Association for Information Systems
AIS Electronic Library (AISeL)

PACIS 2000 Proceedings

Pacific Asia Conference on Information Systems (PACIS)

12-31-2000

Introducing Formal Qualitative Reasoning Techniques to System Dynamics Modelling and Analysis

Neil Smith
Cranfield University

Follow this and additional works at: http://aisel.aisnet.org/pacis2000

Recommended Citation
Smith, Neil, "Introducing Formal Qualitative Reasoning Techniques to System Dynamics Modelling and Analysis" (2000). *PACIS 2000 Proceedings*. Paper 61.
http://aisel.aisnet.org/pacis2000/61

This material is brought to you by the Pacific Asia Conference on Information Systems (PACIS) at AIS Electronic Library (AISeL). It has been accepted for inclusion in PACIS 2000 Proceedings by an authorized administrator of AIS Electronic Library (AISeL). For more information, please contact elibrary@aisnet.org.

Introducing Formal Qualitative Reasoning Techniques to System Dynamics Modelling and Analysis

Neil Smith
Department of Informatics and Simulation
Cranfield University
RMCS, Shrivenham
Wiltshire, SN6 8LA, UK
neil.smith@rmcs.cranfield.ac.uk

Abstract
System dynamics techniques, such as influence diagrams, are used to model poorly understood systems. Qualitative analysis of these models is often extremely vague. More rigorous analysis requires quantitative simulation and can only occur after an additional conversion to a 'stock and flow' model. In this paper we present some augmentations to influence diagrams that allow analysis and simulation to proceed directly from influence diagrams. In addition, we show how, in some limited situations, reliable and formal qualitative predictions of stability can be drawn from such models using the notion of parametric variation.

Keywords: Systems dynamics, qualitative reasoning, influence diagrams, qualitative system dynamics.

1. Introduction

'System dynamics' is the term given to the study of the dynamic behaviour of a variety of complex systems, generally in the domain of human activity systems such as organisational management (Coyle, 1996; Goodman, 1989). These systems are characterised by a lack of explicit knowledge about the fundamental mechanisms as work in the systems, as well as a lack of quantitative information on how such mechanisms operate.

The main tool of system dynamics is representing the system being studied as an influence graph such as that shown in figure 1. The influence graph indicates the major variables in a system and what influences these have on each other. Traditionally, this model is used solely

as a sense-making device, allowing an analyst to organise and communicate his understand-

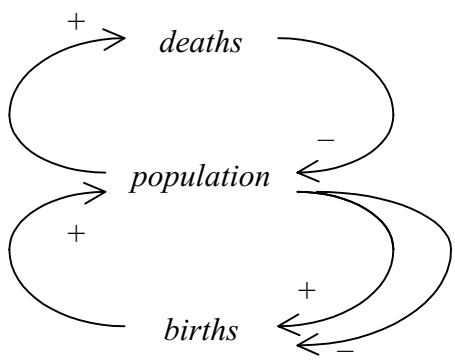


Figure 1. Traditional system dynamics model of controlled population growth.

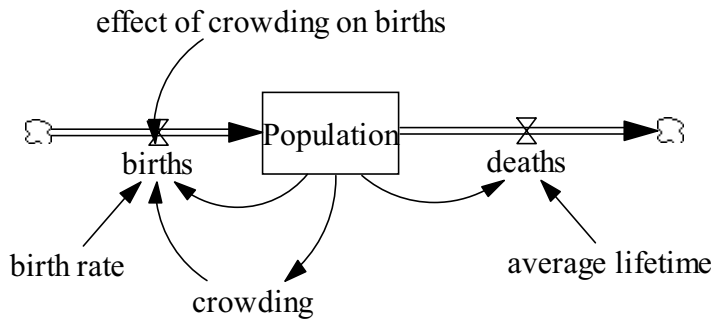


Figure 2. Stock and flow model derived from figure 1.

ing of a complex problem domain. The influence diagram is then manually converted into a 'stock and flow' diagram (figure 2) that shows how the system's components interact. Generally, the stock and flow diagram is more complex than the influence diagram as it includes nodes for each of the model's parameters. The stock and flow diagram is used to develop a set of equations which is used in a numerical simulator to generate the behaviour of the system.

