Technology, Interoperability, and Provision of Public Safety Networks

Completed Research Paper

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Abstract

Public safety networks are crucial for ensuring effective communications among first responders in emergency situations. This paper provides a comprehensive framework for analyzing the key tradeoffs between centralized and decentralized provisions of public safety networks. We extend the classic fiscal federalism model to capture a critical unique property of public safety networks – interoperability. Under decentralized provision, individual districts’ technology choices jointly determine the interoperability of public safety networks. Counterintuitively, the interoperability level is lower when the spillover effect is stronger. Under centralized provision, one uniform technology is chosen to maximize interoperability while accommodating local needs. When comparing output levels under centralized versus decentralized provision, we identify two countervailing effects: the spillover effect and the interoperability effect. In contrast to common public opinion, we find that a decentralized system may provide better quality of services than a centralized system when technology preferences are highly heterogeneous across different districts.

Keywords: Public safety networks, interoperability, centralized and decentralized provisions
Introduction

Modern consumer communication systems and devices provide reliable coverage, resilient dependability, and dynamic capabilities. The same cannot be said about most of the United States’ public safety communication systems, where the safety of its citizens is contingent on the ability of first responders to communicate effectively in emergency situations (Newman et al. 2010). New York City Police Commissioner Raymond Kelly bears witness to this statement, admitting that a teenager with a smartphone has more capabilities in the field than the average emergency responder does with a radio (Kelly 2011). Most current public safety systems provide responders with only voice communication services, but lack data driven services found on typical consumer networks, such as video capable devices to record condition of a patient or geo-location to guide a firefighter to a forest fire, and interoperability of systems to allow synchronized communications between other responders like police and fire fighters. The addition of these types of data driven services would allow for many other beneficial tools and could make the difference between life and death for those in distress and first responders.

Although why public safety agencies’ communication systems have fallen so far behind consumer systems is unclear, the increasing variance between them is becoming more evident. Case in point, the lives of hundreds of first responders could have been saved during the 9/11 attack if first responder agencies had synchronized communication devices to broadcast the building evacuation call. Furthermore, the effects of Hurricane Katrina verified the need for a broader and more dependable public safety network where the absence of basic communication operability proved devastating (Victory 2006). Overwhelming occurrences like these demonstrate that the fragmented network approach previously adopted for public safety networks is inadequate. Moreover, compared to modern consumer communication technologies where provisions of the network is distributed evenly among users, the current provision of public safety networks are decentralized based on regional boundaries, leaving tax payers with extremely costly and spectrally inefficient networks (Hallahan and Peha 2008). The cost is undoubtedly a result of policies regarding the networks being left to the decision of local and state agencies.

Accordingly, in March of 2010, the Federal Communications Commission (FCC) released the National Broadband Plan (NBP) which advocates for a nationwide interoperable wireless broadband network (Newman et al. 2010). The NBP suggests significant changes to current systems, claiming to bring about a centralized and highly dependable nationwide broadband network that is cost effective, yet will still meet the public safety’s stringent network requirements. The NBP calls for responders to work on a centralized network, providing interoperability and simultaneously creating a means for a shared federal funding approach.

Unlike the decentralized approaches, the NBP suggests “giving public safety access to a cohesive nationwide broadband network that leverages the commercial technology and infrastructure,” ultimately lowering the tax payers’ expenses and providing a centralized system that serves both public safety and the general public (Newman et al. 2010). Policymakers must investigate the impact of decentralized and centralized provisions on quality of services and how the cost of the provisions should be shared. Put simply, should there be a centralized system in which a single governing body makes the spending decisions about public safety networks that are financed by the general public, or a decentralized system in which local governments make the spending decisions that are financed by local citizens?

The centralized approach attempts to allocate quality of services evenly throughout the United States to promote interoperability while financing the network with general public taxation. The obvious benefit is that there will be an interoperable network with a uniform service provided by an equal tax; however, the concern is that it does not reflect specific local needs (e.g., different technology and output preferences). This centralized approach is in contrast to the traditional approach where the allocation of quality of services is determined by local governances and financed by local taxation, but leaving fragmented networks. The decentralized approach meets local public safety needs, but the drawback is that it neglects to consider the effects of interoperability to neighboring cities (e.g., Katrina and 9/11).

This paper builds upon the Oates (1972) model, a classic model of fiscal federalism, to investigate the provision problems of public safety networks. We model two districts with heterogeneous preferences for both output and technology. In a decentralized system, each district chooses its own output and technology levels. Consequently, individual districts’ technology choices jointly determine the interoperability of public safety networks. In a centralized system, the central government chooses
individual output levels for each district and a uniform technology to maximize the interoperability. With this model we address several research questions: what are the equilibrium technology choices and output levels in a centralized or a decentralized system? What is the impact of spillovers and heterogeneous technology preferences on government agencies’ choices? How do spillovers and technology heterogeneity jointly determine the interoperability of public safety networks? And most importantly, comparing the centralized and the decentralized systems, which system is superior? On the one hand, if the public safety networks are provided through a decentralized system, we find that the technology choices depend on the degree of technology heterogeneity, but not affected by citizens’ output preference. A district’s equilibrium output level is mainly determined by the output preference of its own citizens rather than the citizens of the other district. Counterintuitively, the level of interoperability between the public safety networks of the two districts is lower when the spillover effect is stronger. On the other hand, if the public safety network is provided through a central government, the equilibrium technology choice is no longer affected by the degree of spillovers. The central government is capable of internalizing the externalities the two districts impose on each other purely through the choice of output levels. Finally, we identify two countervailing effects – the spillover effect and the interoperability effect – in comparing output levels under centralized versus decentralized provision. In contrast to common public opinion, we find that a decentralized system may provide better quality of services than a centralized system when technology preferences are highly heterogeneous across different districts.

**Literature Review**

This paper is related to three literatures – communication networks, fiscal federalism, and technology compatibility and interoperability.

**Communication Networks**

Public safety agencies have produced a highly fragmented infrastructure consisting of many thousands of independent systems using a variety of technologies (Peha 2007). In terms of dependability, the most widely discussed issues are communications whenever multiple agencies or agencies in different geographical areas attempt to cooperate. The use of multiple, potentially incompatible, technologies leads to interoperability problems (Newman et al. 2010).

