</td>
<td>[0.18]</td>
<td>[0.18]</td>
<td></td>
</tr>
<tr>
<td>Dummy2005</td>
<td>-0.60***</td>
<td>-0.61***</td>
<td>-0.62***</td>
<td>-0.60***</td>
</tr>
<tr>
<td></td>
<td>[0.12]</td>
<td>[0.12]</td>
<td>[0.12]</td>
<td>[0.13]</td>
</tr>
<tr>
<td>$D_{2005} - D_{I1}$</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.04]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observations: 47  47  45  47

R-squared: 0.53  0.54  0.53  0.57

#### Panel B: Game Supply

<table>
<thead>
<tr>
<th>Model</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln b_{jt}$</td>
<td>0.81***</td>
<td>0.87***</td>
<td>0.83***</td>
<td>0.87***</td>
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<tr>
<td></td>
<td>[0.25]</td>
<td>[0.30]</td>
<td>[0.28]</td>
<td>[0.22]</td>
</tr>
<tr>
<td>DummyE</td>
<td>0.49</td>
<td>0.56</td>
<td>0.53</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>[0.31]</td>
<td>[0.36]</td>
<td>[0.36]</td>
<td>[0.40]</td>
</tr>
<tr>
<td>DummyHoliday</td>
<td>0.33*</td>
<td>0.33*</td>
<td>0.31*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.20]</td>
<td>[0.20]</td>
<td>[0.20]</td>
<td></td>
</tr>
<tr>
<td>DummyE x Dummy2005</td>
<td>-0.06</td>
<td>-0.04</td>
<td>-0.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.39]</td>
<td>[0.36]</td>
<td>[0.36]</td>
<td></td>
</tr>
<tr>
<td>Dummy2005</td>
<td>-0.50**</td>
<td>-0.53**</td>
<td>-0.58**</td>
<td>-0.74**</td>
</tr>
<tr>
<td></td>
<td>[0.28]</td>
<td>[0.29]</td>
<td>[0.29]</td>
<td>[0.31]</td>
</tr>
<tr>
<td>Dummy2004</td>
<td>-1.03**</td>
<td>-1.03**</td>
<td>-1.06**</td>
<td>-1.04**</td>
</tr>
<tr>
<td></td>
<td>[0.36]</td>
<td>[0.39]</td>
<td>[0.39]</td>
<td>[0.40]</td>
</tr>
<tr>
<td>Dummy2005</td>
<td>-1.08***</td>
<td>-1.21***</td>
<td>-1.08***</td>
<td>-1.20***</td>
</tr>
<tr>
<td></td>
<td>[0.42]</td>
<td>[0.47]</td>
<td>[0.42]</td>
<td>[0.47]</td>
</tr>
</tbody>
</table>

Observations: 94  94  94  94

R-squared: 0.26  0.22  0.26  0.25

Standard errors in brackets. * significant at 10%; ** significant at 5%; *** significant at 1%.
To sum up, our empirical results show that Xbox has a small quality advantage ($Q \approx 1.3$), $e = 0.62$ and $\varphi = 0.31$ in this market. We substitute these data into our dynamic model and find that the value of $e$ and $\varphi$ lie in the range in which the market dynamics are driven by quality advantage in this market.

**Counterfactual Experiments**

We now consider what changes to $e$ and $\varphi$ are needed for PlayStation 2 to drive Xbox out of the market. We use January 2002 as the initial period and simulate the market dynamics by holding one factor at the estimated level and changing the other factor. We find that given $\varphi = 0.31$, $e$ needs to be greater than 1.49 for PlayStation 2 to drive Xbox out of the market. We also find that given $e = 0.62$, the market dynamics are driven by installed base when $\varphi$ is greater than 0.47 but smaller than 0.87. In this case, the market share of Xbox decreases over time. The market dynamics are driven by consumer expectations when $\varphi$ is greater than 0.87.

**Conclusions**

**Managerial Implications**

Our paper provides answers to several important questions with direct managerial implications.

*Does a new platform need to have a huge quality advantage to succeed?*

The conventional wisdom (e.g. Schilling 2003) suggests that if a new platform is unable to make its technology compatible with the incumbent, for it to be successful its technical advantage must offer so much value to consumers that it exceeds the combination of functionality, installed base, and complementary goods value offered by the incumbent. Our results indicate that a huge quality advantage may not be necessary for success. Platforms with a small advantage can also be successful when the market is driven by quality, as in this case both indirect network effects and forward-looking behavior enhance quality advantage.

