Risk and Return in an Electronic Market: An Examination of Investments in Online Rare Coin Auctions Over a Short Horizon

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RISK AND RETURN IN AN ELECTRONIC MARKET:
AN EXAMINATION OF INVESTMENTS IN ONLINE RARE COIN AUCTIONS OVER A SHORT HORIZON

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Abstract

Collectables are often considered as an alternative to stocks for investing. Technological advancements have facilitated collectible investment, allowing information to be transferred worldwide between millions of investors in electronic markets such as online auctions. This research examines the behaviors of these electronic market investors, especially as they compare to traditional stock market investors. This research examines risk and returns in 13,263 auctions involving 755 distinct rare coins collected over 128 days. While stock investors typically ignore risk that can be eliminated through diversification, this research finds evidence that collectible investors who trade in online auctions do not consider diversification over short horizons, but rather they their bid levels include diversifiable risk adjustments, supporting a contention that typical online auction collectible investors view collectibles as stand-alone assets, differing from typical the stock investor’s perspective that diversifiable risk can be ignored when an investment is added to a collection or portfolio.

RISQUES ET BÉNÉFICES SUR UN MARCHÉ ÉLECTRONIQUE: ÉTUDE DES INVESTISSEMENTS DANS LES SITES D'ENCHÈRES NUMISMATIQUES À UN HORIZON DE COURT TERME

Les progrès technologiques ont facilité les investissements dans les articles de collection en permettant à des millions d’investisseurs d’échanger des informations à travers des marchés électroniques telles que les enchères en ligne. Cette recherche examine les risques et les rendements lors de 13 263 enchères de 755 pièces rares, observées sur 128 jours.

Keywords: Asset Pricing, Electronic Markets, Investment, Online Auctions, Portfolio, Rare Coins
Introduction

Periodic bearish leanings of the stock market often leave many investors considering non-stock assets, such as rare coins, as important additions to a diversified portfolio. For example, Swiatek (2001) points out how the cyclical valuation of rare coins can be appealing in a down market. Costello (2002) discusses how coins are a viable alternative to stocks, often acting as a hedge against inflation, much like gold bullion, with some coins having the ability to increase in value. Costello gives examples of coins from a sought-after series often doubling, tripling, or even quadrupling in value after several years.\(^1\)

Internet technology has allowed a transformation of auctions more toward a type of exchange, where millions of people can transact at any time of day. These transactions can be tracked, and large online datasets can be used to examine millions of transactions and facilitate collectible research. Online auctions, particularly eBay, have made investing in collectables much easier. Some businesses target sales solely through the eBay channel, while other businesses, such as Dell and Disney, supplement sales through eBay listings. eBay’s 2005 annual report lists $44.3 billion in revenues involving 71.8 million active buyers and sellers and 546.4 million listings, most of which are available at some point for public viewing and scrutiny. Though not all listings are collectibles (e.g., listings for computers, clothing, CDs, etc.), many of the largest categories have traditionally concentrated on the selling and reselling of collectibles. There has been much research on how various factors that can affect sales price (e.g., reputation (Dellarocas and Wood 2008; Ba and Pavlou 2002), Bidder Type (Bapna, et al. 2000, 2003), previous bids (Kauffman and Wood 2005), just to name a few.)

This research proposes to take price analysis in online auctions to a different level by examining the risk and return in online auctions through the use of financial research tools that track variance in asset pricing. This is the first time that such theory has been applied to auctions, primarily because auction data from traditional (offline) auctions is so thin that it makes the use of these tools impractical. Online auctions, on the other hand, allow millions of collectors to interact. Thus, technology allows the market depth so that collectibles to be compared to the entire collectible market, thus alleviating the need to compare collectibles to the stock market as is done in traditional research. This research examines prices and returns of 755 distinct rare coins sold in 13,263 auctions collected over a 128-day period. Data collected in online auctions can be used to answer difficult research questions pertaining to rare coin investing, especially through online markets, such as:

- What is the appropriate model for risk measurement, as it affects returns, for non-stock investment vehicles such as rare coins purchased in an electronic market?
- How do rare coin collectors who purchase coins in online auctions treat diversification over a short time horizon?
- Has Internet technology transformed online auctions into a type of financial exchange, where financial theory can be applied?

By allowing trading of investment vehicles, online auctions become a type of investment exchange, although clearly not a stock exchange. Rather, Internet technology facilitates investment outside of the typical stock exchange, but with some very large differences. Thus, relatively new technologies make the differences between investing through an exchange and investing through an online auction important for research.

This is believed to be the first study that empirically investigates risk and return for investment-grade assets, specifically rare coins, that are purchased in an electronic market. This is also believed to be the first study to apply an empirical risk-measuring stochastic methodology, typically used to examine stock purchases in the finance literature, to assess how different risks affect purchase prices for rare coins in online auctions.

In this research, returns from distinct rare coins purchased in online auctions are examined, particularly with respect to the relationship to overall stock market returns and overall coin market returns. These findings show that risk

\(^1\) However, Costello (2002) also depicts the risks in rare coin investing, as some coins have flat or declining returns. Burton and Jacobson (1999) dismiss collectibles as hedge investments and find that collective returns, on whole, are less than the risk-free rate. Costello acknowledges this possibility of flat or negative returns, but points out that certain coin investments have to potential for atypical growth, just as prices of certain stocks can rise in a bear or flat market. Some research disagrees with Burton and Jacobson. For instance, Ashenfelter and Graddy (2003) examine collectible investing research and point out how many authors illustrate how collectible returns often surpass that of bond returns.
from online auction purchases affects investors differently than risk from stock market purchases. This research shows that online coin collectors who purchase rare coins in online auctions adjust for idiosyncratic risk, and consider each rare coin investment separately, and not solely in conjunction with the risk the coin adds to an investment portfolio or a collection. This research presents evidence that (a) coin collectors that participate in online auctions do consider variation in prices (financial risk), demanding greater returns for coins with higher variance in price, but that these rare coin collectors do not ignore idiosyncratic risk, but rather consider each coin purchase as a stand-alone investment.