The public safety community has recognized public safety agencies’ interoperability and the limited and fragmented radio spectrum as main concerns related to operations of public safety wireless communications. Previous studies mainly focus on the simulation and analysis of traffic in deployed communication networks to determine their operational status, their performance, and to identify and locate possible network congestion (Cackov et al. 2005; Cackov et al. 2004; Sharp et al. 2004; Song and Trajkovic 2005). Traffic modeling is the most commonly adopted approach for network provisioning, predicting utilization of network resources, and for planning network developments. These studies are used to improve network reliability, which is particularly important for networks used by public safety agencies. Detailed overviews regarding the technical aspects of public safety networks can be found in Peha (2006; 2007) and Newman et al. (2010).

Existing literature on communication networks in general, and public safety networks in particular, focuses on technical issues. We contribute to the literature by taking the policy perspective and analyzing the provisions of public safety networks at the managerial level.

**Fiscal Federalism**

The traditional theory of fiscal federalism, first formulated in Oates (1972), lays out a general normative framework for the assignment of functions to different levels of government. At the most general level, this theory contends that the central government should have the basic responsibility for macroeconomic stabilization and for resource redistribution in the form of assistance to regions of need. Specifically, the central government must provide certain “national” public goods (like national defense, flood control systems, etc.) that provide services to the entire population of the country.

Decentralized levels of government have their raison d’être in the provision of public goods and services whose consumption is limited to their own jurisdictions. Fiscal federalism, under which provision of
public goods is decentralized to subnational governments, allows public consumption levels to be tailored to suit the preferences of a heterogeneous population. This beneficial outcome is achieved via sorting of individuals into demand-homogeneous jurisdictions, each of which provides a different amount of the public good (Brueckner 2004; Brueckner 2006; Cerniglia and Longaretti 2012). The drawbacks of decentralized federalism, which have also been noted in the literature, include the sacrifice of scale economies due to smaller jurisdiction sizes (Alesina and Spolaore 1997; Oates 1972), losses from inter-jurisdictional tax competition when government revenue comes from taxation of a mobile tax base (Brueckner 2004), and failure to properly account for public-good spillovers across jurisdictions (Besley and Coate 2003; Oates 1972). Recent empirical studies of fiscal federalism explore the impact of decentralized public spending on economic growth. This inquiry was inspired in part by the work of Oates (1993), who conjectured that better targeting of growth-enhancing infrastructure investment under federalism could raise an economy's growth rate. In a related argument, Davoodi and Zou (1998) show that, if national and subnational public-goods enter as separate inputs in a Cobb-Douglas aggregate production function, then growth maximization requires an appropriate degree of fiscal decentralization, with the subnational spending share matching its Cobb-Douglas exponent. The initial contributions to the empirical literature, which include Davoodi and Zou (1998), Zhang and Zou (1998), Woller and Phillips (1998), and Xie et al. (1999) disconfirm Oates' conjecture by finding a zero or negative connection between fiscal decentralization and growth, with the latter result possibly consistent with excessive decentralization under the Davoodi-Zou framework. However, other papers by Yilmaz (1999), Lin and Liu (2000), Akai and Sakata (2002), Thiessen (2003), Stansel (2005) and Iimi (2005) all find a positive relationship between decentralization and growth, suggesting that Oates may have been right after all.

We extend the classic Oates model of fiscal federalism to study the provision of public safety networks, and make three contributions to the literature of fiscal federalism: (i) In modeling heterogeneity of citizens, existing literature in fiscal federalism focuses on citizens' preferences for output. We contribute to the literature by introducing heterogeneous local jurisdictions' preferences for technology and modeling both preferences simultaneously. (ii) We investigate the tradeoffs among heterogeneous preferences for output, heterogeneous preferences for technology, and spillovers. (iii) We identify the critical unique feature of public safety networks – interoperability. Unlike common public goods, multiple incompatible technologies are available for public safety networks, which may lead to potential interoperability issues. Consequently, interoperability has important moderating effect over the externality one district imposes on other districts. We model the interoperability feature as the result of technology choices by government agencies and explicitly study this moderating effect.

**Technology Compatibility and Interoperability**

Prior studies in systems competition and the economics of network externalities have explicitly considered compatibility. The standard framework of modeling compatibility was first explored by Katz and Shapiro (1985) and Farrel and Saloner (1985). A system of compatible components is treated as a single good characterized by positive consumption externalities. Such network externalities arise because the utility a consumer obtains from a system increases with the number of others using compatible products. With network externalities, the firms' incentives to produce compatible systems have been shown to depend on the firms' relative size and on how compatibility can be enforced. Cremer et al. (2000) model network externalities such that customers benefit from an increase in network size, and furthermore, the positive network effect is a function of the degree of compatibility (interoperability). They find that the firms may have incentives to degrade interconnection under a market sharing equilibrium. Mason (2000) models ISP-competition with both horizontal and vertical differentiated customers. Mason finds that interoperability results in reduced competitive pressure. The strategic effect of interoperability also has many similarities with the strategic effect of interconnect prices in telephony networks. The literature on two-way access pricing among telecommunication networks, initiated by Armstrong (1998) and Laffont et al. (1998a; 1998b; 2003) studies how access prices affect retail competition and interoperability through telecommunication networks’ choice of retail tariffs. Whether their logic will prevail over politics and corporate strategies in other industries will require further research and analysis such as the one we present.

Existing literature models technology compatibility as a discrete variable. Compatibility (interoperability) analysis is driven by the effect of network externalities due to asymmetric installed bases. Thus a general finding in the literature is that when networks are asymmetric, large networks might have an incentive to
make the networks incompatible because complete compatibility means that large and small networks become equal. We contribute to the literature by endogenizing technology choices in the presence of heterogeneous technology preferences and, hence, the resulting interoperability is a continuous variable. Consequently, interoperability is driven by the tradeoff between catering to heterogeneous technology preferences among local jurisdictions and enjoying the benefit of spillovers.

**Model**

In this section, we first introduce the classic Oates model of fiscal federalism (Oates 1972). We then extend the Oates model by introducing heterogeneous preferences for technology and the resulting interoperability to capture the unique properties of public safety networks.