This process suffers from the problem that qualitative reasoning was developed to solve: the system being examined is only known in the most general sense, while the numerical simulation can only accommodate strictly quantitative information. This paper shows a first step in resolving this dichotomy by applying qualitative reasoning techniques to system dynamics.

One existing approach to resolving the qualitative/quantitative tension has been the development of 'qualitative system dynamics' (Senge, 1990; Senge *et al.*, 1994). The approach here is to compare the original influence graph to a number of 'archetypes'; each archetypes exhibits a specific qualitative behaviour. If a given influence diagram resembles that of an archetype, the supposition is that the model will exhibit similar behaviour. However, this process is purely intuitive and has no formal basis, which means that there can be no reliance on the results of qualitative system dynamics.

The field of qualitative reasoning (Weld and de Kleer, 1990; MQandD, 1995) is a vigorous area of current research in the artificial intelligence community. These topics were developed to address analogous problems in engineering domains (de Kleer, 1984; Smith, 1998), though these approaches have recently spread their effective use into other areas, such as ecological modelling (Struss, 1998). The essence of qualitative reasoning is to identify the qualitatively significant or interesting values, variables, and phenomena within a system, and to use only these to understand and predict the system's behaviour. This approach allows a balance to be struck between the very vague and informal qualitative approach, where little if anything is known with certainty, and a purely quantitative approach where precise values and functional forms are required before any progress can be made.

As a first step towards this goal, we present a method for analysing these models in a more formal way, using some qualitative reasoning techniques. First, we show how some simple

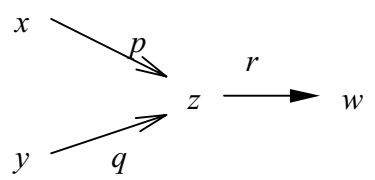
```
carrying capacity = 500
effect of crowding on births lookup([(0,20)-(10,0)], (0,20),
     (3,11), (6,6), (8,2.5), (10,0.75))
birth rate = 2
births = Population * birth rate * effect of crowding on births
     lookup
crowding = Population/carrying capacity
deaths = Population / average life
Population = INTEG(births - deaths, population)
```

Figure 3. Equations derived from figure 2

augmentations to the influence diagram notation allow the simple and automated derivation of state space equations for a model. We also show how feedback caused by parametric variation can be represented in an influence diagram. Finally, we describe how, in some limited circumstances, to evaluate whether this feedback is sufficient to stabilise the model.

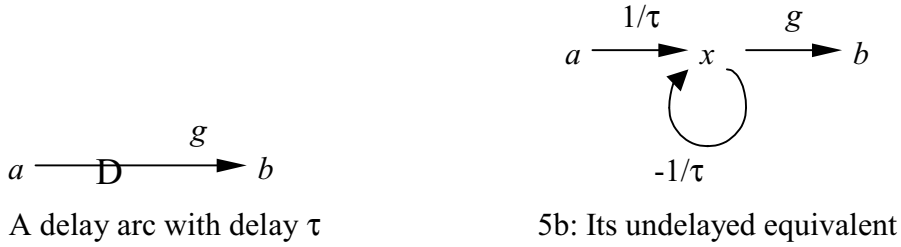
2. Formalising Influence Diagrams

The traditional approach in system dynamics has been to develop an influence diagram model as a 'sense making' step, where an analyst identifies the major features of the problem situation. This influence diagram is then converted to a 'stock and flow' model, equations are manually developed, and simulation is carried out. Indeed, Wolstenholme (1999) states that "no way has yet been established to directly convert [an influence diagram] representation directly to a simulation model." Figure 1 shows an influence diagram developed for a model of population growth limited by overcrowding and figure 2 shows the corresponding stock and flow model (there is a 'stock' of population and flows caused by births and deaths). The equations used for simulation, derived from the stock and flow model, are shown in figure 3. This process of conversion, from influence diagram to stock and flow model to equations, has to be performed manually and intuitively. Fundamentally, this is because of the lack in information contained within an influence diagram. In this section, we show how additional information process can simplify and formalise the conversion process. This is done by augmenting the influence diagram to allow state space equations to be derived automatically from such a model. In this paper, we restrict our attention to linear systems. However, in order to capture the richness inherent in system dynamics influence diagrams, we do not require these models to be parametrically invariant.