The conclusions from this research suggest possible profitable arbitrage opportunities in collectible online auctions. Coin collectors who ignore diversifiable risk will be willing to pay a more appropriate price for coins than those who refuse to adjust for diversifiable risk and thus be at an advantage when buying and selling collectible investments through online auctions. In addition, this research can be valuable to rare coin sellers and buyers when estimating future prices in collectibles that are purchased online, and call for further research into risk, return, efficiency, and pricing mechanisms for collectibles purchased in online auctions and other electronic markets.

Finally, please note that for those who are unaware of financial research, the appendix contains some financial terms that are used in this study.

### Auction Data and Statistical Power

Using a customized automated data-collecting Internet agent written in Java, a computer programming language well-suited to Web data-collecting tasks, a database was built using data that was collected from eBay online auctions from April 24, 2002 through September 10, 2002. Only auctions selling for over $10 were considered. After the data collection was complete, this study selected 13,263 auctions where the most popular 755 distinct rare coins were repeatedly sold. A distinct rare coin is defined as a coin minted in a specific denomination (e.g., Indian head penny, Buffalo nickel etc.) with specific mint marks or characteristics (e.g., red vs. brown (for copper pennies), D (for Denver mint), doubled die, etc.) in a specific grade (e.g., a numeric number from 3 to 70 accepted by coin collectors as describing a coin’s condition). In cases where a distinct rare coin was sold in more than one auction on the same day, only the last auction of the day was considered, giving a measure akin to the closing price of the stock market. To be considered popular, only coins sold in at least 21 days during the 128-day period of this study were considered, although some sold much more frequently. Figure 1 shows the breakdown of how many days all 755 rare coins sold throughout this study.

Figure 1 divides the 755 coins in this study to show the frequency of trades made by each coin. Because of the detailed division of coins by denomination, grade, year, and mint marks, a large amount of data is required to examine trading patterns. Coins are only considered for this research if they have traded more than 20 times during the period of this study, but because of the ability to gather data on thousands of auctions, a large number of rare coins can be identified that meet these prerequisites.

Note, however, that while 755 coins are examined in this research, each daily trade of each rare coin is examined, resulting in 13,263 observations. A large dataset this large has enough statistical power to detect even extremely small relationships. However, this study investigates which measures of risk are valid, and which are not. Regression tests are not sufficient to reject any model or coefficient. Rather, researchers can only accept the alternative hypothesis as true due to evidence allowing the rejection of the null hypothesis, or fail to accept the

---

2 Proof coins, which are uncirculated coins that are specially treated at the mint, are excluded from this set. Outlier analysis suggested by Neter, et al. (1996) identified 17 outliers that were originally considered in a set of 772 unique rare coins, leaving us with 13,263 repeat sales of 755 unique rare coins to analyze.

3 While larger sample sizes are always preferable when examining asset prices with respect to risk, it is often difficult to find time-series data for non-stock assets, such as collectibles, even though such research is important to those who investigate how non-stock assets affect portfolio returns. This sample size compares favorably to other datasets that examine risk and returns with collectibles. This research examines 13,263 repeat sales of rare coins over 128 periods. Baumol (1986) analyzes 640 repeat sales of paintings over 410 years, (1652-1961). Pesando (1993) analyzes 27,961 sales of painting prints for 32 semi-annual periods (1977-1992). Mei and Moses (2002) examine 4,896 repeat sales of rare art pieces from 1875-2000, and in a later study (2005), 4,957 repeat sales of rare art pieces from 1875-2002 were examined.
alternative hypothesis, by not finding enough evidence to reject the null hypothesis. When empirical results fail to reject the null hypothesis, this does not necessarily stipulate that the null hypothesis is true and the alternative hypothesis is false. It becomes not necessarily accurate to conclude a priori that a failure to detect a relationship directly indicates that no relationship exists.

Andrews (1989) points out that a common problem facing econometricians is that of interpreting results when a test fails to reject a hypothesis. He recommends a power test that enables the econometrician to test a model for the likelihood of failing to reject a false null hypothesis, called a Type II error. Cohen (1992) discusses how power tests can help describe the possibility of a Type II error, where a relationship predicted by a model but then rejected (e.g., that a certain type of risk has an effect on return) is, in actuality, a valid relationship.\footnote{Normal statistical inferences test for a Type I error. (e.g., When a cutoff of p-value < 5% is used, there is a 5% probability of a Type I error and a 95% probability of no Type I error.)}

Table 1 reports power analysis retrieved from GPower (Faul and Erfelder 1992), a free and easy-to-use software package used to perform these power tests. Table 1 describes how the sample size of the dataset can detect small effects (i.e., Cohen’s (1992) criteria for a “small effect” is $R^2 >= 0.01$) at more than 99.99% of the time. Power analysis allows interpretation of failure to reject to mean one of three things: (a) that there is a model misspecification (e.g. a nonlinear relationship exists when a linear relationship is predicted); (b) that the relationship is so small ($R^2 < 0.01$) as to be practically insignificant; or (c) that this is a Type II error, where $R^2 >= 0.01$, which in this research’s case can occur less than .01% of the time.

<table>
<thead>
<tr>
<th>Table 1. Post Hoc Power Tests for Models Used in This Research</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Power Criteria</strong></td>
</tr>
<tr>
<td>$\alpha$ (P-value criterion required for significance)</td>
</tr>
<tr>
<td>Effect Size ($R^2$)</td>
</tr>
<tr>
<td>Sample Size ($N$)</td>
</tr>
<tr>
<td>Power (Prob. of NO Type II error)</td>
</tr>
</tbody>
</table>

Thus, a lack of significance in the empirical model does not prove the null hypothesis, but due to the large sample size in the dataset used for this research, a rejection indicates that either there is no relationship (the null hypothesis is true) or the relationship is so slight as to be practically insignificant or the model is incorrectly specified (e.g., nonlinear relationships fail the linear tests, etc.). The likelihood of misspecification can be reduced by incorporating non-linear tests such as those employed by Fama and MacBeth (1973), and by using methodologies that can handle nonlinearities, such as generalized method of moments (GMM) suggested by Hansen (1982). Both techniques are employed in this research, thus reducing the likelihood of misspecification. Thus, an insignificant test can be interpreted as either there is no relationship, or that the relationship is so minute as to be both undetectable and of little practical value.
Methodology for Risk Examination

In this research, sales data is used from individual rare coins to estimate \( \mu \) on a daily basis over the entire period of this study, presumably as coin collectors react to information both within the rare coin market (e.g., previous sales) and outside the rare coin market (e.g., stock market returns).