**The Oates Model**

In the classic Oates model of fiscal federalism, the economy is divided into two distinct districts, indexed by \( i = 1, 2 \). In each district, the local government maximizes the aggregate surplus of its citizens. There are three goods in the economy: two local public goods \( g_1 \) and \( g_2 \) (e.g., roads, parks, etc.), associated with the two districts respectively, and a single private good \( x \) (i.e., personal income). Each citizen is endowed with some of the private good. Producing one unit of the public goods requires \( p \) units of the private good. Each citizen in district \( i \) is characterized by a public good preference parameter \( \lambda \). A citizen with a higher \( \lambda \) values the public goods more. Citizens value all three goods and the utility of a type \( \lambda \) citizen in district \( i \) is:

\[
x + \lambda \left[ (1 - \kappa) \ln g_i + \kappa \ln g_i \right]
\]

where the parameter \( 0 \leq \kappa \leq 1/2 \) denotes the degree of spillovers. When \( \kappa = 0 \), citizens only care about the public good in their own district; when \( \kappa = 1/2 \), they care equally about the public goods in both districts.

In each district, the range of output preference type \( \lambda \) is \([0, \Lambda]\) with a Probability Density Function (PDF) of \( h_i(\lambda) \). The mean type of output preference in district \( i \) is denoted by \( m_i \). As a result, the aggregate benefits from public goods for all citizens in district \( i \) is:

\[
\int_{0}^{\Lambda} \left\{ x + \lambda \left[ (1 - \kappa) \ln g_i + \kappa \ln g_i \right]\right\} h_i(\lambda) d\lambda = x + m_i \left[ (1 - \kappa) \ln g_i + \kappa \ln g_i \right]
\]

Without loss of generality, the Oates model and its extensions make the following assumption to differentiate the two districts.

**Assumption 1:** \( m_1 \geq m_2 \).

Under assumption 1, citizens in district 1 prefer a higher output level of public safety network than those in district 2. For the citizens in the two districts, this assumption captures their heterogeneous preferences for output levels.

In the original Oates Model, under a decentralized system, the level of public good output in each district (i.e., \( g_1 \) and \( g_2 \)) is chosen by the local government to maximize the aggregate surplus of its constituent. In a centralized system, central government chooses a uniform level of public spending (i.e., output \( g \)) for each district.

**Modeling Technology Choices and Interoperability**

In order to analyze the provision of public safety networks, we extend the Oates model by introducing local jurisdictions within each district (e.g., counties within a state) that have heterogeneous preferences for technology and endogenizing the choices of technology. Specifically, we take into account the heterogeneous tastes of local technology preferences within a district in a Hotelling setting, denoted by \( \theta \in [0, 1] \), for various available network technologies. A technology that works well in an expansive wooded area is unlikely to work well in a subway. This has led to many officials squabble over technology preferences. For example security cameras within a public safety network usually rely on robust broadband, but jurisdictions cannot extend fiber and cabling everywhere to support the devices. Large cities with dense population would prefer fiber, cabling and trenching installations to areas where crime is increasing or a disaster is imminent. Suburban areas however prefer technology such as mobile IP broadband cameras that switch between 3G and 4G wireless functionality, which they can position the
cameras temporarily or permanently wherever they're needed. Multiple systems from different vendors deployed across local jurisdictions make it difficult for dispatchers and emergency personnel to access all information from a single device or see all camera views from a single location, which would help them better coordinate their efforts.

In a decentralized system, each district independently chooses a technology \( t_i \in [0, 1] \), where \( i = 1, 2 \) for its public safety network. Let \( o(t_i, t_j) = 1 - |t_i - t_j| \) denote the degree of interoperability\(^1\) for public safety networks across the two districts. If the technology selected by both districts are similar and hence intends to interoperate, i.e., \( o(t_i, t_j) \) is large, then citizens from both districts would derive more utilities from the neighboring district under the presence of spillover. This effect is reversed if technology choices are disparate, i.e., \( o(t_i, t_j) \) is small.

Depending on the technology choices \( t_1 \) and \( t_2 \), a citizen of type \( \lambda \) in district \( i \) derives the following utility from the public safety network:

\[
x + \lambda \left[ (1 - \kappa o(t_i, t_j))(\ln g_i - \delta |t_i - \theta|) + \kappa o(t_i, t_j) (\ln g_i - \delta |t_i - \theta|) \right]
\]

where \( \delta \) is the unit fit cost incurred when the technology \( t_i \) selected by district \( i \) and the local jurisdiction’s technology preference \( \theta \) are different (i.e., the technology misfit). We maintain the private good, \( x \), in (3) from the Oates model, recognizing it is not active in our model.

Local jurisdictions in district \( i \) have different preferences for technology, captured by a PDF \( f_i(\theta) \). Two districts also have different tastes in technology preferences, captured by different PDFs. We consider linear PDFs for both districts. Specifically, \( f_i(\theta) = 1 - \alpha + 2\alpha\theta \) for district 1 and \( f_i(\theta) = 1 + \alpha - 2\alpha\theta \) for district 2. To ensure \( f_i(\theta) \) and \( f_j(\theta) \) are valid PDFs, the range of parameter \( \alpha \) is \([-1, 1]\). Parameter \( \alpha \) captures the degree of technology preference heterogeneity in two districts. Note that our model would reduce to the original Oates model if and only if the local public safety networks are perfectly compatible, i.e., the interoperability is 1. Without loss of generality, we make the following assumption to differentiate the technology preferences of the two districts.

**Assumption 2:** \( 0 \leq \alpha \leq 1 \).

Under assumption 2, the majority of jurisdictions in district 1 favor the technology located at 1, whereas a greater proportion of jurisdictions in district 2 favor the technology located at 0. Next we demonstrate the variation of \( \alpha \) with two special scenarios.

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\(^1\) Compatibility is binary in many contexts outside the IS domain, e.g., two sub-fleets of vehicles requiring different fuels.

\(^2\) We use Greek letters for parameters and Roman letters for decision variables.
As shown in Figure 1, $\alpha = 1$ denotes the scenario where the heterogeneity in technology preference is at its maximum. In reality, this heterogeneity is exemplified by the investment choices faced by many local governments. For instance, municipalities are turning to mobile computing and other networked applications to improve the efficiency of their public safety workforce. Consequently, the need to access and share this vital new flow of data and images is driving local districts investing in a new kind of network: broadband wireless mesh networks using either Wi-Fi, WiMAX, LTE, Satellite or 4.9GHz public safety radio frequencies. The final technology choice made by the local district is most likely driven by local jurisdictions’ technology needs. Consequently, this may lead to the undesirable outcome where neighboring districts adopt incompatible technologies (e.g., WiMAX vs. LTE) and public safety organizations are limited in their ability to communicate and share information with other agencies from different districts.