$$\dot{w} = r.z$$
$$z = p.x + q.y$$

Figure 4. Direct and indirect influences



5a: A delay arc with delay τ

5b: Its undelayed equivalent

Figure 5. Converting delayed arcs

The first extension of the influence diagram notation is to differentiate between direct influences (acting on a variable's derivative) and indirect influences (that act on a variable's value) (Forbus, 1984). We do this by following the notation of Rickel and Porter (1994) (figure 4), where direct influences are represented by solid arrows and indirect influences by open arrows. This distinction allows the identification of state variables in the model, as state variables can only be affected by direct influences, while non-state variables can only be affected by indirect influences.

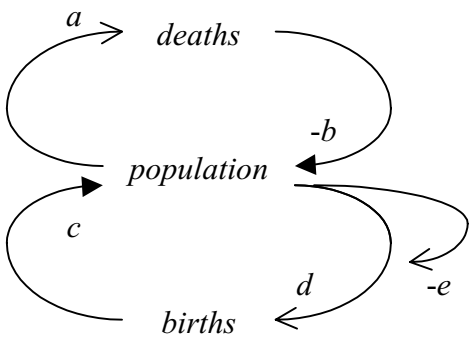
Once this is done, the objective is to eliminate all non-state variables and all indirect influences, combining their effects into direct influences. This simplification is performed directly on the graph, in a manner similar to signal flow diagrams (Wilson and Watkins, 1990; Richards, 1993). However, there are two complications to this approach: some arcs will be annotated with a delay (described in section 2.1) and parametric variation is represented by controlled arcs (described in section 2.2). These simplification techniques have been implemented in a program what will take an influence graph and produce the corresponding minimal set of either state space equations or qualitative differential equations (Kuipers, 1986, 1994).

2.1 Delay arcs

System dynamics influence diagrams often contain delay arcs (figure 5a), which represent an influence taking effect after some delay. This is used to represent such phenomena as delays caused by the transportation of materials, or organisational inertia in response to changes in management policy. Such delays must be eliminated from the model if linear state space equations are to be derived. This is easily done by realising that delayed effects effectively represent effects via hidden state variables. This equivalence is exploited in a well-known identity (Coyle, 1996) which allows delay arcs to be eliminated from an influence graph (figure 5b). It is convenient to eliminate all delay arcs in this way before any other simplifications take place.

2.2 Controlled Arcs

System dynamics influence graphs are traditionally designed to show the 'influences' that can affect a variable. These influences can take many forms which are normally only resolved at the time of the production of the stock and flow model. For instance, in the raw model of controlled population growth (figure 1), two influences affect the number of births. The positive influence represents the notion that a larger population will have a greater number of



$$\dot{p} = -a.b.p + c(d - e.p)p$$

Figure 6. Population model with a controlled arc

sexually mature females; as each such female will have a certain probability of reproducing, the size of the population will influence the number of births.

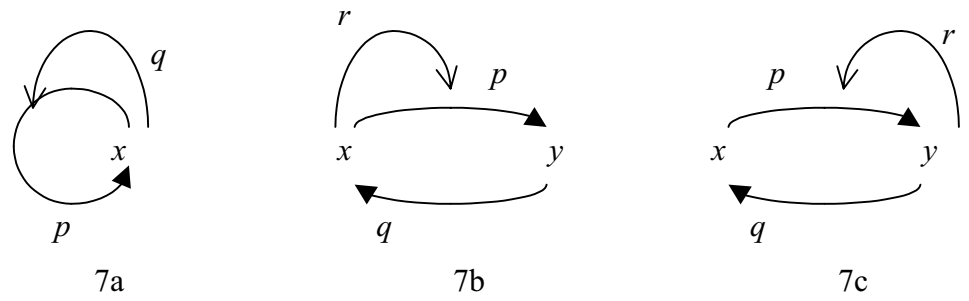
The negative influence on the number of births represents the reduction in fecundity due to population pressures. Figure 1 suggests that this influence affects the number of births in the same manner as the size of the population. However, closer consideration shows that population pressures act to reduce the fecundity of the females. It is this reduced fecundity that causes the lower number of births. This is the relationship shown in the corresponding stock and flow diagram (figure 2). The notion of one influence mediating the effect of another prompts the introduction of controlled arcs as a way of showing this on the influence diagram. The gain of a controlled arc is mediated by the controlling arc or arcs. This extension allows parametric variation to be included in the model. A revised population model, including a controlled arc, is shown in figure 6.