Repeat-sale regression (RSR) was first advanced by Bailey et al. (1963) as a means to estimate the value of real estate. RSR has proven to be a valuable tool in analyzing returns on collectible investments as well, such as rare coin returns analyzed in this study. Mei and Moses (2005; 2002), Pesando (1993), and Goetzmann (1993) all employ a RSR methodology to measure collectible returns, specifically in the art market. Mei and Moses (2005) point out that a benefit of using the RSR is that the resulting index is based upon relatives of the same coin, thus controlling for intrinsically different valuations between rare coins, and thus, it does not suffer from arbitrary specifications of a hedonic model.

RSR is implemented by assuming a continuously compounded excess return \(^5\) for a specific rare coin may be represented by a regression equation based upon previous returns. To begin, daily return rates are defined for observations of repeat coin sales, similar to models used by Mei and Moses (2005) and Pesando (1993):

\[
\text{\( r_t \) = \ln \left( \frac{1}{d_{it}} \left( \frac{\text{price}_{it}}{\text{price}_{it-d_{it}}} - r_f \right) \right) }
\]

where:

- \( r_t \) = The daily return rate for rare coin \( i \) selling on day \( t \)
- \( \text{price}_{it} \) = The final selling price for rare coin \( i \) in on day \( t \)
- \( d_{it} \) = The number of days between an observed sale of rare coin \( i \) at time \( t \) and the most recent observation of a sale of rare \( i \) prior to time \( t \).
- \( r_f \) = The daily risk free rate based on the annual 10-year treasury bill rate during week \( t \).

Controlling for Risk

The goal of this research is to examine the effect of different types of risk on rare coin return. The Capital Asset Pricing Model (CAPM) has played an important role in the research of risk and return of collectibles, just has it has with stocks. For example, Pesando (1993) uses CAPM regression to implement his RSR regression to investigate how risk-adjusted returns of rare art prints compare to the stock market, and Mei and Moses (2002 and 2005) use CAPM to control for risk when examining underperformance of masterpiece artworks. The CAPM works well for collectibles since there is no assumption of firm-specific investments (e.g., book value, dividends, etc.), but rather simply that assets sales are being transacted.

However, while stock market information and traditional auction information(e.g., auction prices received at Sotheby’s) both are available for years and decades after each close, online auction data is often purged following 90 days after the sale. As such, an a priori assumption of static risk for all items sold in online auctions is problematic, since much information about past returns is unavailable to the coin auction bidders. However, researchers (e.g., Jagannathan and Wang 1996; Lettau and Ludvigson 2001) demonstrate that the empirical inconsistencies of CAPM (e.g., those demonstrated by Fama and French (1992)) may be resolved by allowing \( \beta \) to vary over time, thus allowing risk estimates to adjust for new information. Equation (2) combines Equation (1) and the time-varying \( \beta \).

\[
\text{\( R_{it} = \beta_{it} \text{MKT}_t + \epsilon_{it} \)}
\]

\[
\text{\( \beta_{it} = \frac{\text{cov}(r_{it}, \text{MKT}_t)}{\sigma^2(\text{MKT}_t)} \)}
\]

\(^5\) For simplicity, the terms “excess return” and “return” are used interchangeably throughout this research. The risk free rate is factored in whenever returns are calculated.
This research follows the methodology of Mei and Moses (2002), as well as the extensions of Jagannathan and Wang (1996), to apply the time-varying $\beta$ to the analysis of online rare coin auctions, allowing risk estimates to adjust to changing conditions within the rare coin online auction market. In this way, this research examines the effect that different types of risk have on returns in online auctions.

**Adjustment for Thinline Traded Investments**

One econometric challenge with much research into collectibles deals with comparing investments that are *infrequently traded* (or *thinline traded*) compared with other investments within the study (Baumol 1986; Pesando 1993). Thin trading results in a misspecification of beta estimates when using regression models like the CAPM. To alleviate the problem resulting from thin trading, Dimson and March (1983) and other research (e.g., Pesando 1993; Scholes and Williams 1983) recommend adjusting empirical models to account for infrequent trades. This research incorporates a thinly traded investment weighting system recommended by Dimson and March when estimating $\beta$:

$$
\beta_{DM}^{it} = \frac{\text{COV}\left(\frac{r_{it}^{NonDM}}{\sqrt{d_{it}}}, \frac{\text{MKT}_{it}^{NonDM}}{\sqrt{d_{it}}}\right)}{\sigma^2\left(\frac{\text{MKT}_{it}^{NonDM}}{\sqrt{d_{it}}}\right)} 
$$

where:

\begin{align*}
\text{MKT}_{it}^{NonDM} &= \text{the market return at time } t \text{ before any adjustment for thin trading} \\
r_{it}^{NonDM} &= \text{the return of investment } i \text{ at time } t \text{ before any adjustment for thin trading} \\
\beta_{DM}^{it} &= \text{the time-varying beta using a thin-trading adjustment recommended by Dimson and March (1983)} \\
d_{it} &= \text{the number of periods since the last trade of the investment } i \text{ at time } t.
\end{align*}

Without adjustment, thinly traded investments have a corrupting effect on $\beta$ estimates. The Dimson-March correction reduces the weight of thinly traded investments in the regression formula, and thus reduces the inflating effect that thinly traded investments have on beta. Note that when investments are traded in consecutive periods, as is often the case with stocks, then the Dimson-March thinly trading adjustment has no effect since, in that circumstance, $\sqrt{d_{it}} = 1$.

**Multiple Risk Premiums in Online Rare Coin Auctions**

Coin collectors, coin investors, and coin dealers usually keep many coins within a collection or inventory. A coin collection or coin inventory, then, can be viewed as an investment portfolio consisting entirely of rare coins. From an investor’s standpoint, coin collectors, dealers, and investors should be aware of overall returns in both the overall **coin market** and the **stock market** that can possibly impact returns of a rare coin. It is feasible that many collectors consider rare coin investments as either (a) part of a collection, (b) part of a larger portfolio consisting of traditional stock investments, or (c) both. Equations (4), (5), and (6) show a time-varying $\beta$ examining how returns of a coin are affected by the **coin market**.