As $\alpha$ decreases from 1 to 0, this heterogeneity diminishes and eventually the distributions of technology preferences $f_1(\theta)$ become indistinguishable (i.e., $f_1(\theta) = f_2(\theta)$ when $\alpha = 0$) for the two districts as shown in Figure 2. Consequently, local governments have more incentive to embrace similar technology choices. Thus, public safety networks would become more interoperable among different districts.

![Figure 2: Technology preference distributions by districts (low technology heterogeneity, i.e., $\alpha = 0$)](image)

### Decentralized Provision

In a decentralized system, local governments in two districts make their output and technology decisions to maximize the total surpluses within their districts. The optimization in (4) presents the local government of district 1’s decision problem.

$$\begin{align*}
\text{Maximize} & \quad S_1(g_1, t_1| g_2, t_2) = x + m_1 \int_0^1 \left\{ \left[1 - \kappa (1 - |t_1 - t_2|) \right] (\ln g_1 - \delta |t_1 - \theta|) \right. \\
& \quad \left. + \kappa (1 - |t_1 - t_2|) (\ln g_2 - \delta |t_2 - \theta|) \right\} f_1(\theta) d\theta - pg_1
\end{align*}$$ (4)

Subject to: $g_1 \geq 0$, $0 \leq t_1 \leq 1$

The objective of local government of district 1 is to maximize the overall utilities of its citizens, which is the sum of individual utilities as given in (3) minus the cost of producing the output level $g_1$ (e.g., the cost of building $g_1$ cell towers). Given the output level and technology choice of district 2 ($g_2$ and $t_2$), district 1 selects $g_1$ and $t_1$ by balancing the tradeoffs among heterogeneous technology preferences of its own district, i.e., $f_1(\theta)$, output preferences of its citizens, i.e., $m$, and the spillover effect captured by $\kappa (1 - |t_1 - t_2|) (\ln g_2 - \delta |t_2 - \theta|)$. Note that, the spillover effect is a function of the interoperability between the two public safety networks, i.e., $o(t_1, t_2) = 1 - |t_1 - t_2|$. As the two public safety networks become more interoperable, i.e., $o(t_1, t_2)$ increases, the spillover effect is more salient.
Similarly, the local government of district 2’s decision problem is represented by the optimization in (5).

\[
\text{Maximize } S_2 \left( g_2, t_2 \mid g_1, t_1 \right) = x + m_2 \left\{ \left[ 1 - \kappa \left( 1 - |t_1 - t_2| \right) \right] \left( \ln g_2 - \delta |t_2 - \theta| \right) \right\} \int_0^1 f_2 (\theta) d\theta - pg_2
\]

Subject to: \( g_2 \geq 0, 0 \leq t_2 \leq 1 \)

**Lemma 1:** In equilibrium, district 1 always chooses a technology located closer to 1, whereas district 2 chooses a technology located closer to 0, i.e., \( t_1 \geq t_2 \).

The proofs of all lemmas and propositions and the detailed derivations of the equilibrium results are relegated to the appendices.

Under assumption 2, greater proportion of local areas in district 1 favor the technology located at 1, whereas majority regions in district 2 favor technology located at 0. Lemma 1 confirms the intuitive result that local governments will choose a technology that is consistent with its local preference, i.e., \( t_1 \geq t_2 \).

Under decentralized provision, the equilibrium output levels \((g_{1,d}^*, g_{2,d}^*)\) and technology choices \((t_{1,d}^*, t_{2,d}^*)\) for the two districts are:

\[
\begin{align*}
  g_{1,d}^* &= \frac{m_1 \left[ 1 + \alpha + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2} \right]}{p \left[ 1 + \alpha + 2\kappa + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2} \right]} & \text{and} & t_{1,d}^* = \frac{-1 + \alpha + 6\kappa + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2}}{2(\alpha + 4\kappa)} & \text{for district 1}; \\
  g_{2,d}^* &= \frac{m_2 \left[ 1 + \alpha + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2} \right]}{p \left[ 1 + \alpha + 2\kappa + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2} \right]} & \text{and} & t_{2,d}^* = \frac{1}{1 + \alpha + 2\kappa + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2}} & \text{for district 2}.
\end{align*}
\]

Under assumption 1, citizens in district 1 favor higher output level for their public safety network than citizens in district 2, i.e., \( m_1 \geq m_2 \). Intuitively, the local government of district 1 provides a higher output level than district 2 does in equilibrium, i.e., \( g_{1,d}^* \geq g_{2,d}^* \).

**Proposition 1 (Decentralized Provision):**

Under decentralized provision, the equilibrium output levels \((g_{1,d}^*, g_{2,d}^*)\) and technology choices \((t_{1,d}^*, t_{2,d}^*)\) for the two districts have the following properties:

a. The equilibrium technology choices \((t_{1,d}^* \text{ and } t_{2,d}^*)\) are not affected by citizens’ mean output preferences \((m_1 \text{ and } m_2)\). Meanwhile, the equilibrium output level of district 1 \( g_{1,d}^* \) is independent of the mean output preference level \( m_2 \) of citizens in district 2, and vice versa.

b. The interoperability level \( o(t_{1,d}^*, t_{2,d}^*) \) decreases in the degree of spillovers \( \kappa \) and the technology heterogeneity \( \alpha \). The interoperability level is maximized, i.e., \( o(t_{1,d}^*, t_{2,d}^*) = 1 \), if and only if \( \alpha = 0 \).

When deciding its output level, each district only takes into account the benefits received by its own citizens, but not the benefits going to the other district. Hence, the equilibrium output level of one district is independent of the mean output preference level of the other district. Because output preferences are accommodated through local output levels, two districts consider only the technology heterogeneity and spillover while choosing different technologies to accommodate their local needs.