3. Formal Qualitative Analysis of Influence Diagrams

As described in section 1, there are two major strands of analysis within system dynamics. The quantitative approach can be supplemented with qualitative and semi-quantitative simulation packages such as QSIM and its peers (Kuipers, 1986; Kuipers and Berleant, 1988; Coghill, 1996). However, a more interesting avenue to explore is to use qualitative reasoning techniques to augment the qualitative system dynamics of Senge (1990). This requires formal analysis of the structure of the influence graph to determine, with some degree of rigour, the behaviour that will be expected from the model.

The first and most basic behavioural question to be asked is whether the model represents a stable system or not. System dynamics models are generally autonomous (i.e. without exogenous inputs) and do not necessarily have globally conserved quantities such as energy. Stability is defined as having a bounded behaviour in response to a sufficiently small perturbation[1]. For instance, ignoring the action of the controlling arc, the simple population model of figure 6 is stable around the point $p = 0$ if $a.b > c.d$, but unstable otherwise.

[1] Generally, the systems being modelled are non-linear and the models are linearised around an equilibrium point. If the initial perturbation is too large, the linearisation approximation will not hold and model will no longer be an accurate representation of the system.



7a 7b 7c

Figure 7. Some controlled loops

Puccia and Levens (1985) describe how the stability criteria of a model can be determined by applying the Routh-Hurwitz criteria. Each distinct cycle in an (fully simplified) influence graph is identified; its gain is the product of the gains of its constituent arcs. These loop gains can be combined to yield the graph determinant. If various subgraphs are taken, the various graph determinants can be combined to give an evaluation of the overall stability of the model. Such an analysis was performed to yield the stability condition for the population model of figure 6. While this approach is simple and easy to apply, it does have several limitations. The most significant of these is that the Routh-Hurwitz criteria can only be applied to linear or linearised models. The means that a modeller must identify the various equilibrium points and, for each point, linearise the model around that point and assess its stability. The detection of such points is not trivial (Khalil, 1996). Also, the linearisation means that the stability criteria cannot take account of possible changes in the loop gains caused by controlled arcs as the system moves from the equilibrium point.

3.1 Stability of Systems with Controlled Loop Gains

Controlling arcs represent feedback mechanisms in the model, and it is well know that feedback of the correct form is capable of ensuring the stability of an otherwise unstable system (D'Azzo and Houpis, 1960). However, the complex systems generally addressed by systems dynamics practitioners do not have clearly defined inputs and outputs and any feedback mechanisms only operate within small regions of the model. These conditions prevent the straightforward application of traditional control theory techniques for determining if feedback stabilises a particular model.

Given that the stability of the uncontrolled model is determined by loop analysis, and the action of controlled arcs is to alter the gains of these loops, a more promising avenue of investigation is to examine the effects if feedback from the point of view of loop gains. Unfortunately, the gain of a controlled arc depends on the value of one or more state variables, and loop analysis is silent on the transient response of these variables (Puccia and Levens, 1985). At present, no general solution to this problem has been identified, but the effect of controlled arcs on the stability of models has been determined when loops of only one or two state variables contain arcs that are controlled by variables within those loops. Such an example is the controlled population model (figure 6). In self-controlled loops of length one or two, the effect of the controlled arc on the loop gain is independent of the direction of movement of the controlling state variables. In such a case, the loop gain will change as the model moves away from its original equilibrium point. By extrapolating this change it is possible to determine if the system will reach another stable equilibrium point.