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6 Bradfield (2003) classifies methods that adjust for infrequent trading into two categories: *leading-lagging methods* (called *Cohen methods* by Bradfield) use aggregation of lagged and leading regression coefficients (e.g., Scholes and Williams 1977, Dimson 1979, Cohen, et al., 1983), and *trade-to-trade* methods that weight trades based upon the number of periods between the last trade (e.g., Dimson and March 1983). Following Bradfield, this research uses Dimson and March’s methodology. The Scholes and Williams (1977) leading-lagging method was also tested for consistency, and resulted in similar results in direction of coefficients and significance.
\[ E(r_i) = \beta_{i,\text{Coin}} (E(COINMKT_t)) \]  
\[ \beta_{i,\text{Coin}} = \frac{\text{cov}(r_i, COINMKT_t)}{\sigma^2(COINMKT_t)} \]  
\[ s_{i,\text{Coin}} = \sqrt{\sigma^2(r_i) - \beta_{i,\text{Coin}}^2 \text{cov}(r_i, COINMKT_t)} \]

where:

- \( COINMKT_t \) = the Dimson-March weighted return of the coin market at time \( t \)
- \( \beta_{i,\text{Coin}} \) = the Dimson-March adjusted measure of non-diversifiable risk that coin \( i \) adds to the rare coin market at time \( t \)
- \( s_{i,\text{Coin}} \) = the Dimson-March adjusted measure of risk that is idiosyncratic (diversifiable) by combination with the rare coin market at time \( t \), expected to be eliminated due to diversification

By contrast, Equations (7), (8), and (9) show a time-varying \( \beta \) examining how returns of a coin are affected by the stock market.

\[ E(r_i) = \beta_{i,\text{Stock}} (E(STOCKMKT_t)) \]  
\[ \beta_{i,\text{Stock}} = \frac{\text{cov}(r_i, STOCKMKT_t)}{\sigma^2(STOCKMKT_t)} \]  
\[ s_{i,\text{Stock}} = \sqrt{\sigma^2(r_i) - \beta_{i,\text{Stock}}^2 \text{cov}(r_i, STOCKMKT_t)} \]

where:

- \( STOCKMKT_t \) = the Dimson-March weighted return of the stock market as measured by the S&P 500 at time \( t \)
- \( \beta_{i,\text{Stock}} \) = the Dimson-March adjusted measure of non-diversifiable risk that coin \( i \) adds to the stock market portfolio at time \( t \)
- \( s_{i,\text{Stock}} \) = the Dimson-March adjusted measure of risk that is idiosyncratic (diversifiable) by combination with the stock market at time \( t \), expected to be eliminated due to diversification

In addition to examining the impact of each market on the returns of a rare coin in separate tests, a third test also needs to be implemented that examines impacts from both markets simultaneously on the return of a rare coin in a single empirical model, in case a collection of rare coins and a collection of stocks are combined in a single portfolio:

\[ E(r_i) = \beta'_{i,\text{Coin}} (E(COINMKT_t)) + \beta'_{i,\text{Stock}} (E(STOCKMKT_t)) \]  
\( \beta'_{i,\text{Coin}} \) and \( \beta'_{i,\text{Stock}} \) denote the betas derived using returns from the coin market and the stock market, respectively. Multiple risk premia within a single model need to be adjusted for correlation between the returns of two markets which can cause a misspecification of the betas, leading to errors in the estimation of the beta coefficients (used for non-diversifiable risk estimation) and the standard error (used for idiosyncratic risk estimation). Equations (11) and (12) show models that are mathematically equivalent to Merton’s (1973), but the multiple risk premia betas are defined in terms of the single risk premium betas. Equation (13) shows a measure for idiosyncratic risk when multiple risk premia are considered.\(^7\)

\(^7\) As can be easily determined by Equations (8) and (9), when correlations are equal, there is no change in correlation

\[ \begin{cases} 
\rho_{i,\text{Coin}} = \rho_{i,\text{Coin}} \\
\rho_{i,\text{Stock}} = \rho_{i,\text{Stock}} 
\end{cases} \]  
iff \( \rho_{i,\text{Coin}} = 0 \). Otherwise, estimates for betas are adjusted for correlation between the risk premia of the respective markets. Note that, in this research, each investment is weighted for value, which is common to
Risk and Return in an Electronic Market

Charles A. Wood

Cross-Serial Correlation and Nonlinearities of Returns

The ubiquitous nature of online auctions brought about by Internet technology allows massive data collection over shorter periods to achieve similar statistical power when compared to other studies where data collection took decades or even centuries (e.g., Mei and Moses 2002; Baumol 1986; Pesando 1993). However, there are challenges to the short horizon nature of this study that go beyond statistical power. There is strong evidence that short-horizon returns are serially correlated.

In addition to cross-serial correlation brought about by short-term nature of this study, there is also an issue of non-normality of returns. In this study, coin returns also fail normality tests. Ordinary least squares (OLS) tests show a high degree of heteroskedasticity as well as serial correlation.\(^9\) Hansen (1982) resolves the problem of non-normality and heteroskedasticity by describing a Generalized Method of Moments (GMM) technique that greatly relaxes the strict distributional assumptions relative to OLS and corrects for unknown sources of heteroskedasticity.\(^10\) Many investment datasets exhibit some sort of serial correlation over a short horizon.
(e.g., Lo and MacKinlay 1988), and returns in this study also shows serial correlation. Box and Jenkins (1970) describe techniques to adjust for serial correlation, and test statistics developed by Box and Pierce (1972) indicate significant serial correlation for up to three lagged periods in this study. Newey and West (1987) recommend a standard errors adjustment, employed here, to correct for non-normalities resulting from serial correlation. Their technique reduces coefficient instability that may result from such serial correlation. Further, since GMM is non-linear, fit measures, such as $R^2$, are suspect in GMM. For non-linear least squares, Greene (2002) recommends a Wald test to test model fit with nonlinear procedures such as GMM.