Surprisingly, we find that the interoperability level in equilibrium, \( o(t_{1,d}^*, t_{2,d}^*) \), decreases in the degree of spillovers \( \kappa \). In other words, the degree of interoperability between local public safety networks is lower when there is more spillover. This result highlights the tradeoff faced by local governments between satisfying heterogeneous technology preferences among local jurisdictions and achieving a higher degree of interoperability. The spillover effect moderates this tradeoff. When the spillover effect is strong, it is more beneficial for local governments to sacrifice some degree of interoperability in exchange for a technology favored by its local jurisdictions. In addition, the equilibrium interoperability increases as the
technology preferences of the two districts become more similar. The public safety networks are fully interoperable, i.e., \( o(t'_{1,d}, t'_{2,d}) = 1 \) with \( t'_{1,d} = t'_{2,d} = 1/2 \), when the jurisdictions of the two districts have the same preferences for technology, i.e., \( \alpha = 0 \).

**Centralized Provision**

Following recent literature in fiscal federalism (Besley and Coate 2003; Stansel 2005), in a centralized system, the central government chooses different output levels of public safety networks, \( g_1 \) and \( g_2 \), for the two districts. Furthermore, the central government chooses a uniform technology \( t \) for both districts. In reality, even though the central government has options to choose from multiple technology providers for public safety networks, we rarely observe any multi-source procurement decisions being made on a single project, especially when the interoperability is essential to the success of the project under concern.

In a centralized system, the central government simultaneously chooses \( g_1, g_2, \) and \( t \) to maximize total surplus for both districts and its decision problem is shown below:

Maximize \( S(g_1, g_2, t) = S_1(g_1, g_2, t) + S_2(g_1, g_2, t) \) (6)

Subject to: \( g_1 \geq 0, g_2 \geq 0, 0 \leq t \leq 1 \)

where \( S_1(g_1, g_2, t) = x + m_1 \int_0^1 [(1 - \kappa)(\ln g_1 - \delta|t - \theta|) + \kappa(\ln g_2 - \delta|t - \theta|)] f_1(\theta)d\theta - pg_1 \) and

\[
S_2(g_1, g_2, t) = x + m_2 \int_0^1 [(1 - \kappa)(\ln g_2 - \delta|t - \theta|) + \kappa(\ln g_1 - \delta|t - \theta|)] f_2(\theta)d\theta - pg_2
\]

Under centralized provision, the optimal output levels \( (g_{1,c}^*, g_{2,c}^*) \) for the two districts and technology choice \( (t_c^*) \) are:

\[
g_{1,c}^* = \frac{(1 - \kappa)m_1 + \kappa m_2}{p}, \quad g_{2,c}^* = \frac{(1 - \kappa)m_2 + \kappa m_1}{p}, \quad t_c^* = \frac{m_1 + m_2}{m_1 + m_2 - \alpha(m_1 - m_2) + \sqrt{(m_1 + m_2)^2 + \alpha^2(m_2 - m_1)^2}}.
\]

Similar to the decentralized system, the central government provides higher level output of public safety network to the district whose citizens value the output more, i.e., \( g_{1,c}^* \geq g_{2,c}^* \).

**Proposition 2 (Centralized Provision):**

Under centralized provision, the equilibrium output levels \( (g_{1,e}^*, g_{2,e}^*) \) for the two districts and technology choice \( (t_e^*) \) have the following properties:

a. The equilibrium output levels \( (g_{1,e}^* \) and \( g_{2,e}^* \) are independent of the degree of technology heterogeneity \( \alpha \).

b. The equilibrium technology choice \( (t_e^*) \) is independent of the degree of spillovers \( \kappa \).

In contrast to the decentralized system, the central government selects the output levels by simultaneously considering the output preferences of both districts. The equilibrium output levels \( (g_{1,e}^* \) and \( g_{2,e}^* \) are no longer dependent on the degree of technology heterogeneity \( \alpha \). Under centralized provision, one uniform technology is chosen to maximize the interoperability while accommodating citizens’ heterogeneous preferences for output and local jurisdictions’ preferences for technology. Thus, the central government is capable of internalizing the externalities the two districts impose on each other purely through the choice of output levels. Meanwhile, the equilibrium technology choice \( (t_e^*) \) is not affected by the degree of spillovers \( \kappa \). Compared to decentralized system, centralized system reduces the complexity in decision making for the central government. The central government may first select a technology without considering the spillover effect, and then decide the output levels free of concern with technology heterogeneity.
Centralized versus Decentralized Provision

In this section we compare the performance of centralized versus decentralized provisions in terms of the output level of public safety networks. In the context of public safety networks, the output level can be interpreted as the quality of service, which can be measured by the coverage of wireless network signal, spectral efficiency, dependability, advanced capability, security etc. Proposition 3 delineates conditions regarding whether a centralized or decentralized system provides better quality of service for public safety networks.

Proposition 3: Comparing the centralized system and the decentralized system, we find that:

a. For district 1, the decentralized system provides higher output than the centralized system, i.e.,
\[ g^*_{1,c} < g^*_{1,d} \] if and only if \( \alpha > \left( \frac{4m_2}{m_1 + m_2} + \frac{m_1}{2(m_1 - m_2)} - \kappa \right) \); otherwise, \( g^*_{1,c} \geq g^*_{1,d} \).

b. For district 2, the centralized system always provides higher output than the decentralized system, i.e., \( g^*_{2,c} \geq g^*_{2,d} \).

Proposition 3 demonstrates that the comparison of output levels (i.e., the quality of services) under centralized versus decentralized provision critically depends on the degree of spillovers \( \kappa \), the degree of technology heterogeneity \( \alpha \), and the output preferences \( m_1 \) and \( m_2 \) of the two districts. There are two countervailing effects in determining output levels: the spillover effect and the interoperability effect. On the one hand, the local government in a decentralized system only takes into account the benefits received by citizens in its own district and neglects the spillover benefits going to other districts. Thus, the local government might under-provide a public safety network with a lower quality of service due to this spillover effect. On the other hand, a centralized system maximizes the interoperability by selecting a uniform technology at the expense of local technology preferences, while a decentralized system sacrifices interoperability to better satisfy local technology needs. This interoperability effect becomes more salient as the technology heterogeneity level \( \alpha \) increases. When \( \alpha \) is higher than a threshold, the interoperability effect outweighs the spillover effect leading to the centralized system under-providing in district 1 compared to the decentralized system.

Comparing the social surplus under centralized and decentralized provision, we find that the centralized provision yields a higher total surplus, i.e., \( S^*_c \geq S^*_d \), when the degree of spillovers \( \kappa \) is higher than a threshold, or when the degree of technology heterogeneity \( \alpha \) is higher than a threshold.