*Is there first-mover advantage in platform-based markets?*

The traditional theory of network effects and business advice based on this theory suggest that first-movers in platform-based markets are likely to be the winners and that their leadership positions will be difficult to dislodge. Our study shows that this is true only when the market dynamics are installed-base or expectations driven. When the dynamics are quality-driven, however, installed-base advantages do not provide a safety shield for the incumbent. In addition, time to convergence does not vary significantly with installed-base advantages. Therefore, in this case, platform quality is more critical than being the first to market.  

*Are platform-based markets always “winner-takes-all” markets?*

Our study shows that when the strength of indirect network effects and the discount factor are small, multiple platforms can co-exist in the long run. Furthermore, when the market is a “winner-takes-all” market, the winner does not have to the incumbent. For instance, when the market dynamics are driven by consumer expectations, new entrants could use various strategies to gain favorable expectations from the consumers. For example, they could use advertising to promote an impression that their installed bases will soon become very large. They could also use credible commitments to signal that they are determined to win the battle.

*What roles should the government play in platform-based markets?*

Our study shows that in the quality driven scenario, the market is efficient—the platform with superior quality will achieve larger market shares than it would without indirect network effects. Therefore, government intervention may be counter-productive. When the market is installed-base driven, an inefficient outcome will occur. As a result, government intervention such as enforcing standardization and interoperability can help rescue the market from an inefficient outcome. In the expectations driven scenario, government support of the superior platform may help it gain favorable expectations and hence market dominance.

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14 Here we do not consider other potential first-mover advantages such as technological leadership and preemption of assets (Lieberman and Montgomery 1988).
Future Research

In our study, we treat the strength of indirect network effects \((e)\) and the consumer discount factor \((\varphi)\) as exogenously given. Future research could examine the determinants of the two parameters. For example, \(e\) is likely to correlate with the degree of differentiation between the two platforms. When the two platforms have little differentiation, \(e\) should be very large as adoption decisions will be strongly influenced by application variety. Therefore, platform providers could strategically design and position their products to change \(e\).

In addition, we do not consider dynamic pricing of the platforms and assume that platform prices are the same. While this assumption applies to many markets, in some markets prices could be a critical factor that influences market evolution. For example, as the number of applications increases, the incumbent might find it optimal to increase the price of its platform. In this case, the entrant is more likely to survive. In addition, while platform providers can use the insights from our model to decide whether and when to upgrade platforms, our work does not explicitly consider quality upgrade. An account of how firms dynamically set their prices and adjust platform quality, and how consumers’ forward-looking behavior is transmitted into these endogenized price and quality decisions is an empirical challenge even for markets without indirect network effects (e.g. Melnikov 2001; Song and Chintagunta 2003; Gowrisankaran and Rysman 2007). We leave these topics for future research.

Appendix: Proof of Proposition 1

It is convenient to consider a heuristic approach using \(dr_d/dt = b_d\) and \(dd_d/dt = d_d\). We first rewrite the differential equations and then use phase diagrams to illustrate the trajectories of the market. Define the ratio of the number of developers in the two networks as \(r_d = d_d/d_t\) and the ratio of the number of consumers as \(r_b = b_d/b_t\). Then

\[
\frac{dr_d}{dt} = \left(\frac{dd_d}{dt}\right) - \left(\frac{(dd_d)(dd_d)}{dt}\right) = \frac{\alpha b_d}{d_t d_d} \left( Fr_b - r_d \right).
\]

Similarly,

\[
\frac{dr_b}{dt} = \frac{M d_t^e}{b_t (Q \cdot d_d^e + d_b^e)} \left( Qr_b^e - r_b \right).
\]

The directions of the trajectories are determined by the signs of \(dr_d/dt\) and \(dr_b/dt\), which are in turn determined by \(Fr_b - r_d\) and \(Qr_b^e - r_b\). By examining the signs of \(dr_d/dt\) and \(dr_b/dt\), we obtain the phase diagrams in Figure 3. We use arrows to indicate the directions of the trajectories for states in each region. In the cases of \(e > 1\) and \(e < 1\), the two curves intersect at point \(H\), \((r_d = (QF)^\frac{1}{e}, r_b = (QF)^\frac{1}{e} F^{\pi})\).

**Case (a):** \(e > 1\). We have three fixed points: \((0, 0)\), \(H\) and \((\infty, \infty)\). It is easy to see that \(H\) is an unstable fixed point. Therefore, the trajectories naturally converge to \((\infty, \infty)\) or \((0, 0)\). That is, a monopoly emerges.

**Case (b):** \(e < 1\). In this case, only \(H\) is a stable fixed point. The trajectories naturally converge to \(H\) so that the system eventually reaches a state where both platforms exist and have fixed market shares on the two sides. The market shares are determined by \(F\) and \(Q\). In the case where \(F = Q = 1\), the system converges to a state in which the number of consumers and the number of developers are equal for both platforms.

![Figure 3. Phase Diagrams for Different Values of \(e\)](image-url)
References