**Stochastic Empirical Model Formulation and Empirical Analysis**

In this section, the data and insights from these models are used to examine the entire rare coin market as well as individual coins within the market

**Market Indices**

To develop a coin market index, first a coin index for each coin is calculated. Daily coin indices are calculated as that day’s final selling price as a percentage increase of the first day’s final selling price for each coin. On days where a coin does not trade, it is assumed that there is no price change from the previous trade. The coin market index for each day is then calculated using the average index of each coin. Each coin initially has an equal weight on the index, regardless of nominal price charged for the coin. The closing S&P 500 on the day prior to this study (4/24/2002) is 1093.14, and the stock market index is calculated as the S&P 500 closing price as a percentage increase or decrease from 1093.14. Daily indices are used in this study, but for brevity, Table 2 shows weekly measures of the coin market index and the stock market index.

**Table 2. Online Coin Auction Market Index vs. the S&P 500**

<table>
<thead>
<tr>
<th>Week</th>
<th>Closing Date</th>
<th>Coin Market Index</th>
<th>Closing S&amp;P 500</th>
<th>Stock Market Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>04/24/02</td>
<td>0.000</td>
<td>1093.14</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>05/01/02</td>
<td>0.105</td>
<td>1086.46</td>
<td>-0.006</td>
</tr>
<tr>
<td>2</td>
<td>05/08/02</td>
<td>0.103</td>
<td>1088.85</td>
<td>-0.004</td>
</tr>
<tr>
<td>3</td>
<td>05/15/02</td>
<td>0.099</td>
<td>1091.07</td>
<td>-0.002</td>
</tr>
<tr>
<td>4</td>
<td>05/22/02</td>
<td>0.140</td>
<td>1086.02</td>
<td>-0.007</td>
</tr>
<tr>
<td>5</td>
<td>05/29/02</td>
<td>0.083</td>
<td>1067.66</td>
<td>-0.023</td>
</tr>
<tr>
<td>6</td>
<td>06/05/02</td>
<td>0.122</td>
<td>1049.90</td>
<td>-0.040</td>
</tr>
<tr>
<td>7</td>
<td>06/12/02</td>
<td>0.119</td>
<td>1020.26</td>
<td>-0.067</td>
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<tr>
<td>8</td>
<td>06/19/02</td>
<td>0.124</td>
<td>1019.99</td>
<td>-0.067</td>
</tr>
<tr>
<td>9</td>
<td>06/26/02</td>
<td>0.140</td>
<td>973.53</td>
<td>-0.109</td>
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<tr>
<td>10</td>
<td>07/03/02</td>
<td>0.099</td>
<td>953.99</td>
<td>-0.127</td>
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<tr>
<td>11</td>
<td>07/10/02</td>
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<td>920.47</td>
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<tr>
<td>12</td>
<td>07/17/02</td>
<td>0.120</td>
<td>906.04</td>
<td>-0.171</td>
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<tr>
<td>13</td>
<td>07/24/02</td>
<td>0.089</td>
<td>843.43</td>
<td>-0.228</td>
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<tr>
<td>14</td>
<td>07/31/02</td>
<td>0.127</td>
<td>911.62</td>
<td>-0.166</td>
</tr>
<tr>
<td>15</td>
<td>08/07/02</td>
<td>0.078</td>
<td>876.77</td>
<td>-0.198</td>
</tr>
<tr>
<td>16</td>
<td>08/14/02</td>
<td>0.087</td>
<td>919.62</td>
<td>-0.159</td>
</tr>
<tr>
<td>17</td>
<td>08/21/02</td>
<td>0.145</td>
<td>949.36</td>
<td>-0.132</td>
</tr>
<tr>
<td>18</td>
<td>08/28/02</td>
<td>0.134</td>
<td>917.87</td>
<td>-0.160</td>
</tr>
</tbody>
</table>

Figure 2 graphically compares the rare coin and stock market indices shown in Table 2. Although Figure 2 shows that the coin market has outperformed the stock market in the 128-day term of this study, Pesando (1993), Mei and Moses (2002), and Baumol (1986) all report in longer-term studies that collectibles tend to underperform the stock market.

The coin market index shows a slight, though significant 0.01% positive daily trend (p-value < .001) during this study. Conversely, the S&P 500 shows a significant -0.17% negative trend (p-value < .001) during this study. Further, there is much variability during this period in both the coin market (as Baumol (1986) and Pesando (1993) indicate is common in collectible markets) as well as atypical variance and negative returns during this period in the
stock market. Thus, this counter-cyclical may be due to atypical decreases in stock prices over this period, making collectible investments more appealing during bear markets.

Figure 2. Coin Market Index vs. S & P 500

Stochastic Model Development

When analyzing the relationship between risk and return, Fama and MacBeth (1973) describe several testable implications. Three of their tests are of particular interest to this research. First, Fama and MacBeth cite a test based on Sharpe (1964) and Linter (1965) to examine if the intercept in the market model is equal to the risk free rate. The returns in this paper have been adjusted for the risk free rate, and thus, theoretically, the market models shown in Equation (2) do not include and intercept term, and there should be no significant intercept in any of the market models in this research. Second, Fama and MacBeth specify that in a market of risk-adverse investors, higher risk should be associated with higher expected returns, or $E(MKT_t) > 0$ and $\beta_i > 0$. Third, $\beta_i$ is considered a complete measure of risk. Therefore, the expected returns of an investment ($E(R_{it})$) can be completely determined by non-diversifiable risk ($\beta_{it}$) and idiosyncratic risk ($s_{it}$) will have no impact upon market returns since idiosyncratic risk is completely eliminated through diversification.\(^{11}\)

Fama and MacBeth point out that if returns can be predicted by non-diversifiable risk, then a generalized stochastic model can be developed to empirically examine cross-sectional data to test the three implications of interest to this study:\(^{12}\)

$$ r_{it} = \gamma_0 + \gamma_1 \beta_{it-1} + \gamma_3 s_{it-1} + \eta_{it} $$  

(14)

Empirical Models

The stochastic model shown in Equation (14) can be used to examine the non-diversifiable risk ($\beta_{it}$) and idiosyncratic risk ($s_{it}$) for each specific rare coin on each day. Fama and MacBeth show how $\gamma_3$ can be used to examine the effect of idiosyncratic risk on coin returns and that the test of $\gamma_3 = 0$ can be used to test for the absence of any idiosyncratic risk effect. To test that higher risks lead to higher returns, Fama and MacBeth show that if $\gamma_1 > 0$, then the market covariance does have a positive effect on risk. Finally, since excess returns are used in this research, as opposed to nominal returns, returns in the empirical analysis have already been adjusted for the risk-free

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\(^{11}\) This stochastic model is based on Fama and MacBeth’s (1973) model. They also include a CL implication asserting that no non-linear relationship exists between market return and investment return. To test this implication, they include $\beta_i^2$ term in their model, which is not done in this research, but an external duplication of their research shows no non-linear relationship.