![Figure 3: Centralized versus Decentralized Provision](image)

Notes: \( S_c < S_d \) in regions 1 and 2 while \( S_c \geq S_d \) in regions 3 and 4; \( g_{1,c} < g_{1,d} \) in regions 1 and 3 while \( g_{1,c} \geq g_{1,d} \) in regions 2 and 4. Figure 3 is based on \( m_1 = 2.2, m_2 = 1, d = 0.1, \) and \( p = 0.1 \). Other parameter values generate qualitatively similar figures.
Figure 3 demonstrates that the comparison of total surpluses under centralized versus decentralized provision critically depends on both the degree of spillovers \( \kappa \) and the degree of technology heterogeneity \( \alpha \). As \( \kappa \) increases from 0 to 1/2, total surpluses under both centralized and decentralized systems decrease. However, the total surplus decreasing rate under centralized provision is smaller than that under decentralized provision. When \( \kappa \) is higher than a threshold, the centralized system becomes superior. As \( \alpha \) increases from 0 to 1, cross-district technology preference heterogeneity decreases while within-district technology preference heterogeneity increases. When \( \alpha \) is higher than a threshold, increased within district heterogeneity outweighs the benefits of the reduced cross-district heterogeneity, resulting in a higher social surplus under centralized provision.

Essentially, this paper addresses a general class of decision-delegation problems and the results can be generalized to diversified contexts other than public safety networks. For example, compatibility is a critical factor when firms make the software adoption and implementation decisions such as a unified ERP installation or a best-local-solution with middleware.

**Concluding Remarks**

**Theoretical Implications**

This paper extends the classic fiscal federalism model (Oates 1972) by introducing one important dimension – technology – to the problem of providing public goods. From the technology perspective, public safety networks are unique compared to other public goods. For most public goods such as highways, available technologies are standard and, thus, local jurisdictions have the same technology preferences. Interoperability is not an issue for such public goods. For public safety networks, however, multiple available technologies exist (e.g., radio frequencies allocated, wireless communication standards, communication devices, etc.) and local jurisdictions’ preferences for these technologies are highly heterogeneous. This paper captures the unique properties of public safety networks in terms of technology preferences and the interoperability of the resulting systems due to technology compatibility.

**Managerial Implications**

This paper provides a comprehensive framework for analyzing the key tradeoffs between centralized and decentralized provision of public safety networks. Findings in this paper shed light upon provision policies regarding public safety networks. The majority of public opinion suggests maximizing interoperability through centralized provision. Our findings identify a potential drawback of this implementation strategy – local technology needs are compromised. This drawback from centralized provision must be weighed against the benefits of improved interoperability in determining provision policies. A decentralized system may provide better quality of services when technology preferences are highly heterogeneous across different districts.
Appendices

Proof of Lemma 1

We first analyze two cases, $t_1 \leq t_2$ and $t_1 \geq t_2$, individually. We then show that the $t_1 \leq t_2$ case is dominated by the $t_1 \geq t_2$ case.

Case 1: $t_1 \leq t_2$

\[
S_1 = m_1 \left[ \int_{t_1}^{t_2} C_1 (1 - \alpha + 2 \theta \alpha) d\theta + \int_{t_1}^{t_2} C_2 (1 - \alpha + 2 \theta \alpha) d\theta + \int_{t_1}^{t_2} C_3 (1 - \alpha + 2 \theta \alpha) d\theta \right] - pg_1,
\]

where

\[
A_1 = x + \left[ 1 - \kappa (1 - t_1 + t_2) \right] \left[ \ln g_1 - (t_1 - \theta) \right] + \kappa (1 - t_1 + t_2) \left[ \ln g_2 - (t_2 - \theta) \right],
\]

\[
A_2 = x + \left[ 1 - \kappa (1 - t_1 + t_2) \right] \left[ \ln g_2 - (t_2 - t_1) \right] + \kappa (1 - t_1 + t_2) \left[ \ln g_3 - (t_2 - \theta) \right],
\]

\[
A_3 = x + \left[ 1 - \kappa (1 - t_1 + t_2) \right] \left[ \ln g_3 - (\theta - t_1) \right] + \kappa (1 - t_1 + t_2) \left[ \ln g_3 - (t_2 - \theta) \right].
\]

\[
S_2 = m_2 \left[ \int_{t_1}^{t_2} D_1 (1 - \alpha - 2 \theta \alpha) d\theta + \int_{t_1}^{t_2} D_2 (1 - \alpha - 2 \theta \alpha) d\theta + \int_{t_1}^{t_2} D_3 (1 - \alpha - 2 \theta \alpha) d\theta \right] - pg_2,
\]

where

\[
B_1 = x + \left[ 1 - \kappa (1 - t_1 + t_2) \right] \left[ \ln g_1 - (t_1 - \theta) \right] + \kappa (1 - t_1 + t_2) \left[ \ln g_2 - (t_2 - \theta) \right],
\]

\[
B_2 = x + \left[ 1 - \kappa (1 - t_1 + t_2) \right] \left[ \ln g_2 - (t_2 - t_1) \right] + \kappa (1 - t_1 + t_2) \left[ \ln g_3 - (t_2 - \theta) \right],
\]

\[
B_3 = x + \left[ 1 - \kappa (1 - t_1 + t_2) \right] \left[ \ln g_3 - (\theta - t_1) \right] + \kappa (1 - t_1 + t_2) \left[ \ln g_3 - (t_2 - \theta) \right].
\]