When a controlled arc is a member of a loop of length one or two, there are only two variables that can control the arc: the arc's originating variable, or its destination variable. In the former case (figure 7a or figure 7b), the relevant state equation is:

$$\begin{aligned}\dot{y} &= p.x \\ &= (p_0 + r.x).x \\ &= p_0.x + r.x^2\end{aligned}$$

(the values of x and y are taken to be the deviation of these state variables from their equilibrium positions). As the gain of the controlled arc, and therefore the gain of the controlled loop, varies with the square of the deviation from equilibrium, we can see that the effect of the controlling arc on the loop gain will strictly increase as the model moves from equilibrium, regardless of whether x moves above or below its equilibrium value.

When an arc is controlled by its destination variable (figure 7c)

$$\begin{aligned}\dot{y} &= p.x \\ &= (p_0 + r.y).x \\ &= p_0.x + r.x.y\end{aligned}$$

Let us assume that $\dot{y}(0) = p_0.x > 0$, therefore $x > 0$. If the model is stable, x will soon return to zero. If it is unstable, x will continue to move away from the equilibrium point; therefore, at all later t, $x > 0$ and $y > 0$, therefore $r.x.y > 0$. Again, the effect of the controlling arc on the loop gain will increase with time. Similar arguments leading to the same results can be followed for different signs of either of the arcs or the initial variation in the controlling variables.

These results are only applicable to self-controlled loops of length one or two because of the possibility of ambiguity arising from the interaction of several variables on the controlled arc, none of which may be the direct influence on the variable in question. These results show that for self-controlled loops of length one or two, the effect of a controlling arc will increase over time. In order to see whether this will have any effect on the eventual stability of the model, we allow the effect of the controlled arc to move to ∞ as $t \rightarrow \infty$. If the gain of the controlling arc is positive, the effect of the feedback will be to increase the magnitude of the loop gain; conversely, a negative controlling arc gain will move the loop gain towards zero. This allows the effect of the parametric variation to be included in the stability analysis, and therefore show whether the system will ever reach a stable point.

This treatment of parametric variation is limited to linear systems. In another paper (Smith, 2000) we describe how this approach can be extended to accommodate a wider variety of systems. This is achieved by extending the formalism of qualitative differential equations introduced by Kuipers (1986) to include modulated monotonic functions.

Referring back to the population model (figure 6), recall that the model is stable if $a.b > c.d$. If this is not the case, the population will move from its equilibrium value of $p = 0$. As it does so, the controlling arc will act to reduce the gain of the births loop until it becomes less than the gain of the deaths loop. At this point, the model will become stable. Notice that this method does not identify where this point may be: it simply states that such a point exists. Algebraic analysis of this model shows that an initially unstable model becomes stable when

the population reaches $p^* = \frac{s}{t} - \frac{p.q}{r.t}$. Again, this process has been implemented as a program that will identify if the feedback present in a model is sufficient to ensure the stability of a model.

4. Further work

The work presented here represents the first steps in the formalisation of qualitative systems dynamics. This work could progress down several avenues. The role of controlled arcs in maintaining stability in more complex situations needs should be addressed; however, this will require an understanding of the transient response of state variables after a perturbation. Ishida (1989) has had some success in this area. More generally, qualitative system dynamics depends on the identification of structural cliches to predict the behaviour exhibited by a model (Senge *et al.*, 1994). The limitations of this approach are obvious and well known (Lane and Smart, 1996) but qualitative reasoning approaches might provide appropriate tools for deriving useful results. For instance, the complexity of identification problem could be reduced through the use of order of magnitude reasoning (Raiman, 1991) to eliminate loops with insignificant gains. The easy identification of such loops is hampered by the action of controlled arcs and the relationship between loop gains and delays.

5. Conclusions

The major contributions of this work apply to both quantitative and qualitative system dynamics. Firstly, for quantitative system dynamics, we have described some simple augmentations to the influence diagram notation and shown that these augmented diagrams contain all the information needed to produce a set of state equations. This has been demonstrated by deriving such state equations without the need for the intervening stock and flow diagram.

Secondly, we have introduced some rigour into the study qualitative system dynamics. The objective of qualitative system dynamics, to predict the qualitative behaviour of a model from simply inspecting its structure (subject to some assumptions about the magnitude of effects) is an attractive one. However, existing techniques are entirely without rigour; this paper has addressed this issue. We have introduced the concept of controlled arcs to represent feedback mechanisms acting in a model and we have shown how the combination of these augmentations presented here can be used to predict qualitatively whether the feedback present in a model is sufficient to ensure the model's stability.