\(^{12}\) For consistency with equations later in the paper, no $\gamma_2$ parameter is included in Equation (11), and parameter numbers jump from $\gamma_1$ to $\gamma_3$. The $\gamma_2$ parameter will be used to examine S&P 500 effects on coin returns.
rate. Thus, to test for the risk-free rate as beta, the intercept should equal zero (i.e., $\gamma_0 = 0$). To form the empirical model, three empirical models are implemented to test the implications from Equations (4), (7), and (10).

This research contains three empirical models derived from the stochastic model presented in Equation (14). In Empirical Model 1, the coin market is examined in isolation, based on the model defined in Equations (4), (5), and (6), to determine the effects of the coin market on the returns of individual coins without considering any other risk premia:

**Empirical Model 1: Coin Market Beta**

$$r_t = \gamma_0, Coin + \gamma_1, Coin \beta_{t-1}, Coin + \gamma_3, Coin s_{t-1}, Coin + \eta_t, Coin \quad (15)$$

Table 3 shows GMM empirical results from Empirical Model 1, and examines the effect of the covariance of individual rare coin returns against the overall coin market return.

<table>
<thead>
<tr>
<th>Table 3. GMM Estimation of the Effect of the Coin Market Beta on Coin Market Returns</th>
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</thead>
<tbody>
<tr>
<td>Coeff</td>
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<tr>
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</tr>
<tr>
<td>Intercept ($\gamma_0, Coin$)</td>
</tr>
<tr>
<td>Non-diversifiable Risk from the Coin Market ($\gamma_1, Coin$)</td>
</tr>
<tr>
<td>Idiosyncratic Risk ($\gamma_3, Coin$)</td>
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<tr>
<td>Wald Statistic</td>
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</tbody>
</table>

*** = p-value < .001; 13,263 Auctions Analyzed; Dependent variable is Coin Return

The effect that time-varying coin market $\beta$ and idiosyncratic risk that is not explained by the coin market is examined in Table 3. These results indicate that idiosyncratic risk has a significant impact on returns, whereas the impact that diversifiable risk has on returns is slight or non-existent. This is evidence that coin collectors purchase coins as stand-alone investments, with little or no attention given to that coin’s impact on the overall value of a coin collection.

Both Mei and Moses (2005) and Pesando (1993) describe how stock market and collectible market returns should be related to risk. Individual stocks are related to other stocks in a market, and thus the covariance between the previous stock returns and the previous market returns should predict the current stock price. Similarly, the covariance of previous collectible returns and previous stock returns should determine the price of collectibles, whether the collectibles are paintings (in Pesando’s research) or rare coins (as examined in this research). Based on Pesando’s insights and on the Equations (7), (8), and (9), Empirical Model 2 is developed to determine the effects of the stock market on the returns of individual coins without considering any other risk premia:

**Empirical Model 2: Stock Market Beta**

$$r_t = \gamma_0, Stock + \gamma_2, Stock \beta_{t-1}, Stock + \gamma_3, Stock s_{t-1}, Stock + \eta_t, Stock \quad (16)$$

Table 4 shows results from Empirical Model 2, and examines the effect of the covariance of individual rare coin returns against the overall stock market return.

<table>
<thead>
<tr>
<th>Table 4. GMM Estimation of the Effect of the Stock Market Beta on Coin Market Returns</th>
</tr>
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<tbody>
<tr>
<td>Coeff</td>
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<tr>
<td>Wald Statistic</td>
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</tbody>
</table>

*** = p-value < .001; 13,263 Auctions Analyzed; Dependent variable is Coin Return

The effect that time-varying coin market $\beta$ and idiosyncratic risk that is not explained by the stock market is examined in Table 3. These results also indicate that idiosyncratic risk has a significant impact on returns, whereas the impact that diversifiable risk has on returns when a coin is added to a stock portfolio is insignificantly different from zero. This is further evidence that coin collectors purchase coins as stand-alone investments, with little or no attention given to that coin’s impact on the overall value of an investment portfolio.
In Empirical Model 3, the both coin market and stock market are jointly examined to determine the effects of both risk premia on the returns of individual coins, based on the models defined in Equations (10), (11), (12), and (13):

**Empirical Model 3: Combined Risk Premia of Coin Market and Stock Market**

\[ r_{it} = \gamma_0 + \gamma_1 \beta_{Coin} + \gamma_2 \beta_{Stock} + \gamma_3 \sigma_i + \eta_i \]  

(17)

Table 5 shows the results from Empirical Model 3, and examines the effect that both the overall coin and stock market, combined, have on individual rare coin returns.

| Table 5. GMM Estimation of the Effect of both Coin Market & Stock Market Betas on Coin Market Returns |
|-------------------------------------------------|-----------------|-----------------|
| Coeff | Std Err | p-value |
| Intercept \((\gamma_0')\) | -0.0061 | 0.0082 | 0.4543 |
| Non-diversifiable Risk from the Coin Market \((\gamma_1,\text{Coin}')\) | 0.0000 | 0.0003 | 0.9326 |
| Non-diversifiable Risk from the Stock Market \((\gamma_2,\text{Stock}')\) | -0.0004 | 0.0003 | 0.2680 |
| Idiosyncratic Risk \((\gamma_3')\) | 0.3537 | 0.0327 | 0.0000*** |
| Wald Statistic | 37,703*** |

This table examines both coin and stock market betas concurrently to ascertain how the coin market and stock market, concurrently affect returns on rare coins. As with the previous empirical models, these results also indicate that idiosyncratic risk has a significant impact on returns, whereas the impact that diversifiable risk has on returns when a coin is added to either a stock portfolio or a coin collection is insignificantly different from zero. These results further support that coin collectors purchase coins as stand-alone investments, with little or no attention given to that coin’s impact on either the overall value of an investment portfolio or the overall value of a coin collection.