Since \( \frac{\partial^2 S_1}{\partial y^2} = \frac{m_1}{g_1^2} \left[ 1 - \kappa (1 + t_1 - t_2) \right] < 0 \) and \( \frac{\partial^2 S_2}{\partial y^2} = \frac{m_2}{g_2^2} \left[ 1 - \kappa (1 + t_1 - t_2) \right] < 0 \), we know that \( g_1 = \frac{m_1}{p} \left[ 1 - \kappa (1 + t_1 - t_2) \right] \) and \( g_2 = \frac{m_2}{p} \left[ 1 - \kappa (1 + t_1 - t_2) \right] \). Define \( H = t_1 + t_2 \) and \( L = t_1 - t_2 \). Then \( t_1 = (H + L)/2 \) and \( t_2 = (H - L)/2 \). Substituting \( g_1, g_2, t_1, \) and \( t_2 \) back to \( S_1 \) and \( S_2 \) and computing the second derivatives, we get

\[
\frac{\partial^2 S_1}{\partial H^2} = -\frac{1}{2} \delta m_1 \left[ 1 - \alpha \left[ 1 - H - (1 - \kappa - \kappa L) \right] \right] < 0 \] \] and

\[
\frac{\partial^2 S_2}{\partial H^2} = -\frac{\delta m_2}{2} \left[ 1 - \alpha \left[ H - (1 + L)(1 - 2 \kappa L) \right] \right] < 0 \] \] since \( 0 < \alpha < 1, \) \( 0 \leq \kappa \leq 1/2, \) \( 0 \leq H \leq 1, \) and \(-H \leq L \leq 0.\)

FOC yields that \( H^* = 1 \) and \( -1 \leq L^* = \frac{-1 + 2 \kappa + \sqrt{4 - 4 \kappa}}{\alpha - 4 \kappa} \leq 0.\)

Case 2: $t_1 \geq t_2$

\[
S_1 = m_1 \left[ \int_{t_1}^{t_2} C_1 (1 - \alpha + 2 \theta \alpha) d\theta + \int_{t_1}^{t_2} C_2 (1 - \alpha + 2 \theta \alpha) d\theta + \int_{t_1}^{t_2} C_3 (1 - \alpha + 2 \theta \alpha) d\theta \right] - pg_1,
\]

where

\[
C_1 = x + \left[ 1 - \kappa (1 - t_1 + t_2) \right] \left[ \ln g_1 - (t_1 - \theta) \right] + \kappa (1 - t_1 + t_2) \left[ \ln g_2 - (t_2 - \theta) \right],
\]

\[
C_2 = x + \left[ 1 - \kappa (1 - t_1 + t_2) \right] \left[ \ln g_2 - (t_2 - t_1) \right] + \kappa (1 - t_1 + t_2) \left[ \ln g_3 - (t_2 - \theta) \right],
\]

\[
C_3 = x + \left[ 1 - \kappa (1 - t_1 + t_2) \right] \left[ \ln g_3 - (\theta - t_1) \right] + \kappa (1 - t_1 + t_2) \left[ \ln g_3 - (t_2 - \theta) \right],
\]

\[
S_2 = m_2 \left[ \int_{t_1}^{t_2} D_1 (1 - \alpha - 2 \theta \alpha) d\theta + \int_{t_1}^{t_2} D_2 (1 - \alpha - 2 \theta \alpha) d\theta + \int_{t_1}^{t_2} D_3 (1 - \alpha - 2 \theta \alpha) d\theta \right] - pg_2,
\]

where
$$D_i = x + [1 - \kappa(1 - t_i + t_z)] \ln g_z - \delta(t_z - \theta) + \kappa(1 - t_i + t_z) \ln g_i - \delta(t_i - \theta)$$

$$D_2 = x + [1 - \kappa(1 - t_i + t_z)] \ln g_z - \delta(t_z - \theta) + \kappa(1 - t_i + t_z) \ln g_i - \delta(t_i - \theta)$$

$$D_3 = x + [1 - \kappa(1 - t_i + t_z)] \ln g_z - \delta(t_z - \theta) + \kappa(1 - t_i + t_z) \ln g_i - \delta(t_i - \theta)$$

Since \( \frac{\partial^2 S_1}{\partial g_1^2} = -\frac{m_1 [1 - \kappa(1 - t_i + t_z)]}{g_1^2} \) and \( g_i = \frac{m_1 [1 - \kappa(1 - t_i + t_z)]}{p} \), we know that \( g_i = \frac{m_1 [1 - \kappa(1 - t_i + t_z)]}{p} \). Define \( H = t_i + t_2 \) and \( L = t_i - t_2 \). Then \( t_i = (H + L)/2 \) and \( t_2 = (H - L)/2 \). Substituting \( g_i, g_z, t_i, t_2 \) back to \( S_1 \) and \( S_2 \) and computing the second derivatives, we get:

\[
\frac{\partial^2 S_1}{\partial H^2} = -\frac{\partial m_1}{\partial H} \left[ 1 + \alpha \left( H - (1 - L)(1 + 2\kappa L) \right) \right] < 0
\]

and

\[
\frac{\partial^2 S_2}{\partial H^2} = \frac{1}{2} \delta m_2 \left[ 1 + \alpha \left( 1 - H + L(1 - 2\kappa + 2\kappa L) \right) \right] < 0
\]

since \( 0 \leq \alpha \leq 1 \), \( 0 \leq \kappa \leq 1/2 \), \( 0 \leq H \leq 1 \), and \( 0 \leq L \leq H \).

FOC yields that \( H^* = 1 \) and \( 0 \leq L^* = \frac{-1 + 2\kappa + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2}}{\alpha + 4\kappa} \leq 1 \).

Comparing case 1 and case 2 yields \( S_i(t_1 \geq t_2) - S_i(t_1 \leq t_2) \geq 0 \) and \( S_i(t_1 \geq t_2) - S_i(t_1 \leq t_2) \geq 0 \). Therefore, the \( t_1 \geq t_2 \) case is weakly dominated by the \( t_1 \geq t_2 \) case.

Q.E.D.

**Derivation of the equilibrium results under decentralized provision**

Based on results from Lemma 1, we know that \( H^* = 1 \) and \( L^* = \frac{-1 + 2\kappa + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2}}{\alpha + 4\kappa} \), where \( H = t_i + t_2 \) and \( L = t_i - t_2 \). Substituting \( H^* \) and \( L^* \) into \( t_i = (H + L)/2 \), \( t_2 = (H - L)/2 \), \( g_i = \frac{m_1 [1 - \kappa(1 - t_i + t_z)]}{p} \), and

\[
g_z = \frac{m_z [1 - \kappa(1 - t_i + t_z)]}{p}
\]

yields:

\[
t^*_1 = \frac{-1 + \alpha + 6\kappa + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2}}{2(\alpha + 4\kappa)}
\]

and

\[
t^*_2 = \frac{1}{1 + \alpha + 2\kappa + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2}}
\]

\[
g^*_1 = \frac{m_1 [1 + \alpha + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2}]}{p [1 + \alpha + 2\kappa + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2}]};
\]

and

\[
g^*_2 = \frac{m_2 [1 + \alpha + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2}]}{p [1 + \alpha + 2\kappa + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2}]}.
\]

Q.E.D.
Proof of Proposition 1

Based on the equilibrium under decentralized provision, the resulting interoperability \( o(t_{1,d}^*,t_{2,d}^*) \) can be specified as: \( o(t_{1,d}^*,t_{2,d}^*) = 1 - \frac{1}{t_{1,d}^* - t_{2,d}^*} = 1 - t_{1,d}^* + t_{2,d}^* = \frac{2}{1 + \alpha + 2\kappa + \sqrt{1-4\kappa + (\alpha + 2\kappa)^2}} \).