References

Coghill, G. M. "Mycroft: a Framework for Constraint-Based Fuzzy Qualitative Simulation," *PhD thesis*, Heriot–Watt University, 1996.

Coyle, R. G. *System Dynamics Modelling: a Practical Approach*, 1996, London: Chapman and Hall.

D'Azzo, J. J., and Houpis, C. H. *Feedback Control System Analysis and Synthesis*, 1960, Singapore: McGraw–Hill.

de Kleer, J. "How Circuits Work," *Artificial Intelligence* (24), 1984, pp. 205–280.

Forbus, K. D. “Qualitative Process Theory,” *Artificial Intelligence* (24), 1984, pp. 85–168.

Goodman, M. R. *Study Notes in System Dynamics*, 1989, Portland, Oregon: Productivity Press.

Ishida, Y. “Using Global Properties for Qualitative Reasoning: a Qualitative System Theory,” *Proceedings of the 11th International Joint Conference on Artificial Intelligence*, 1989, pp. 1174–1179.

Khalil, H. K. *Nonlinear Systems,* (2ed.), Upper Saddle River, 1996, NJ: Prentice Hall.

Kuipers, B. “Qualitative Simulation,” *Artificial Intelligence* (29), 1986, pp. 289–388.

Kuipers, B. *Qualitative Reasoning: Modeling and Simulation with Incomplete Knowledge*, 1994, Cambridge, Massachusetts: MIT Press.

Kuipers, B., and Berleant, D. “Using Incomplete Quantitative Knowledge in Qualitative Simulation,” *Proceedings of the 7th National Conference on Artificial Intelligence (AAAI-88)*, 1988, pp.324–329.

Lane, D. C., and Smart, C. “Reinterpreting ‘generic structure’: evolution, application and limitations of a concept,” *System Dynamics Review* (12:2), 1996, pp. 87–120.

MQ and D. “Qualitative Reasoning: a Survey of Techniques and Applications,” *AI Communications* (8:3-4), 1995, pp. 119–192.

Puccia, C. J., and Levens, R. *Qualitative Modeling of Complex Systems: an Introduction to Loop Analysis and Time Averaging*, 1985, Cambridge, MA: Harvard University Press.

Raiman, O. “Order of Magnitude Reasoning,” *Artificial Intelligence* (51), 1991, pp. 11–38.

Richards, R. J. *Solving Problems in Control*, 1993, Harlow: Longman.

Rickel, J., and Porter, B. “Automated Modelling for Answering Prediction Questions: Selecting the Time Scale and System Boundary,” *Proceedings AAAI-94*, 1994, pp. 1191–1198.

Senge, P. M. *The Fifth Discipline: the Art and Practice of the Learning Organization*, 1990, London: Century Business.

Senge, P. M., Kleiner, A., Roberts, C., Ross, R. B., and Smith, B. J. *The Fifth Discipline Fieldbook*, 1994, London: Nicholas Brealey.

Smith, N. “A New Architecture for Automated Modelling,” *Proceedings AAAI-98*, 1998, pp. 225–231.

Smith, N. “First Steps in Formalising Qualitative Systems Dynamics,” *Working Notes of the Fourteenth International Workshop on Qualitative Reasoning (QR-00)*, 2000, to appear.

Struss, P. “Artificial Intelligence for Nature — Why Knowledge Representation and Problem Solving Should Play a Key Role in Environmental Decision Support.” in Herzog, O., Günter, A. (eds.), *KI-98: Advances in Artificial Intelligence*, 1998, Berlin: Springer Verlag.

Wilson, R. J., and Watkins, J. J. *Graphs: an Introductory Approach*, 1990, New York: John Wiley and Sons.

Weld, D. S., and de Kleer, J. (eds.) *Readings in Qualitative Reasoning About Physical Systems*, 1990, San Mateo: Morgan Kaufmann.

Wolstenholme, E. F. “Qualitative vs. Quantitative Modelling: the Evolving Balance”, *Journal of the Operational Research Society* (50), 1999, pp. 422–428.