In all three empirical models, the effect of betas from the coin and stock markets on returns is not significantly different from zero, supporting a contention that diversifiable risk has little (if any) significant impact on returns in the rare coin market. However, the effect of idiosyncratic risk is significant in all three models, indicating that while coin collectors may diversify, they do not consider diversification when determining the appropriate price for rare coins. Rather consider each coin investment separately, apart from existing holdings in a portfolio or collection. These tests show a reasonable fit with the empirical model, showing a p-value < .001 for all three models.

Since the returns used in this study are adjusted for the risk free rate, research from Sharpe (1964) and Lintner (1965) suggest that intercepts in the empirical models should not vary significantly from zero. All three of the empirical models support that the risk free rate of return is considered by coin collectors, and that investors consider returns above the risk free rate.

The results of the examination of idiosyncratic risk and non-diversifiable risk are surprising. The estimates of \(\gamma_1\) and \(\gamma_2\) are used to examine the effect of the coin market betas and the stock market betas, respectively, on rare coin returns, thus testing the effect of non-diversifiable risk on returns. Prior research into stock markets suggests that one or both of these should be significantly positive. However, these tests show them to be insignificantly different from zero in each empirical model. As described earlier in this paper, power tests indicate that the sample size is this research is large enough for an extremely powerful empirical analysis. The GMM technique used in this research adjusts for nonlinearities, and other tests are also employed that indicate that there is no non-linear relationship. Thus, the chance of model misspecification is reduced, and insignificance can be interpreted as either insignificance is the result of no relationship between diversifiable risk and return or the relationship between

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13 This does not imply that coin collectors do not diversify (i.e., they add coins to a multi-coin coin collection or add coins as an investment to a stock portfolio). Rather, this research indicates that online auction rare coin collectors do not consider this diversification when determining a coin prices. It is also possible that other assets (rare stamps, paintings, rare wines, etc.) could be considered for diversification. However, just as stock market studies consider only stocks for diversification, it is reasonable and safe to consider only rare coins, along with stocks, when examining the effect of diversification on rare coin prices, that diversification would primarily occur in these two areas for most, if not all, coin collectors.
diversifiable risk and return is infinitesimal as to be practically nonexistent, and thus should have little effect on purchase decisions in online auctions.\(^\text{14}\)

The estimates of \(\gamma_3\) are used to examine the effect of idiosyncratic risk on rare coin returns, thus testing the effect of diversifiable risk on returns. Since idiosyncratic risk can be eliminated by diversification, one would expect that the effect of idiosyncratic risk on returns should be insignificantly different from zero. However, these results show a strong positive relationship between idiosyncratic risk and coin returns, which is opposite of these expectations. This research suggests that participants in rare coin auctions consider each rare coin as a stand-alone investment, and not as a part of a portfolio or collection. Returns of each coin are considered independently of other investments in other rare coins or stocks.

**Further Analysis**

In Figure 3, the 13,263 auctions are divided into quintiles of 2,652 or 2,653 auctions, grouped by their measure of idiosyncratic risk derived from Empirical Model 3, which measures idiosyncratic risk after risk premium from the coin market and the stock market are both simultaneously taken into consideration. Figure 3 shows that greater idiosyncratic risk typically leads to greater returns.

![Figure 3: Quintile Breakdown Averages Daily Returns with Respect to Idiosyncratic Risk](image)

This figure illustrates a key point of this research. This figure supports a contention that collectors adjust for different levels of idiosyncratic variance in prices. The variance levels may seem quite high, but note that high variance with collectibles is not a new finding, but rather conforms to the findings of Pesando (1993) and Baumol (1976), both of whom discuss how returns of collectibles have higher variances than returns in stocks.

It is possible that large idiosyncratic variances that are found in the rare coin market have the effect of masking the covariance with the market so that non-diversifiable risk is virtually ignored by rare coin collectors. Tables III, IV, and V shows that the non-diversifiable risk does not function well as a predictor with the short-horizon rare coin data during this period, and both the empirical results in these tables and Figure 3 illustrate how an increasing idiosyncratic risk leads to an increasing return over the short horizon when investing in rare coins in online auctions.

**Limitations**

This study has some limitations. First, while online auction markets are thicker than offline auction markets (thus allowing this type of research), the online coin market is not as thick as the stock market. As with any research on collectibles as investments, this makes studying risk and return in the rare coin market more difficult than studying risk and return in the stock market. However, since risk and return still are considered when investors speculate outside of the stock market, research such as this, though difficult, is vital to bring forward. This research addresses

\(^{14}\) CAPM-based models like the model in this research assumes linear effects of beta on returns. I did test for (and did not find) nonlinear relationships using the method suggested by Fama and MacBeth (1973).
the thinness of the online coin market by examining several thousand different types of coins and only choosing those coins that are the most traded. Thin trading adjustments are accomplished using Dimson and March’s (1983) methodology, to ensure that betas are not misstated. These two considerations make collectible research more feasible, despite thinness of the market.

Another limitation is that, due to data constraints, this research only examines data over a short horizon. While some financial researchers also examine returns using short horizons (e.g., Lo and MacKinlay 1988, 1999), most asset pricing researchers typically investigate the effect of idiosyncratic risk and non-diversifiable risk by examining stock prices often over a period of several decades rather than several months, using monthly returns rather than daily returns (e.g., Fama and MacBeth 1973; Fama and French 1992; Lettau and Ludvigson 2001). However, the online auction market differs from the stock market with respect to the availability of past information. Daily information is available from the stock market over decades from many publications and online information sources, but online coin auction information is usually unavailable after 90 days, requiring a large effort from the coin investor to retrieve adequate information from past periods. Thus, it will be difficult for some rare coin investors to retrieve information from returns from periods long past, such as a year, and as such, a shorter time window may be more viable than when analyzing stock returns. Also, short horizon data constraints that affect non-diversifiable risk \( \beta \) will have similar effects on idiosyncratic risk \( \sigma \) when performing out-of-study examinations where variability from previous periods predicts returns in the current period, such as this study. The short-horizon of this analysis may make significance difficult to detect in both non-diversifiable risk \( \beta \) and idiosyncratic risk \( \sigma \), yet despite this, there is a significant effect of idiosyncratic risk \( \sigma \) on returns. Nevertheless, while this research should not be used to inform on behavior of long-horizon returns, a contention of this research and other researchers (e.g., Lo and MacKinlay 1988, 1999; Blume and Friend 1978) is that short-horizon phenomenon are interesting, important to understand, and potentially profitable. Lo and MacKinlay (1999, p 141), in particular, stress this point and claim that research in short-horizon phenomenon both describes possible profitable investment opportunities and also is suggestive of long-horizon relationships.