Since \( \frac{\partial o}{\partial \kappa} = \frac{2(1-\alpha - 2\kappa)}{\sqrt{1-4\kappa + (\alpha + 2\kappa)^2}} \leq 0 \) and \( \frac{\partial o}{\partial \alpha} = \frac{2(1 + \alpha + 2\kappa)}{1 + \alpha + 2\kappa + \sqrt{1-4\kappa + (\alpha + 2\kappa)^2}} \leq 0 \), the interoperability decreases in the degree of spillovers \( \kappa \) and the technology heterogeneity \( \alpha \). Since \( \frac{\partial o}{\partial \alpha} \leq 0 \), \( o(t_{1,d}^*,t_{2,d}^*) = \frac{2}{1 + \alpha + 2\kappa + \sqrt{1-4\kappa + (\alpha + 2\kappa)^2}} \leq o(t_{1,d}^*,t_{2,d}^* | \alpha = 0) = 1 \). The maximum level of interoperability is achieved at \( \alpha = 0 \).

Q.E.D.

Derivation of the optimal results under centralized provision

In a centralized system, the central government maximizes the overall surplus \( S = S_1 + S_2 \), with

\[
S_1 = m_1 \left[ \int_0^1 E_1 (1 - \alpha + 2\theta \alpha) d\theta + \int_0^1 E_2 (1 - \alpha + 2\theta \alpha) d\theta \right] - pg_1,
\]

\[
S_2 = m_2 \left[ \int_0^1 E_1 (1 - \alpha + 2\theta \alpha) d\theta + \int_0^1 E_2 (1 - \alpha + 2\theta \alpha) d\theta \right] - pg_2,
\]

where

\[
E_1 = x + (1 - \kappa) \left[ \ln g_1 - \delta(t - \theta) \right] + \kappa \left[ \ln g_2 - \delta(t - \theta) \right],
\]

\[
E_2 = x + (1 - \kappa) \left[ \ln g_1 - \delta(\theta - t) \right] + \kappa \left[ \ln g_2 - \delta(\theta - t) \right],
\]

\[
F_1 = x + (1 - \kappa) \left[ \ln g_1 - \delta(t - \theta) \right] + \kappa \left[ \ln g_2 - \delta(t - \theta) \right],
\]

\[
F_2 = x + (1 - \kappa) \left[ \ln g_1 - \delta(\theta - t) \right] + \kappa \left[ \ln g_2 - \delta(\theta - t) \right].
\]

Since \( \frac{\partial^2 S}{\partial t^2} = -2\delta \left[ m_1 + m_2 - \alpha (m_1 - m_2)(1-2t) \right] < 0 \), \( \frac{\partial^2 S}{\partial g_1^2} = \frac{(1 - \kappa)m_1 + \kappa m_2}{g_1^2} < 0 \), \( \frac{\partial^2 S}{\partial g_2^2} = \frac{(1 - \kappa)m_1 + \kappa m_2}{g_2^2} < 0 \), and \( \frac{\partial^2 S}{\partial t \partial g_1} = \frac{\partial^2 S}{\partial t \partial g_2} = \frac{\partial^2 S}{\partial g_1 \partial g_2} = 0 \), Hessian matrix is negative definite and the optimal solution is achieved at the following critical point: \( g_{1,c}^* = \frac{(1 - \kappa)m_1 + \kappa m_2}{p} \), \( g_{2,c}^* = \frac{(1 - \kappa)m_2 + \kappa m_1}{p} \), and \( t^* = \frac{m_1 + m_2}{m_1 + m_2 - \alpha (m_1 - m_2) + \sqrt{(m_1 + m_2)^2 + \alpha^2 (m_1 - m_2)^2}} \).

Q.E.D.
**Proof of Proposition 2**

Based on the equilibrium under centralized provision, the equilibrium output levels \((g_{1,c}^*\text{ and } g_{2,c}^*)\) are dependent on \(\kappa, m_1, m_2,\) and \(p,\) and are independent of \(\alpha.\) The equilibrium technology choice \((t_c^*)\) is dependent on \(\alpha, m_1,\) and \(m_2,\) and are independent of the degree of spillovers \(\kappa.\)

Q.E.D.

**Proof of Proposition 3**

For district 1, \(g_{1,c}^* - g_{1,d}^* = \frac{(1-\kappa)m_1 + \kappa m_2}{p} - \frac{m_1 \left[ 1 + \alpha + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2} \right]}{p \left[ 1 + \alpha + 2\kappa + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2} \right]},\) which is negative if and only if \(\alpha > \left( \frac{4m_2}{m_1 + m_2} \right) \left( \frac{m_1}{2(m_1 - m_2)} - \kappa \right).\) Therefore \(g_{1,c}^* < g_{1,d}^*,\) if and only if \(\alpha > \left( \frac{4m_2}{m_1 + m_2} \right) \left( \frac{m_1}{2(m_1 - m_2)} - \kappa \right);\) otherwise, \(g_{1,c}^* \geq g_{1,d}^*.\)

For district 2, \(g_{2,c}^* - g_{2,d}^* = \frac{(1-\kappa)m_2 + \kappa m_1}{p} - \frac{m_2 \left[ 1 + \alpha + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2} \right]}{p \left[ 1 + \alpha + 2\kappa + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2} \right]},\) which is nonnegative if and only if \(2\kappa \left[ (1-\kappa)m_2 + \kappa m_1 \right] + \kappa(m_1 - m_2) \left[ 1 + \alpha + \sqrt{1 - 4\kappa + (\alpha + 2\kappa)^2} \right] \geq 0,\) which is always true.

Q.E.D.
References