Finally, although these results may well apply for any in online auction collectible market, one cannot claim \textit{a priori} that these specific results (a significant impact of idiosyncratic risk on returns) are generalizable to any non-stock investment vehicle or even assets available in other online auctions or during other time periods. Rather, these results provide an impetus to examine other markets in future research to obtain a more comprehensive and detailed representation of the behavior of non-stock investments.

**Conclusion and Future Research**

Although many researchers have long since held that a primary use of collectibles is for investment (e.g., Ashenfelter and Graddy 2003), this exploratory study is the first research that views an online auction market as a type of investment exchange resulting from the massive increases in participation made possible by Internet technology. While other authors such as Mei and Moses (2002) and Pesando (1993) also compare risk-adjusted collectible returns to the market, this is the first research that examines how market returns inside a collectible market (as opposed to a stock market) can affect returns of individual collectibles in that market. This type of analysis is made possible since millions of buyers and sellers can now contact each other and exchange information, resulting in large datasets that can be collected over a shorter horizon when compared to traditional auctions.

The findings in this research are of interest to researchers and investors. This research uses time-varying risk measurements, both with betas derived from the stock market as well as within the online coin market to show that idiosyncratic risk does affect returns over a short horizon in this market. Thus, while coin collectors do diversify, in that they add coins to a collection, when coin collectors’ form their valuations, they do not consider diversification when making coin purchases. Rather, rare coin collectors insist on being compensated for diversifiable risk over a short horizon when making coin purchases. These results show that idiosyncratic risk plays an important part in expected returns in rare coin online auctions.

This research has several implications for investment managers. Primarily, this research shows that coin collectors consider idiosyncratic risk to be important, thus implying that the typical coin collector views a coin as a stand-alone investment, and does not typically consider the coin’s addition to value in an investment portfolio or a collection. Since idiosyncratic risk is diversifiable, these results indicate that there are some arbitrage opportunities that exist in the online rare coin markets. Thus, results from this research should interest both managers of coin shops as well as collectors, investors, speculators, and rare coin resellers.
In addition, this research shows that coin prices increased slightly across the time period of this study, during a period of sharp declines in the S&P 500. While traditional research (e.g., Mei and Moses 2002, Pesando 1993, Baumol 1989) indicates that collectibles, such as art, traditionally underperform the stock market in multi-year studies, this research illustrates that this may not be true during all periods, and that collectibles, such as rare coins, could have a place inside a well-diversified portfolio—an observation echoed by Ashenfelter and Graddy (2003), among others. Coin market indices in this study show less variance than the S&P 500 during this study, but most research indicates that there is more volatility in collectibles than in stocks in the long term. Even accepting that collectible returns typically have a higher variance than stocks (as is shown in this research), this research indicates that collectibles can be effectively utilized within a portfolio in order to reduce the impact of stock market fluctuations. Future research should examine the optimal mix of collectibles with traditional portfolio investments in different types of markets.

Previous auction research in collectibles typically examines data from auction houses such as Sotheby’s in New York, to which most people had only limited access. It is possible that the translocational nature of the Internet, with online auction houses such as eBay, will change the dynamics of pricing within auctions. This research provides evidence indicating that individuals who purchase rare coins in online markets behave differently than typical stock investors analyzed by Fama and MacBeth (1973), Sharpe, (1964), and Lintner (1965). This research finds that risk is considered when making rare coin investments, but collectors do not tend to consider diversifiable risk when adjusting for returns. Future research should examine efficiency in online markets; whether the behavior in online auctions differs drastically from behavior in traditional auctions or if behavior differs across different types of auctions; and the long-term effects of risk on return in collectible online auction markets.

Appendix – Terms from Research in Finance Used in This Study

- **Return** – The amount of money that an investment makes. Typically expressed in change of price.

- **Risk** – The expectation that a financial investment will return what is expected. The riskier the investment, the less certain the financial returns. This measure is usually proxied by variability in prices or returns within the finance literature. There are two categories of risk that are split by the CAPM framework:
  - **Idiosyncratic Risk or Diversifiable Risk** – The risk that can be removed through the use of diversification. Because every investment has certain idiosyncrasies, some good and some bad, combining several investments together will eliminate idiosyncratic risk (by “zeroing out” the idiosyncrasies). Because it is so easily removed through diversification, theories from finance contend that idiosyncratic risk should not be a factor in investments.
  - **Market Risk or Non-diversifiable Risk** – The risk that cannot be removed through the use of diversification. Because it cannot be removed through diversification, theories from finance contend that market risk should be a factor in investments.

- **The Capital Asset Pricing Model (CAPM)** – This model compares returns from individual investments and the entire market, and splits the risk of an investment into idiosyncratic (diversifiable) risk and market (non-diversifiable) risk.

- **Repeat-sale Regression (RSR)** – A technique for analyzing risk and return of an investment by examining repeated sales of that investment.

- **Risk Free Rate** – The amount of interest you can make in an investment that is completely risk free. Usually, the 10-year treasury U.S. Bond rate is used. This is the lower bound that an investor should expect from an investment. The theory is that the U.S. will never “go out of business,” so it never makes sense to expect less than the risk free rate from an investment, since you can always just buy U.S. bonds with the money and get the interest from the bonds as a guaranteed return.

- **Thinly-trade** – Describes an investment that does not trade as often as other investments. Comparing thinly-traded investments with frequently-traded investments requires an adjustment, since differences in trading can corrupt estimates. With U.S. stock market research, this adjustment is not necessary since all items trade every day. With emerging stock exchanges and with other types of investing, such as with collectibles, this adjustment is important to ensure that coefficients from regression analysis are not corrupted.
References


